SPICE modeling of meminductor based on its constitutive relation

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Abstract: Behavioral model of meminductor is developed and implemented in SPICE simulation program. This model is related to the flux-or TIF (Time-Integral of Flux) – controlled meminductor, the inductance of which is controlled by the amount of magnetic flux associated with the meminductor. The model uses the meminductor constitutive relation as the only input data. Results of transient analyses clearly demonstrate the basic fingerprints of the meminductor.

Keywords: - meminductor, constitutive relation, SPICE model.

1 Introduction

Fabrication of the solid-state memristive device in HP laboratories [1] which resembles the memristor, hypothetical element introduced to the circuit theory by Leon Chua in 1971 [2], led to a rise of interest in memristive [3], memcapacitive and meminductive [4] systems. The latter two represent a generalization of other still hypothetical but promising memcapacitor and meminductor. Since all the above elements are not currently available as off-the-shelf devices, the role of modeling them increases, particularly with the aim of implementing such models in the current programs for circuit simulation.

Till this time, many SPICE models of memristor [5-11] and several models of mem-capacitor and meminductor [11-13] were proposed. Most of them are based on a simple mathematical description of the physical phenomena in the mem-system, having the form of first-order differential equation. Then the corresponding SPICE models are based on the state-space approach. There are few exceptions from this rule. Mutator concept is used in [14-16]. In [7] and [10], memristors are modeled from their constitutive relations (CR) rather than from their physical models. The CR defines in a compact form and unambiguously a concrete circuit component. This means, for example, that CR is related to this component but not to the way this component interacts with its surroundings. For the memristor, the CR is a flux-charge relation where flux is the time-domain integral of voltage (TIV) whereas charge is the time-domain integral of current (TIC). Similarly, the CR of the mem-capacitor is a relation between TIV and TIQ (time-domain integral of charge), and the CR of the meminductor is a relation between TIC and TIF (time-domain integral of flux). Models of the elements based on their CRs have several advantages in comparison with the conventional physical models. They are independent on the concrete implementation of the element. In addition, these models preserve all the important fingerprints which identify the element. It is important for computer experiments which should verify the theory of the operation of ideal mem-elements.

There are known only two works dealing with memristor modeling via its CR [7, 10] and one paper describing such memcapacitor model [17]. Presently, only one SPICE model of meminductor was published [13] but this model is based on the physical model.

In this paper, a CR-based SPICE model of the meminductor is proposed. The methodology of building the CR-based model of the memcapacitor from [17] is followed. The experiments with its model in PSpice demonstate that the model behavior preserves all the basic fingerprints of the meminductor.

2 Constitutive relation of flux/TIF-controlled meminductor

According to [13], the flux/TIF-controlled meminductor is defined axiomatically by its nonlinear constitutive relation

$$q = \hat{q}(\rho), \tag{1}$$

where q and ρ are the time-domain integrals of electric current i (TIC) and of magnetic flux φ

(TIF), associated with the meminductor. The inverse meminductance Λ_M (Λ_M=1/L_M, L_M is meminductance) in the operating point Q can be derived from the constitutive relation as follows (see Fig. 1):

$$\Lambda_M(\rho) = \left.\frac{d\hat{q}(\rho)}{d\rho}\right|_Q . \qquad (2)$$

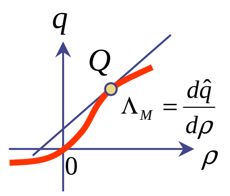

Fig. 1: Example of the constitutive relation of flux/TIF-controlled meminductor.

Note that the CR must hold the following condition which is one of the fingerprints of the meminductor:

$$\hat{q}(\rho) = 0 \text{ for } \rho = 0 . \qquad (3)$$

The current-flux relations of the meminductor can be obtained after differentiating both sides of Eq. (1) with respect to time:

$$\frac{dq}{dt} = i(t) = \frac{d\hat{q}(\rho)}{d\rho}\frac{d\rho}{dt} = \Lambda_M(\rho).\varphi(t) \cdot \qquad (4)$$

Consider the CR (1) in the form of Taylor series

$$q = \hat{q}(\rho) = \sum_{k=1}^{\infty} \lambda_k \rho^k \qquad (5)$$

where λ_k, k=1, 2, … are real coefficients. Note that (5) fulfills the condition (3). Then Eq. (4) describes conventional current-flux relation of the inductor, with the inverse meminductance of the flux/TIF-controlled meminductor being TIF-dependent according to the formulae

$$\Lambda_M(\rho) = \sum_{k=1}^{\infty} k.\lambda_k \rho^{k-1} = \lambda_1 + \sum_{k=2}^{\infty} k.\lambda_k \rho^{k-1} . \qquad (6)$$

Note that when the CR (1) is linear, i.e. when λ_k = 0 for k > 1, then the inverse meminductance is independent of the circuit variables, and the meminductor behaves as a linear inductor. In other words, the memory effect is described by the remaining terms of the Taylor series just for k > 1.

3 PSpice model of flux/TIF-controlled meminductor

The PSpice model of the meminductor with pre-defined CR is based on the following steps:

1] The terminal voltage v of the meminductor is sensed and led to a cascade of two time-domain integrations in order to get the flux (φ) and the TIF (ρ).

2] Based on the knowledge of TIF, the inverse meminductance Λ_M is computed from Eq. (6).

3] Based on the knowledge of Λ_M and φ, the current i of the meminductance can be computed as follows:

$$i(t) = \Lambda_M \varphi(t) = \Lambda_M [\varphi(0) + \int_0^t i(\alpha) d\alpha] \cdot \qquad (7)$$

In PSpice, the latter operation can be accomplished by a controlled current source.

The schematic illustration of the above procedure is shown in Fig. 2 (a).

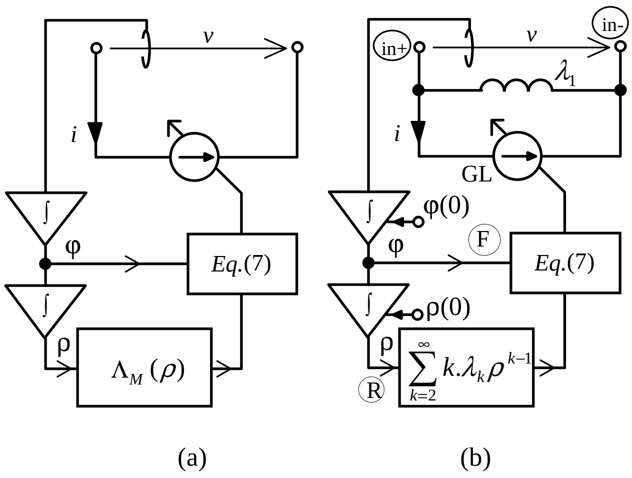

Fig. 2: Proposed models of the meminductor, (a) general, (b) specified for SPICE implementation.

The general model in Fig. 2 (a) is specified in Fig. 2 (b), taking into account the Taylor expansion (5) of the CR. Equation (6) for the inverse meminductance can be interpreted as two inductors in parallel, one with the fixed inverse meminductance λ_1 and the second which is modeled by the sum on the right side of Eq. (6). The second inductor is then modeled via the current controlled source GL. The inductor with fixed inductance connected in parallel to the meminductor terminals prevents any prospective conflicts of ideal sources when driving the meminductor model by external current source.

Figure 2 (b) also indicates the initial states of both integrators, i.e. initial flux and TIF which should be additional input data of the model. The initial flux determines the initial current of the fixed inductor which should be defined in PSpice via IC attribute of the inductor. Combining Equations (6) and (7) yields

$$i(t)=\left[\lambda_1+\sum_{k=2}^{\infty}k.\lambda_k\rho^{k-1}\right]\left[\varphi(0)+\int_0^t i(\alpha)d\alpha\right]= \tag{8}$$

$$=\lambda_1\varphi(0)+\lambda_1\int_0^t i(\alpha)d\alpha+\varphi(t)\sum_{k=2}^{\infty}k.\lambda_k\rho^{k-1}$$

The first two terms on the last row describe the fixed inductor, with the initial current

$$i_1(0)=\lambda_1\varphi(0)\,. \tag{9}$$

The third term describes the variable inductor which is modeled via a current source GL in Fig. 2 (b).

As an example, consider a meminductor defined by the CR

$$q=\lambda_1\rho+\lambda_3\rho^3,\ \lambda_1 = 50\ \mathrm{H}^{-1},\ \lambda_3 = 50\ \mathrm{MA\ V^{-3}s^{-5}}. \tag{10}$$

The inverse meminductance depends on the TIF as follows:

$$\Lambda_M=\frac{dq}{d\rho}=\lambda_1+3\lambda_3\rho^2\,. \tag{11}$$

Note that it is positive for arbitrary values of TIF, thus the necessary condition of the meminductor passivity is fulfilled.

The complete PSpice code of the meminductor subcircuit which corresponds to Fig. 2 (b) is given below:

```
.subckt FC_meminductor in+ in- params: Finit=0 Rinit=0
.param la1 50 la3 50meg
EF F 0 value={Finit+SDT(v(in+,in-))}
ER R 0 value={Rinit+SDT(v(F))}
L1 in+ in- {1/la1} IC={la1*Finit}
GL in+ in- value={3*la3*v(R)^2*v(F)}
.ends FC_meminductor
```

EF- and ER-controlled sources provide time-domain integrations via the PSPICE internal function SDT. The parameters Finit and Rinit represent the initial values of flux and TIF.

The above meminductor was excited by a current source with sinusoidal waveform, with the amplitude of 100 mA and frequency of 1 Hz. The results of PSpice analysis are in Fig. 3.

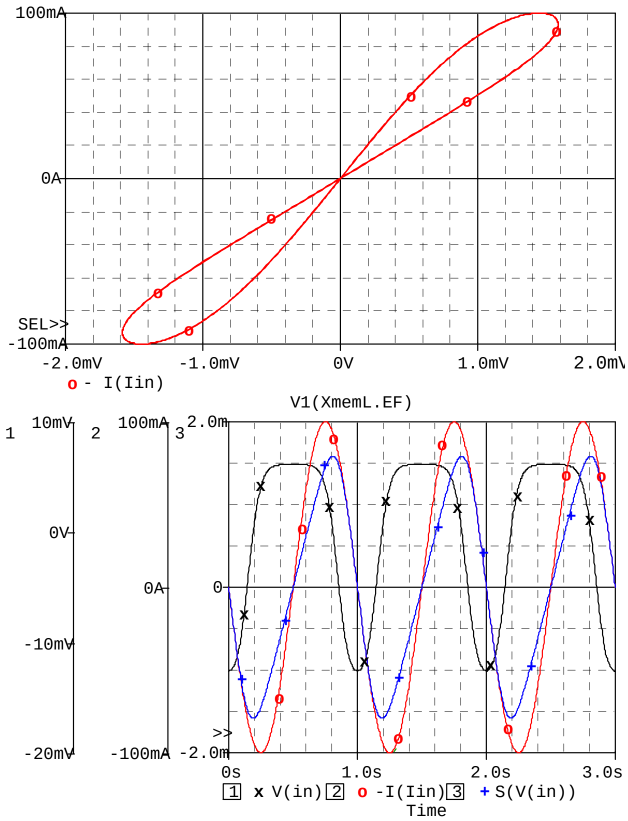

Fig. 3: PSpice analysis of meminductor excited by harmonic 100 mA/1 Hz current source.

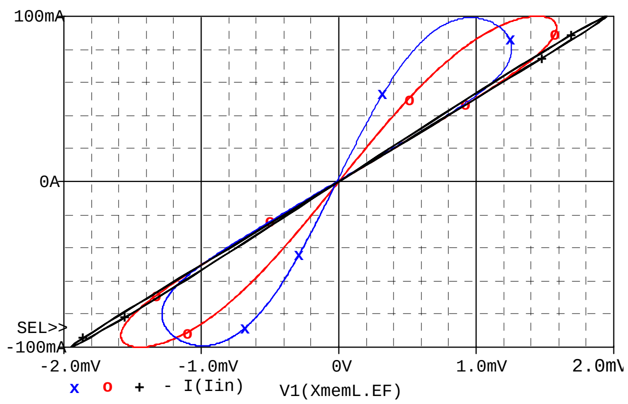

Fig. 4: PSpice analysis of meminductor excited by harmonic 100 mA current source. The memory effect decreases with increasing frequency (x 0.5 Hz, o 1 Hz, + 5 Hz).

As shown in Fig. 3, the current (-I(Iin)) and flux (S(V(in))) waveforms cross zero levels at identical time instants. This is one of the well-known meminductor fingerprints. The upper Figure also demonstrates the hysteretic loop in the flux-current characteristic. This loop is suppressed with increased frequency of applied signal as obvious from Fig. 4.

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4 Conclusion

A method described in the paper enables PSpice modeling of general flux/TIF-controlled meminductor which is defined by its flux-current constitutive relation. The mathematical description of the inverse meminductance, which is a nonlinear function of time-domain integral of flux (TIF), is necessary for such a modeling. Analogical procedure can be applied to modeling of current/charge-controlled meminductor.

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