Generation of Voltage-Mode OTRA-Based Multifunction Biquad Filter
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Abstract: With the dual input-and-output characteristics of the well-known operational transconductance amplifier (OTA), the operational transresistance amplifier (OTRA) is attractive to be used in circuit design. Applying a new analytical synthesis method, a low-pass, band-pass, and high-pass biquad filter without capacitors in feedback loops using three OTRAs, two capacitors, and several resistors is presented. Based upon component sensitivity tendency and variation amount, just properly adjusting one or two resistances by a small difference, or giving approximate component values for achieving precise output responses is investigated and developed in this paper. Null group sensitivities are also simulated and validated in this paper.

Key Words: Active filters, analog circuit design, continuous-time filters, low-pass filters, band-pass filters, high-pass filters, operational transresistance amplifiers

1. Introduction
An operational transconductance amplifier (OTA) [1], simultaneously representing both an active element and an electronically tunable conductor, becomes one of the most prospective active elements in the field of analogue circuit design. Its input-and-output characteristics are (i) the two + and – terminal input currents, I+ = I− = 0, (ii) the output current, Io = (V+ − V−)gm in which gm is the transconductance of an OTA, and V+ and V− are the two + and − terminal input voltages, (iii) very high input impedance, and (iv) very high output impedance. On the other hand, another active element called operational transresistance amplifier (OTRA) [2-6] has the following input-and-output relationships: (i) the two + and − terminal input voltages, V+ = V− = 0, (ii) the output voltage, Vo = (I+ − I−)Rm in which Rm is the transresistance of an OTRA, and I+ and I− are the two + and − terminal input currents, (iii) very low input impedance, and (iv) very low output impedance. Based upon the above two sets of input-and-output characteristics, it is apparent that an OTRA is the dual of an OTA and vice versa. In addition to this, the main advantage of eliminating the parasitics at the input port of an OTRA due to virtually grounded input terminals is an extra benefit and very attractive for designers. Note that the OTAs cannot enjoy this benefit. Therefore, we may make the predictions as follows.

(i) OTRA-based circuit structures may be the dual of OTA-based circuit structures.
(ii) OTRA-based circuits may have better performance through some particular design methods than OTA-based circuits.

Hence, several distinct OTRAs have been proposed in the literature since 1992 [2-6]. So do many different kinds of analog circuits, such as integrators [7], immittance simulators [8], oscillators [9-12], square or triangular waveform generators [13-14], monostable [15-16] and bistable [17] multivibrators, first- [12, 18-20] and second-order [21] all-pass filters, and other second-order single- [22, 23] and multi-function [4, 24] and high-order [25] filters. A low-pass Tow-Thomas biquad filter was presented [22] using two pseudo-differential OTRAs, four capacitors, and several resistors. Four single-function biquad filters were also proposed [23] using a single OTRA, two or three (one more) capacitors, in addition to some resistors. A universal biquad filter [24] employing four (two more) capacitors was presented. An OTRA-based low-pass, band-pass, and high-pass biquad filter using only two capacitors, was also published [4]. Since the two capacitors were synthesized and put in feedback loops, the non-ideal analysis exhibits that the ideal sC has been replaced by practical sC + Gm where Gm = 1/Rm, and Rm is the trans-resistance function of an OTRA. This arrangement produced additional poles in the transfer function. Note that this main disadvantage has been improved in this paper.
2. Analytical Synthesis

Give a generic voltage-mode second-order high-pass filter transfer function as below.

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{b_2 s}{a_2 s^2 + a_1 s + a_0}
\]

(1)

Cross multiplying (1), dividing it by \(a_2 s^2\), yield

\[
V_{\text{out}} \left(1 + \frac{a_1}{a_2 s} + \frac{a_0}{a_2 s^2}\right) = \frac{b_2}{a_2} V_{\text{in}}
\]

(2)

Since \(\frac{a_0}{a_2} = \left(\frac{a_0}{a_1}\right) \frac{a_1}{a_2}\)

(3)

(2) becomes

\[
V_{\text{out}} \left(1 + \frac{a_1}{a_2 s} + \left(\frac{a_0}{a_1}\right) \frac{a_1}{a_2 s}\right) = \frac{b_2}{a_2} V_{\text{in}}
\]

(4)

namely,

\[
V_{\text{out}}(1) + V_{\text{out}} \left(\frac{a_1}{a_2 s}\right) + V_{\text{out}} \left(\frac{a_0}{a_1}\right) \frac{a_1}{a_2 s} = \frac{b_2}{a_2} V_{\text{in}}
\]

(5)

which is equivalent to

\[
V_{\text{out}}(1) - V_{\text{out}} \left(-\frac{a_1}{a_2 s}\right) + V_{\text{out}} \left(-\frac{a_0}{a_1}\right) \frac{a_1}{a_2 s} = \frac{b_2}{a_2} V_{\text{in}}
\]

(6)

We may let

\[
V_1 = V_{\text{out}} \left(-\frac{a_1}{a_2 s}\right) = V_{\text{out}} \left(-\frac{a_1/a_2}{s}\right)
\]

(7)

(6) is then simplified as

\[
V_{\text{out}}(1) - V_1(1) + V_1 \left(-\frac{a_0}{a_1 s}\right) = \frac{b_2}{a_2} V_{\text{in}}
\]

(8)

And let \(V_2 = V_1 \left(-\frac{a_0}{a_1 s}\right) = V_1 \left(-\frac{a_0/a_1}{s}\right)\)

(9)

(8) becomes

\[
V_{\text{out}}(1) - V_1(1) + V_2(1) = \frac{b_2}{a_2} V_{\text{in}}
\]

(10)

Each of Eqs. (7), (9), and (10) may be realized using one OTRA, several resistors, and one or without capacitor(s). The combination of these three sub-circuitries is shown in Fig. 1.

From (7) and (9), we obtain

\[
\frac{V_1}{V_{\text{in}}} = -\left(\frac{a_1 b_2}{a_2} / a_2\right) s
\]

(11)

\[
= \frac{a_2 s^2 + a_1 s + a_0}{a_2 s^2 + a_1 s + a_0}
\]

}\[
= \left(\frac{a_0 b_2}{a_2}\right)
\]

\[
\frac{a_2 s^2 + a_1 s + a_0}{a_2 s^2 + a_1 s + a_0}
\]

(12)

Hence, Fig. 1 is a band-pass (from output voltage \(V_1\)) and low-pass (from output voltage \(V_2\)) in addition to a high-pass filter.

Since the two input terminals of an OTRA are virtually grounded, the resistor shown in Fig. 1 may be replaced by two parallel NMOS transistors \(M_1\) and \(M_2\), shown in Fig. 2, both of which are matched and operating in ohmic region. The current passing through an NMOS transistor is given by

\[
I = K_n (V_{\text{gs}} - V_T)(V_D - V_S) + \sum a_i V_D^I - V_S^I
\]

(13)

The transistors \(M_1\) and \(M_2\) have the same drain and source voltages leading to the cancellation [4] of the odd and even nonlinearities shown in (13). The current difference shown in Fig. 2 is

\[
I_1 - I_2 = \mu_n C_m \frac{W}{L} (V_1 - V_2)
\]

(14)
NMOS transistor circuit.
Applying the replacement of a resistor with two parallel NMOS transistors, Fig. 1 may be transformed to another second-order OTRA-MOS-C low-pass, band-pass, and high-pass filter shown in Fig. 3.

\[ S_{G_i}^{1H} = \frac{G_5 G_2 G_5}{\Delta_D}, \quad S_{G_i}^{H} = 1 \]  

(18)

Thus, we obtain that

\[ S_{G_i}^{H} + S_{G_i}^{H} = 0, \quad S_{G_i}^{H} + S_{G_i}^{H} = 0, \]

\[ S_{G_i}^{H} + S_{G_i}^{H} + S_{G_i}^{H} = 0, \]

\[ S_{i}^{H} + S_{i}^{H} + S_{i}^{H} = 0 \]  

(19)

There are four null group sensitivities in (19). It shows that if the components in a null group have the same sensitivity tendency (increment or decrement) and identical variation rate, the group sensitivity equals null. This result may offer a prospective research work for eliminating output deviations due to component variations in an integrated circuit.

### 4. Non-ideal Analysis

Considering the non-ideal input-and-output characteristic of an OTRA in a circuit structure, that is, \( V_o = (1\pm 1)R_m \), the non-ideal transfer function of the second-order LP, BP, and HP filter shown in Fig. 1 is presented below in which the non-ideal numerator and denominator are

\[ \Delta_N(HP) = s^2 C_1 C_2 G_1 \]

\[ \Delta_N(BP) = s C_2 G_6 (G_4 + G_{m3}) \]

\[ \Delta_N(LP) = G_6 (G_4 + G_{m3}) (G_5 + G_{m2}) \]

\[ \Delta_D = s^2 C_1 C_2 G_1 + s C_2 G_2 (G_4 + G_{m3}) + (G_3 + G_{m1}) (G_4 + G_{m3}) (G_5 + G_{m2}) \]  

(20)

Comparing the above three non-ideal results, we may say that the most accurate numerator is the high-pass one, the worst numerator is the low-pass one, and the band-pass one has the medium accuracy in numerator. However, the deviations of the denominators for three different cases are the same. Note that the total accuracy of whole transfer function from theoretical point of view depends on the deviation percentages of both the numerator and the denominator. If both the deviation percentages are identical, then the whole transfer function has perfect output responses without any errors. A comparison with the previous OTRA-based low-pass, band-pass, and high-pass biquad filter (Fig. 11 in [4]) is essential. The non-ideal numerator and denominator are

\[ \Delta_N(HP) = (s C_1 + G_{m2}) (s C_2 + G_{m3}) G_1 \]

\[ \Delta_N(BP) = (s C_2 + G_{m3}) G_2 G_1 \]

\[ \Delta_N(LP) = G_1 G_2 G_3 \]
(21) has more serious non-ideal effect than (20) due to the following main reason: “in (21), sC involves in G; since sC and G are two different kinds of elements, it leads to the distortion of output signals. However, there are no such involvements in (20).”

5. H-Spice Simulations

To verify theoretical predictions, the second-order high-pass, band-pass, and low-pass filter shown in Fig. 1 is simulated using the H-Spice with 0.35μm process (+1.65V supply voltages), and the CMOS Implementations, shown in Fig. 4, of the OTRA (with V_B_f=0.97V) presented in [5]. The W/L of the MOSFET in the OTRA [5] is shown in Table I.

Table I: W/L of the MOSFET in the OTRA [5]

<table>
<thead>
<tr>
<th>MOSFET</th>
<th>W/ L (μm)/L (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1-M3, M12-M13</td>
<td>70/1.75</td>
</tr>
<tr>
<td>M4, M7</td>
<td>7/1.75</td>
</tr>
<tr>
<td>M5-M6</td>
<td>21/1.75</td>
</tr>
<tr>
<td>M8-M11</td>
<td>35/1.75</td>
</tr>
<tr>
<td>M14</td>
<td>35/0.35</td>
</tr>
</tbody>
</table>

The component values are given by C_1=C_2=C_3= 10pF (for 100kHz) or 1pF (for 1MHz), and R_1=R_2 =R_3=R_6=100kΩ, R_4=112.54kΩ, R_5= 225.08kΩ. The simulated amplitude-frequency responses are respectively shown in Figs. 5, 6, and 7 having the simulated f_{3db}=100.84kHz with peak 0.9869 (LP), 102.33kHz with peak 0.9868 (BP), and 102.82kHz with peak 0.9868 (HP) when the theoretical operating f_{3db}=100kHz with peak 1.0000, and f_{3db}=0.9588MHz with peak 0.9869 (LP), 1.0186 MHz with peak 0.9472 (BP), and 1.0729MHz with peak 0.9472 (HP) when the theoretical operating f_{3db}=1MHz with peak 1.0000.

We notice that the error, 2.816%, of the f_{3db} of the high-pass response at 100kHz is a little bit higher. This deviation is mainly due to the approximation design rule: approaching all trans-resistances to infinity. It means that the deviation is inevitable under the rule. Therefore, how to improve the output accuracy is an important matter for OTRA-based circuits. And since precise component values lead to the deviations of output signals, a set of proper variations of component values may make a more accurate output signal.

Table I shows the variation percentage of Δf_{3dB} and ΔV_{peak} when each component value is added by 1%. Based upon the sensitivity tendency and variation amount shown in Table I, we choose R_4 to increase by 2.816/0.9834=2.863%, that is, replacing R_4 by 115.76k Ω from 112.54k Ω. Thus, the output parameters, f_{3db} and V_{peak} of the high-pass filter are much improved and shown in Table II. If we just tune R_4 by only 116 k Ω much simpler than the precise value 115.76kΩ for eliminating the difficulty of fabricating such a precise resistor in an integrated circuit, the simulated results are also shown in Table II. As can be seen, the approximation value still produces very precise output signals.

Table I Variation percentage of Δf_{3dB} and ΔV_{peak} when the component value is added by 1% (for HP)

<table>
<thead>
<tr>
<th>Comp.</th>
<th>Δf_{3dB} (%)</th>
<th>ΔV_{peak} (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_1 (+1%)</td>
<td>-0.0094</td>
<td>0.0008</td>
</tr>
<tr>
<td>R_2 (+1%)</td>
<td>-0.9790</td>
<td>0.9876</td>
</tr>
<tr>
<td>R_3 (+1%)</td>
<td>-0.0024</td>
<td>0.0101</td>
</tr>
<tr>
<td>R_4 (+1%)</td>
<td>-0.9834</td>
<td>-0.0014</td>
</tr>
<tr>
<td>R_5 (+1%)</td>
<td>0.0017</td>
<td>0.0060</td>
</tr>
<tr>
<td>R_6 (+1%)</td>
<td>1.0218</td>
<td>-0.9901</td>
</tr>
</tbody>
</table>

Table II Improved output parameters after adding R_4 by 2.863%

<table>
<thead>
<tr>
<th>Filter</th>
<th>f_{3dB} and V_{peak} (R_4=115.76 kΩ)</th>
<th>f_{3dB} and V_{peak} (R_4=116 kΩ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-Pass</td>
<td>100.85 (0.85%) 0.9869</td>
<td>100.85 (0.85%) 0.9869</td>
</tr>
<tr>
<td>Band-Pass</td>
<td>100.93(0.93%) 0.9867</td>
<td>100.69(0.69%) 0.9867</td>
</tr>
<tr>
<td>High-Pass</td>
<td>99.986(0.014%) 0.9867</td>
<td>99.785(0.215%) 0.9867</td>
</tr>
</tbody>
</table>
If we feel that the error of the above low-pass response is not good enough, we may do component-value tuning once again. Similar to Table I, we obtain the variation percentage of \( \Delta f_{3dB} \) and \( \Delta V_{\text{peak}} \) when each component value is added by 1\% for the low-pass filter and shown in Table III. Based upon the sensitivity tendency and variation amount shown in Table III, we choose \( R_5 \) to increase by \( 0.8507/0.9823=0.8661\% \), that is, replacing \( R_5 \) by \( 227.03k \Omega \) from \( 225.08k \Omega \). Thus, the output parameters, \( f_{3dB} \) and \( V_{\text{peak}} \), of the low-pass filter are much improved and shown in Table IV. If we just tune \( R_5 \) by only \( 227k \Omega \) much simpler than the precise value \( 227.03k \Omega \) for eliminating the difficulty of fabricating such a precise resistor in an integrated circuit, the simulated results are also shown in Table IV. As can be seen, the approximation value still produces very precise output signals.

Some group sensitivities exhibit the cancellation of component sensitivities in a group. Figs. 8 to 10 show these null group sensitivities for the three groups, (i) \( C_1, C_4, G_1, G_2, G_3, \) and \( G_6 \), (ii) \( C_5 \) and \( G_4 \), and (iii) \( C_3 \) and \( G_5 \), respectively. In this regard, the fabrication of integrated circuits may be oriented to make the group components have identical variation tendency and amount such that the defect due to component variations may be eliminated.

### Table III Variation percentage of \( \Delta f_{3dB} \) and \( \Delta V_{\text{peak}} \) when the component value is added by 1\% (for LP)

<table>
<thead>
<tr>
<th>Com.</th>
<th>( \Delta f_{3dB}(%) )</th>
<th>( \Delta V_{\text{peak}}(%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1(+1%) )</td>
<td>0.0156</td>
<td>0.0005</td>
</tr>
<tr>
<td>( R_2(+1%) )</td>
<td>0.9930</td>
<td>0.0018</td>
</tr>
<tr>
<td>( R_3(+1%) )</td>
<td>0.0220</td>
<td>0.9969</td>
</tr>
<tr>
<td>( R_4(+1%) )</td>
<td>0.0101</td>
<td>0.0007</td>
</tr>
<tr>
<td>( R_5(+1%) )</td>
<td>-0.9823</td>
<td>-0.0000</td>
</tr>
<tr>
<td>( R_6(+1%) )</td>
<td>-1.0270</td>
<td>-0.9898</td>
</tr>
</tbody>
</table>

### Table IV Improved output parameters after adding \( R_5 \) by 2.863\%

<table>
<thead>
<tr>
<th>Filter</th>
<th>( f_{3dB} ) and ( V_{\text{peak}} ) (( R_5=227.03k \Omega ))</th>
<th>( f_{3dB} ) and ( V_{\text{peak}} ) (( R_5=227k \Omega ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-Pass</td>
<td>100.01 (0.01%) 0.9869</td>
<td>100.02 (0.02%) 0.9869</td>
</tr>
<tr>
<td>Band-Pass</td>
<td>100.23 (0.23%) 0.9868</td>
<td>100.23 (0.23%) 0.9868</td>
</tr>
<tr>
<td>High-Pass</td>
<td>99.773 (0.227%) 0.9868</td>
<td>99.774 (0.226%) 0.9868</td>
</tr>
</tbody>
</table>
Fig. 8 Null group sensitivity curves (dashed line, C₁ with 1% tolerance; dotted line, G₄ with 1% tolerance, real line, nominal; circle line, C₁+G₄ with 1% tolerance)

Fig. 9 Null group sensitivity curves (dashed line, C₂ with 1% tolerance; dotted line, G₅ with 1% tolerance, real line, nominal; circle line, C₂+G₅ with 1% tolerance)

Fig. 10 Null group sensitivity curves (dashed line, G₆ with 1% tolerance; dotted line, G₂ with 1% tolerance, real line, nominal or G₁ with 1% tolerance or G₃ with 1% tolerance; circle line, all with 1% tolerance)

6. Conclusions

A new OTRA-based low-pass, band-pass, and high-pass biquad filter is presented without a capacitor in feedback loops using a new analytical synthesis method. The difference between finite tranresistance and infinite transresistance makes output signals with rather errors. Based upon sensitivity tendency and variation amount, appropriately varying some resistor may lead to very precise output responses. H-Spice simulations validate null group sensitivities in addition to filter feasibility and low component sensitivities.

References


