Adaptive Control for Uncertain Systems with Unknown Disturbance

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Abstract: In this paper, we present a new scheme to design adaptive controllers for both linear and nonlinear uncertain systems in the presence of unknown disturbances. The control design is achieved by introducing certain well defined functions and a new linkage term in the adaptive update law. For the design and implementation of the controller, no knowledge is assumed on the unknown system parameters and disturbances. It is shown that the proposed controller can guarantee global uniform ultimate boundedness. A bound for the truncated $L_2$ norm of the tracking error is obtained as a function of design parameters.

Key–Words: Adaptive control, backstepping, nonlinear systems, disturbance, tuning functions

Adaptive control has seen significant development since the appearance of a Lyapunov-based recursive design procedure known as backstepping [1]. A great deal of attention has been paid to tackle both linear and nonlinear systems. A number of results have been obtained to control systems without disturbance, see for example [2, 3, 4, 5, 6, 7]. For the control of systems in the presence of disturbances, it received a lot attention in [8, 9, 10, 11, 12, 13, 14]. In [11], robust adaptive control for nonlinear systems with bounded disturbance was studied by using parameter estimation projection, where the unknown parameters must be inside a known set and the bound of the disturbance is also known. In [12], by using transfer function method based on the VSS theory, a model reference robust control scheme was proposed for systems with disturbances. The effect of the disturbance is bounded by a known function. In [13], the problem of output regulation of switched linear system with known parameters and disturbance was studied, where the disturbance is generated by a known differential equation. In [14], a flat zone was used to handle the problem of controlling nonlinear time invariant systems in the presence of disturbances, where the disturbance is generated by an exosystem. Clearly, these existing schemes have some restrictions on the disturbance involving known bounds of disturbance or the unknown parameters.

In this paper, we develop a new output feedback control design scheme for systems with unknown external disturbance and system parameters. To handle the disturbances, well defined functions are introduced to eliminate their effects in the Lyapunov functions employed in the recursive design steps. In the estimator, a new linkage term is added to handle the parametric uncertainty and the effects of unknown disturbance, in order to enhance the robustness of the adaptive controller with respect to the disturbance. When generalizing the design idea to nonlinear systems, the disturbance considered is multiplied by a function of system output. The effect of this term cannot be assumed bounded and this will make the design and analysis complicated since such a term appears in the state estimation error when observers are used for state estimation and Lyapunov functions during recursive design. To compensate such effects, we design a new $\rho$th order differentiable function and employ it in the recursive backstepping technique. Owing to these new terms and functions, a priori knowledge on system parameters and disturbance is no longer needed. The term multiplying the control and other system parameters are not assumed to be within known intervals. A known bound of the disturbance is also not required. It is shown that the proposed adaptive controller can guarantee all the signals in the closed-loop adaptive system globally uniformly ultimately bounded.
bound for the tracking error is obtained as a function of design parameters, which implies that such a bound can be adjusted by choosing the design parameters.

1 Problem Formulation

The class of single-input single-output linear systems with external unknown disturbance is given by

\[
(p^n + a_{n-1}p^{n-1} + \ldots + a_1 p + a_0)y(t) = (b_m p^m + \ldots + b_1 p + b_0)u(t) + (g_{n-1} p^{n-1} + \ldots + g_1 p + g_0)d(t) \tag{1}
\]

where \( p = \frac{d}{dt} \), the coefficients \( a_i, g_i \) and \( b_j \) \((i = 0, 1, \ldots, n - 1; j = 0, 1, \ldots, m) \) are unknown constants; \( u \) and \( y \) are system input and output; \( d \) is the external disturbance.

The control objective is to reject the disturbance and to force the system output to track a given reference signal \( y_r(t) \) as close as possible. To this end, the following assumptions are made about the system.

**Assumption 1.** The system is minimum phase, i.e. all zeros of the polynomial \( b_n p^m + \ldots + b_1 p + b_0 \) are stable.

**Assumption 2.** The relative degree \( \rho = n - m \) is known.

**Assumption 3.** The reference signal \( y_r \) and its \((\rho - 1)\)th order derivatives are also assumed to be known and bounded.

**Assumption 4.** The sign of high-frequency gain \((sgn(b_m))\) is known.

**Remark 1** Compared with [11] - [13] neither the uncertain parameters \( a_i \) and \( b_i \) are assumed inside known intervals, nor a bound or a function to bound the effect of the disturbance is available. The lack of such priori knowledge makes the controller design more difficult.

In order to design state estimation filters to observe system states, we now represent the plant (1) as in the observer canonical form

\[
\dot{x} = Ax - ya + \begin{bmatrix} 0_{(p-1)\times 1} \\ b \end{bmatrix} u + D(t) \tag{2}
\]

\[
y = e_1^T \dot{x}
\]

where

\[
A = \begin{bmatrix} 0 & I_{n-1} \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}, \quad a = \begin{bmatrix} a_{n-1} \\ \vdots \\ a_0 \end{bmatrix},
\]

\[
b = \begin{bmatrix} b_m \\ \vdots \\ b_0 \end{bmatrix}, \quad D(t) = \begin{bmatrix} g_{n-1} d(t) \\ \vdots \\ g_0 d(t) \end{bmatrix}.
\]

In order to proceed, we rewrite (2) as

\[
\dot{x} = Ax + F(y, u)^T \theta + D(t) \tag{3}
\]

\[
y = e_1^T x
\]

\[
F(y, u)^T = \begin{bmatrix} 0_{(p-1)\times (m+1)} \\ I_{m+1} \end{bmatrix} u, \quad I_n y \tag{4}
\]

and the \( p = n + m + 1 \) - dimensional parameter vector \( \theta = [b, a]^T \). For state estimation, by following the standard procedures as in [1], we can obtain

\[
\dot{\lambda} = A_0 \lambda + e_n u \tag{5}
\]

\[
\dot{\eta} = A_0 \eta + e_n y \tag{6}
\]

\[
\Omega^T = [v_m, \ldots, v_1, v_0, \Xi] \tag{7}
\]

\[
v_j = A_0^j \lambda, \quad j = 0, \ldots, m \tag{8}
\]

\[
\Xi = -[A_0^{n-1} \eta, \ldots, A_0 \eta, \eta] \tag{9}
\]

\[
\xi = -A_0^j \eta \tag{10}
\]

where the vector \( k = [k_1, \ldots, k_n]^T \) is chosen so that the matrix \( A_o = A - ke_1^T \) is Hurwitz. Hence there exists a \( P \) such that \( PA_o + A_o P^T = -2I, \ P = P^T > 0 \). With these designed filters our state estimate is \( \hat{x} = \xi + \Omega^T \theta \), and the estimation error \( \epsilon = x \dot{\hat{x}} \) satisfies

\[
\dot{\epsilon} = A_0 \epsilon + D(t) \tag{11}
\]

Let \( V_\epsilon = \epsilon^T P \epsilon \). It can be shown that

\[
\dot{V}_\epsilon \leq -\epsilon^T \epsilon + \| PD(t) \|^2 \tag{12}
\]

Then system (1) can be expressed as

\[
\dot{y} = b_m v_{m,2} + \xi_2 + \bar{\omega}^T \theta + \epsilon_2 + D_1(t) \tag{13}
\]

\[
\dot{v}_{m,i} = v_{m,i+1} - k_i v_{m,1}, \quad i = 2, \ldots, \rho - 1 \tag{14}
\]

\[
\dot{v}_{m,\rho} = v_{m,\rho+1} - k_\rho v_{m,1} + u \tag{15}
\]

\[
\omega = [v_{m,2}, v_{m,1}, \ldots, v_0, \Xi(2) - y e_1^T]^T \tag{16}
\]

\[
\bar{\omega} = [0, v_{m,1}, \ldots, v_0, \Xi(2) - y e_1^T]^T \tag{17}
\]

and \( v_{1,2}, \epsilon_2, \xi_2 \) denote the second entries of \( v_1, \epsilon, \xi \) respectively. \( D_1(t) \) is the first variable of \( D(t) \). The problem of this paper is to design an adaptive controller to make system (1) BIBO stable.

2 Control Design

In this section, we present the adaptive control design using the backstepping technique with tuning functions in \( \rho \) steps. As usual in backstepping approach, the following change of coordinates is made.

\[
z_1 = y - y_r \tag{18}
\]

\[
z_i = v_{m,i} - \hat{y}_r(i-1) - \alpha_i, \quad i = 2, \ldots, \rho \tag{19}
\]
where \( \hat{e} \) is an estimate of \( e = 1/b_m \) and \( \alpha_{i-1} \) is the virtual control at each step and will be determined in later discussions. We now illustrate the backstepping design procedures with details given for the first step. 

**Step. I** We design the virtual control law \( \alpha_1 \) and parameter updating law

\[
\alpha_1 = \hat{e} \alpha_1 \\
\dot{\alpha}_1 = -(c_1 + l_1 + \frac{1}{4})z_1 - \xi_2 - \omega^T \hat{\theta} \\
\dot{\hat{e}} = -\gamma \text{sgn}(b_m)(\hat{y}_r + \hat{\alpha}_1)z_1 - \gamma l_e(\hat{e} - e_0) \\
\tau_1 = (\omega - \hat{e}(\hat{y}_r + \alpha_1)e_1)z_1
\]

where \( c_1 \) and \( l_1 \) are two positive real design parameters, where \( l_e, e^* \) are positive design constants, \( \gamma \) is a positive design parameter, \( \hat{e} \) denotes the estimate of \( e = 1/b_m \). The following useful property can be obtained:

\[
l_e \hat{e}(\hat{e} - e_0) \leq -\frac{1}{2}l_e e^2 + \frac{1}{2}l_e(\hat{e} - e_0)^2
\]

To proceed, we define the Lyapunov function

\[
V_1 = \frac{1}{2}z_1^2 + \frac{1}{2} \omega^T \Gamma^{-1} \hat{\theta} + \frac{b_m}{2\gamma} e^2 + \frac{1}{2l_1} V_e
\]

where \( \Gamma \) is a positive definite design matrix and \( \hat{\theta} \) denotes the estimate of \( \theta \). Then the derivative of \( V_1 \) along with (13), (18), (20), and (22), is given by

\[
\dot{V}_1 \leq -c_1 z_1^2 + b_m z_1 z_2 + \frac{1}{4}z_2^2 - \frac{b_m}{2} l_e e^2 \\
+ \frac{b_m}{2} l_e(e - e_0)^2 + \hat{\theta}^T (\tau_1 - \Gamma^{-1} \hat{\theta}) \\
- \frac{1}{2 l_1} \| e \|^2 + \frac{1}{2 l_1} \| PD(t) \|^2 + D_1(t)^2
\]

**Step. i (i = 2, ..., \rho).** Define

\[
V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2l_i} V_e \\
\alpha_i = -c_i z_i - l_i \| \frac{\partial \alpha_{i-1}}{\partial y} \|^2 z_i - l_{i-1} + \beta_i - \frac{z_i}{4} \\
+ \frac{\partial \alpha_{i-1}}{\partial \theta} \Gamma \tau_i - \left( \sum_{k=2}^{i-1} z_k \frac{\partial \alpha_{k-1}}{\partial \theta} \right) \frac{\partial \alpha_{i-1}}{\partial y} \omega \\
+ \frac{\partial \alpha_{i-1}}{\partial \theta} \Gamma \tau_i (\hat{\theta} - \theta^*) \\
\tau_i = \tau_{i-1} - \frac{\partial \alpha_{i-1}}{\partial y} \omega z_i
\]

where \( \beta_i \) denotes some known terms and its detailed can be found in [1]. Then the actual adaptive controller is obtained and given by

\[
u(t) = \alpha_p - v_{m,p+1} + \hat{e} y_r \\
\dot{\hat{\theta}} = \Gamma \tau_p + \Gamma \dot{\tau}_p (\hat{\theta} - \theta^*)
\]

The final Lyapunov function \( V_\rho \) satisfies

\[
\dot{V}_\rho \leq -\sum_{k=1}^{\rho} \frac{c_k z_k^2}{2} - \sum_{i=1}^{\rho} \frac{1}{2l_i} \| e \|^2 - \frac{1}{2} \| \hat{\theta} \|^2 \\
- \frac{b_m}{2} l_e e^2 + M_\rho
\]

where

\[
M_\rho = \frac{1}{2}\| \theta - \theta^* \|^2 + \frac{b_m}{2} l_e(e - e_0)^2 \\
+ \sum_{i=1}^{\rho} \frac{1}{2l_i} \| PD(t) \|^2 + \rho \| D_1(t) \|^2
\]

Notice that

\[
-\sum_{k=1}^{\rho} \frac{c_k z_k^2}{2} - \frac{b_m}{2} l_e e^2 - \sum_{i=1}^{\rho} \frac{1}{2l_i} \| e \|^2 - \frac{1}{2} \| \hat{\theta} \|^2 \\
\leq -f_- \dot{V}_\rho
\]

and

\[
V_\rho = \sum_{i=1}^{\rho} z_i^2 + \frac{1}{2} \hat{\theta} \hat{\theta} + \hat{e}^2 + \sum_{i=1}^{\rho} \epsilon \epsilon \epsilon \\
+ \max \left\{ \frac{1}{2}, \frac{1}{2\gamma}, \frac{1}{2l_1} \lambda_{\max}(\Gamma), \frac{b_m}{2\gamma}, \frac{1}{2l_1} \lambda_{\max}(P) \right\}
\]

\[
f_+ = \min \{ c_i, \frac{b_m l_e}{2l_i}, \frac{1}{2l_i} \}
\]

where \( \lambda_{\max}(P) \) and \( \lambda_{\max}(\Gamma) \) are the maximum eigenvalues of \( P \) and \( \Gamma \), respectively. Therefore, from (32) we obtain

\[
\dot{V}_\rho \leq -f^- V_\rho + M_\rho
\]

where \( f^- = f^- / f^+ \), \( M_\rho \) is the bound of \( M_\rho \). By direct integrations of the differential inequality (36), we have

\[
V_\rho \leq V_\rho(0) + \frac{M_\rho}{f^-}
\]

This shows that \( V_\rho \) is uniformly bounded. Thus \( z_i, \hat{\theta} \) and \( e \) are bounded. Since \( z_1 \) and \( y_r \) are bounded, \( y \) is also bounded. In view of the boundedness of \( y \) and Assumption 1, we conclude that the input \( u \) is bounded. Then from (6) and (5) we can show that \( \lambda, \eta \) and \( x \) are bounded as in [1].
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We now derive a bound for the vector $z(t)$ where $z(t) = [z_1, z_2, \ldots, z_p]^T$. Firstly, the following definitions are made.

$$c_0 = \min_{1 \leq i \leq \rho} c_i,$$

$$l_0 = \sum_{i=1}^{p} \frac{1}{2T}$$ (38)

$$||z||_{[0,T]} \leq \sqrt{\frac{1}{T} \int_0^T z(t)^2 dt}$$ (39)

Then from (36), we have

$$V_\rho \leq -c_0 ||z||^2 + M_\rho$$ (40)

Integrating both sides, we obtain

$$||z||_{[0,T]} \leq \frac{1}{c_0} \frac{||V_\rho(0) - V_\rho(T)||}{T} + \frac{\frac{t}{2}}{c_0} \frac{||\theta - \theta^*||^2}{2} + \frac{||P||^2}{T} l_0 \int_0^T \|D(t)||^2 dt$$

$$+ \frac{||b||_m}{l_e} ||e - e^*||^2 + \frac{1}{T} \int_0^T \rho D_1(t)^2 dt$$ (41)

On the other hand, from (36), we have

$$\frac{||V_\rho(0) - V_\rho(T)||}{T} \leq f^* V_\rho(0) + \frac{||b||_m}{l_e} ||e - e^*||^2$$

$$+ \frac{\frac{t}{2}}{c_0} \frac{||\theta - \theta^*||^2}{2} + \frac{||P||^2}{T} l_0 \int_0^T \|D(t)||^2 dt$$

$$+ \frac{1}{T} \int_0^T \rho D_1(t)^2 dt,$$ for all $T \geq 0$,

where we have used the fact that $e^{-f^*(T-t)} \leq 1$ and $\frac{1-e^{-f^*T}}{f^* T} \leq f^*$. By setting $z_i(0) = 0$, the initial value of the Lyapunov function is

$$V_\rho(0) = \frac{1}{2} \frac{||\hat{\theta}(0)||^2}{l_1} + \frac{||b_m||_\infty}{2\gamma} ||\hat{e}(0)||^2 + l_0 ||e(0)||^2$$. (42)

Using (35), the fact that $f^* / c_0 \leq 2$, a bound resulting from (41) - (42) is given by

$$||z||_{[0,T]} \leq \frac{\hat{\theta}(0)||^2}{l_1} + \frac{||b_m||_\infty}{2\gamma} ||\hat{e}(0)||^2 + l_0 ||e(0)||^2$$

$$+ \frac{\frac{t}{c_0}}{2} \frac{||\theta - \theta^*||^2}{2} + \frac{2 l_0 ||P||^2}{T c_0} \int_0^T \|D(t)||^2 dt$$

$$+ \frac{1}{c_0} \frac{||b_m||_e ||e - e^*||^2}{2} + \frac{1}{T c_0} \int_0^T \rho D_1(t)^2 dt$$ (43)

The results obtained from the above analysis are now summarized in the following theorem.

**Theorem 1** Consider the closed-loop adaptive system consisting of the plant (1) satisfying Assumptions 1-4, the controller (30), the parameter updating law (22), (31) and the filters (5) and (6). All the signals in the system are globally uniformly bounded. Furthermore, the tracking error satisfies the bound given in (43).

### 3 Extension to Nonlinear Systems

In this section we will extend the adaptive control design to nonlinear systems with unknown disturbance. Consider the following nonlinear systems:

$$\dot{x} = Ax + \psi_0(y) + b\sigma(y)u + \Psi(y)\alpha + \Phi(y)d(t)$$ (44)

where $y = e^T x \in R$, $x = [x_1, \ldots, x_n]^T \in R^n$, $u \in R$ are system input, states and output respectively. $\Psi(y)\alpha = \sum_{i=1}^{p} \psi_i(y)a_i, \Phi(y)d(t) = \sum_{j=1}^{q} \phi_j(y)d_j(t), \psi_i, i = 0, \ldots, p$ and $\phi_j, j = 1, \ldots, q$ are known smooth nonlinear vectors in $R^n$, $\sigma(y)$ is known smooth function, $b = [0 \ldots b_m \ldots b_0]^T \in R^n$ and $a \in R^p$ are the vectors of unknown constant parameters, $d(t) \in R^n$ denotes unknown time-varying bounded disturbances. Similar class of systems was analyzed in [15]. The assumptions made here are the same as those of the linear systems given in Section 2. We start by representing the plant (44) as in the observer canonical form.

$$\dot{\xi} = A_0\xi + ky + \psi_0(y)$$ (45)

$$\dot{\Xi} = A_0\Xi + \Psi(y)$$ (46)

$$\dot{\lambda} = A_0\lambda + e_n\sigma(y)u$$ (47)

$$\dot{v_j} = A_0v_j$$ (48)

where $A_0 = A - ke_1^T, k = [k_1, k_2, \ldots, k_l]^T$, $k$ chosen so that $A_0$ is Hurwitz. From our designed filters, system (44) can be expressed as

$$\dot{y} = b_m v_m + \xi_2 + \omega^T \theta + e_2 + \Phi_1(y)d(t)$$ (49)

$$\omega = [v_m, v_m - 1, \ldots, v_0, \Xi_2 + \Psi_1]^T$$ (50)

$$\dot{\Xi_2} = [0, v_m - 1, \ldots, v_0, \Xi_2 + \Psi_1]^T$$ (51)

where $e_2, v_2, \xi_2$ and $\Xi_2$ denote the second entries of $e, v, \xi$ and $\Xi$ respectively; $\Phi_1$ and $\Psi_1$ are the first elements of the $\Phi(y)$ and $\Psi(y)$ respectively. With the above filters, a state estimate is given by $\hat{x} = \xi + \Omega \theta$ and the estimation error is defined as $e = x - \hat{x}$. Suppose $P \in R^{n \times n}$ is a positive definite matrix, satisfying $PA_0 + A_0^T P \leq -3I$ and let $V_e = e^T Pe$. It can be shown that

$$\dot{V}_e \leq -2 ||e||^2 + ||P(\Phi(y))d(t)||^2$$ (52)

**Remark 2** Note that the unknown disturbance $d(t)$ is multiplied by a function $\Phi(y)$ and this function may not be bounded. So the term $\|P(\Phi(y))d(t)\|^2$ cannot be assumed bounded. In most existing results including the one developed for linear systems in the previous sections, the disturbance is not multiplied by an unbounded function and this makes the design and analysis simpler. The effect of this term is compensated by introducing a new well defined function.
Firstly we define \( s(x) \) as
\[
s(x) = \begin{cases} 
  x^2 & |x| \geq \delta \\
  (\delta^2 - x^2)^{\rho} + x^2 & |x| < \delta 
\end{cases} \quad (53)
\]
where \( \delta \) is a positive design parameter. It can be shown that \( s(x) \) is \((\rho - 1)\)th order differentiable and bounded below for \( |x| < \delta \). Then based on \( s(x) \), a function \( H(z_1) \) is defined as follows
\[
H(z_1) = \frac{\Phi(y)}{s(z_1)} = \begin{cases} 
  \frac{\Phi(y)}{z_1^\rho} & |z_1| \geq \delta \\
  \frac{\Phi(y)}{(\delta^2 - z_1^\rho)} & |z_1| < \delta 
\end{cases} \quad (54)
\]
Clearly \( H \) is well defined and for \(|z_1| < \delta \), \( H \) is bounded as \( s(z_1) \) is bounded below. With the function \( H \), the term \( \| P\Phi(y)d(t)T \| \) in (52) is divided into two terms as follows:
\[
\dot{V}_t \leq -2\| \epsilon \|^2 + \frac{1}{2} s^4 \| PH \|^4 + \frac{1}{2} \| d(t) \|^4 \quad (55)
\]
**Remark 3** Note that the new function \( H \) is \((\rho - 1)\)th order differentiable, so is the term \( \frac{1}{2} s^4 \| PH \|^4 \). This property will be considered in the controller design. Now the term \( \frac{1}{2} \| d(t) \|^4 \) in (55) is bounded.

As the details of the controller design using the backstepping approach are similar to those Section 4, we now just present the designed control law and the parameter update laws as below.
\[
\alpha_1 = \dot{\hat{\alpha}}_1 \quad (56)
\]
\[
\hat{\alpha}_1 = -(c_1 + l_1)z_1 - \tilde{\omega}^T \hat{\theta} - \frac{1}{4} z_1 \| \Phi_1(y) \|^2
- \xi_2 - \sum_{i=1}^\rho \frac{1}{8l_i} z_1 s^3(z_1) \| PH \|^4 \quad (57)
\]
\[
\alpha_i = -c_i z_i - l_i \frac{\partial \alpha_{i-1}}{\partial y} z_i - z_{i-1} + \beta_i
- \frac{z_i}{4} \frac{\partial \alpha_{i-1}}{\partial y} \Phi_1(y) \| \Phi_1(y) \|^2 + \frac{\partial \alpha_{i-1}}{\partial \theta} \Gamma \theta - \theta^* \\
- \left( \sum_{k=2}^{i-1} \beta_{i-k} \right) \frac{\partial \alpha_{i-1}}{\partial y} \| \Phi_1(y) \|^2 + \frac{\partial \alpha_{i-1}}{\partial \theta} \Gamma \theta_i - \gamma \| \alpha_{i-1} \| \Gamma \theta_i \quad (58)
\]
\[
u(t) = \frac{1}{\sigma(y)} (\alpha_{\rho} - v_{m,\rho+1} + \dot{\theta} y_{r}(\rho)) \quad (59)
\]
\[
\dot{\hat{\theta}} = -\gamma sgn(b_m)(\dot{y}_r + \alpha_1)z_1 - \gamma \| \dot{\hat{\theta}} - \epsilon \| \quad (60)
\]
\[
\dot{\theta} = \Gamma \theta_\rho + \Gamma \theta \quad (61)
\]
**Remark 4** Note that in the control design, a new term \( \sum_{i=1}^\rho \frac{1}{8l_i} z_1 s^3(z_1) \| PH \|^4 \) is introduced in \( \hat{\alpha}_1 \). It is used here to handle the term \( \frac{1}{2} s^4 \| PH \|^4 \) in the error Lyapunov function (55). For \( |z_1| \geq \delta \), the effect of \( \frac{1}{2} s^4 \| PH \|^4 \) is compensated by this new term. On the other hand when \( |z_1| < \delta \), \( \frac{1}{2} s^4 \| PH \|^4 \) is bounded. Also note that the parameter update laws are the same as those in the linear control design.

For the designed adaptive controller, we can also obtain the following theorem similar to that for linear systems. Note that similar results on the tracking performance are also applicable here.

**Theorem 2** Consider the closed-loop adaptive nonlinear system consisting of the plant (44) satisfying Assumptions 1-4, the controller (59), the parameter updating laws (60), (61) and the filters (45) to (47). All the signals in the system are globally uniformly bounded.

### 4 Simulation Studies

In this section, we illustrate the above method using a simple system given in the following.
\[
(p^2 + ap)y(t) = bu(t) + d(t) \quad (62)
\]
where \( b = 1, a = 1.2, d(t) = 0.2 \cos(2t) \). These parameters are not needed to be known in the controller design. The objective is to control the system output \( y \) to follow a desired trajectory \( y_r(t) = 2 \sin(2t) \). The filters from (5) and (6) are implemented. The adaptive control laws \( \alpha_1 \) in (20), \( \nu(t) \) in (30), and parameter update laws in (31) are used, where \( \theta = [b, a]^T \), respectively. The design parameters are chosen as \( c_1 = c_2 = 1.5, l_1 = l_2 = 0.2, \Gamma = 0.2 I_2, \tau = 0.1, k_1 = 6, k_2 = 8 \). The initial values are set as \( y(0) = 0.1, \theta(0) = [0.5, 0.5]^T \). Figure 1 shows the system output \( y \) and the desired trajectory signal. Figure 2 shows the control signal \( u(t) \). Clearly, these simulation results verify that our proposed scheme is effective to cope with unknown disturbance.

![Figure 1: Output y(...) and trajectory y_r(-)](image-url)
5 Conclusion

In this paper, a scheme is proposed to design an adaptive output-feedback controller for uncertain systems in the presence of disturbances. In our design, the term multiplying the control and the system parameters are not assumed to be within known intervals. The bound of the disturbance is not required. To handle the disturbances, well defined functions are introduced to eliminate their effects in the Lyapunov functions employed in the recursive design steps. Furthermore, the overparamterization problem is also solved by using the concept of tuning functions. It is shown that the proposed controller can make the whole adaptive control system stable.

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References:


