DTM Based on an Ellipsoidal Squares

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Abstract: - The paper presents the description of the quality analysis following the most popular world standards used for the description of terrain and sea bottom shape:

- DTED - Digital Terrain Elevation Data, used by NATO,
- SRTM model - Shuttle Radar Topography Mission, worked out within the confines of the international mission of the space shuttle Endeavour,
- DBDB-V - Digital Bathymetric Data Base - Variable Resolution, used for the description of sea and ocean bottoms,
- NMB - Network Model Bathymetry, as one of the AML - Additional Military Layers for WECDIS - Warship Electronic Chart Display and Information Systems.

In the main part of the article, a new method of the description of terrain and sea bottom shape have been shown. The method is based on the matrix model of the GRID type, where DTM array consists of geographical squares.

Key-Words: - Digital Terrain Model, ellipsoidal squares, 3D navigation

1 Introduction

Currently there are many standards worldwide for describing the shape of terrain and sea bottoms, which use the matrix model. From the most important of these, we can recognize: DTED, SRTM, DBDB-V and NMB.

They are defined by a matrix model based on a grid, whose approximated fragments (geographical grid mesh) are ellipsoidal trapezoids with fixed angle dimensions. Because the size of the trapezoids changes along with the change of their parallel position, these models do not allow the maintenance of uniform precision in the projection of the actual surface of the terrain/sea bottom in the whole area on the reference ellipsoid.

In consideration of the above, attempts made to solve this problem through the use of scientific research, resulted in the studied matrix model based on a grid, whose approximated fragments are approximately square ellipsoids determined as ellipsoid.

The model designed takes into consideration its future use in sea navigation and also the necessity to maintain a consistently high level of precision of the projected area of terrain/sea bottom in the whole area on the reference ellipsoid.

2 Worldwide evaluations according to DTM standards

Every DTM - Digital Terrain Model is an approximation of reality, which is why the choice of a most suitable model and the definition of its parameters depends on the demanded requirements for interpreting the phenomenon.

In view of the simplicity of building a matrix model, the most often described phenomenon is the character of the area (also the shape of the terrain/sea bottom).

The faithfulness of the presented actual area of terrain/sea bottom by means of this model depends above all on the quality of the source material and size of aggregate data of height/depth.

The DTED standard classifies the matrix model according to what is known as information resolution. Each of the numerical products is rated at a conventional level:

- Level 0 (DTED 0) suitable with regards to geometric precision at a scale of 1:1 000 000,
- Level 1 (DTED 1) suitable with regards to geometric precision at a scale of 1:1 250 000,
- Level 2 (DTED 2) suitable with regards to geometric precision at a scale of 1:1 50 000,
- Level 3 (DTED 3) suitable with regards to geometric precision at a scale of 1:1 25 000.

In addition, within the confines of each level, the Earth’s area is divided into zones, in which the matrix has various resolutions [1].

Tab. 1 Division into zones and precision levels in the DTED standard

<table>
<thead>
<tr>
<th>Zone</th>
<th>Zone borders [geographic latitude]</th>
<th>Parallel resolution</th>
<th>Meridian resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0° - 50°</td>
<td>0° 30' 3' 1&quot;</td>
<td>0° 30' 3' 1&quot;</td>
</tr>
<tr>
<td>II</td>
<td>50° - 70°</td>
<td>30° 30' 3' 1&quot;</td>
<td>30° 30' 3' 1&quot;</td>
</tr>
<tr>
<td>III</td>
<td>70° - 75°</td>
<td>30° 60' 6' 2&quot;</td>
<td>90° 9' 3&quot;</td>
</tr>
<tr>
<td>IV</td>
<td>75° - 80°</td>
<td>30° 120' 12' 4&quot;</td>
<td>120° 12' 4&quot;</td>
</tr>
<tr>
<td>V</td>
<td>80° - 90°</td>
<td>30° 180' 18' 6&quot;</td>
<td>180° 18' 6&quot;</td>
</tr>
</tbody>
</table>

The numerical model of terrain SRTM (Shuttle Radar Topography Mission) is developed with the angle resolution $\Delta \varphi = \Delta \lambda = 1^\circ = 0.000277771^\circ$. Currently publicly available SRTM -3 data has been divided into segments, of which each one incorporates a sector of the Earth’s surface of the dimensions 1° by 1°, and the exact sides of the sector measure 1.00083333° and as a result these neighboring segments overlap each other negligibly.

The data in each segment makes up a matrix of numbers corresponding to a grid of a size of 1201 rows and 1201 columns composed of cells of dimensions $\Delta \varphi = \Delta \lambda = 3^\circ = 0.00083333^\circ$ [2].

The technical specifications of DBDB-V (Digital Bathymetric Data Base - Variable Resolution) developed by the National Imagery and Mapping Agency, describe a numerical model of the sea bottom by means of geographical grids of the resolutions $\Delta \varphi = \Delta \lambda = 0,5^\prime$, $\Delta \varphi = \Delta \lambda = 1^\prime$, $\Delta \varphi = \Delta \lambda = 2^\prime$, $\Delta \varphi = \Delta \lambda = 5^\prime$ [3].

The technical specifications of NMB developed by the Geospatial Maritime Working Group appointed by the NATO Geographic Conference, describe a numerical model of the sea bottom by means of geographical grids with 9 levels of resolution [4].

Tab. 2 Division of levels of precision in the NBM standard

<table>
<thead>
<tr>
<th>Resolution level</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel resolution</td>
<td>2°</td>
<td>0,5°</td>
<td>5°</td>
<td>1°</td>
<td>15°</td>
<td>3°</td>
<td>0,5°</td>
<td>0,1°</td>
<td>0,05°</td>
</tr>
<tr>
<td>Meridian resolution</td>
<td>2°</td>
<td>0,5°</td>
<td>5°</td>
<td>1°</td>
<td>15°</td>
<td>3°</td>
<td>0,5°</td>
<td>0,1°</td>
<td>0,05°</td>
</tr>
</tbody>
</table>

3 Distortion of projected areas of the terrain/sea bottom in DTED, SRTM, DBDB-V and NMB

All the represented standards define a matrix model based on a grid, whose approximated fragments (grid mesh), are ellipsoidal trapezoids.

The size of the trapezoid changes along with change in its parallel position in the ellipsoid. A change in the position of the trapezoid causes a change in the length of the arcs meridians and parallels, which make up the borders of the trapezoid. The area of the ellipsoidal trapezoid $P_T$ can be determined using the relationship:

$$P_T = b^2 \int_{\lambda_1}^{\lambda_2} \int_{\phi_1}^{\phi_2} \frac{\cos \varphi}{(1 - e^2 \sin^2 \varphi)^{3/2}} d\varphi d\lambda$$

where
- $b$ - length of the semi-minor axis of the ellipsoid,
- $e^2$ - square of the first ellipsoidal eccentricity.
The use of this relationship (1) can determine the area of the ellipsoidal trapezoid $P_r$ in relation to geographic latitude for the chosen level of resolution established by the DTED, SRTM, DBDB-V, NMB standards (this calculation was also done using the Simpson numerical method – which gave approximately the same results).

After analysis of the obtained results, it can be stated that ellipsoidal trapezoids of fixed angle dimensions throughout the whole area of the ellipsoid, change their linear dimensions. The greatest changes undergone are in the length of the trapezoid base. The effects of this are that the area of the approximated trapezoid fragment of terrain/sea bottom area differs at different geographic latitudes. DTED can be recognized as the best model. In this model, the changing base length of the trapezoid is compensated for by the creation of zones of differing angle resolutions. However, this model only minimizes and does not eliminate the changing phenomenon of the ellipsoidal trapezoid area produces change in its parallel position on the ellipsoid.

**4 DTM based on an ellipsoidal squares**

I propose introducing the DTM model, which would enable the maintenance of uniform precision of projections of the actual area of terrain/sea bottom in the whole area on the ellipsoid. This model will be a matrix model of the GRID type in which grid mesh will be ellipsoidal squares, positioned one next to the other covering the whole area of the respective ellipsoid. The linear length of the sides of each ellipsoidal square $\Delta x$ will correspond to the length of the equator arcs defined for each fixed
angle $\Delta \lambda$ and will be calculated by means of the following formula:

$$
\Delta x = a \cdot \Delta \lambda \cdot \text{arc} \, \lambda'[m]
$$

(2)

where

$$
\text{arc} \, \lambda' = \frac{2\pi}{21600},
$$

$a$ - length of the semi-major axis of the ellipsoid in meters,

$\Delta \lambda$ - the fixed angle value expressed in minutes,

$\text{arc} \, \lambda'$ - the angle value 1° expressed in radians.

The starting coordinate system for grid will be found at points specified by the geographical starting coordinate system. The position of the remaining grid nodes will be defined in accordance with these points.

Fig. 5 Relation between geographical and DTM grid

To determine the geographical coordinates of each grid node $P(\varphi_w, \lambda_w)$ the following formula will be applied:

$$
\varphi_w = \sum_{j=1}^{J} \left[ \frac{\Delta x \cdot \left( 1 - e^2 \sin^2(\varphi_{w-1} + \Delta \lambda / 2) \right)^{3/2}}{a(1-e^2)} \right] - \\
- \left[ \left( e^2 \cos^2(\varphi_{w-1} + \Delta \lambda / 2) / 8 \right) \left( 1 - e^2 \sin^2(\varphi_{w-1} + \Delta \lambda / 2) \right) \right] \\
\cdot \left( 1 - \tan^2(\varphi_{w-1} + \Delta \lambda / 2) \right) + \\
e^2 \tan^2(\varphi_{w-1} + \Delta \lambda / 2) \sin^2(\varphi_{w-1} + \Delta \lambda / 2) + \\
\cdot 4e^2 \sin^2(\varphi_{w-1} + \Delta \lambda / 2) \right) \right]^{3/2} + \\
\left[ \left( \frac{\Delta x \cdot \left( 1 - e^2 \sin^2(\varphi_{w-1} + \Delta \lambda / 2) \right)^{3/2}}{a(1-e^2)} \right) \right]^{3/2} + \ldots
$$

(3)

where

$e'$ - second ellipsoidal eccentricity,

$\varphi_w = 0$ for $w = 0$,

geographic latitude $\varphi_w$ is accepted as north for $j > 0$ and south for $j < 0$, 

\[ \lambda_w = \sum_{i=1}^{H} \left[ \Delta x \left/ \left( 1 - e^2 \sin^2 \varphi_w \right)^{1/2} \right. \right] \quad (4) \]

where geographic longitude \( \lambda_w \) is accepted as east for \( i > 0 \) and west for \( i < 0 \).

5 Summary

Considering their simplicity in building, matrix models should be applied to DTM descriptions. Until now, applied world standards of DTM have not guaranteed uniform precision of actual projections of the area of terrain/sea bottom in the whole area of the reference ellipsoid. I propose the application of a DTM model based on ellipsoidal squares. This model allows the maintenance of a uniform level of precision with projections of the terrain/sea bottom in the whole area of the respective ellipsoid. To use this in navigation, it is recommended to apply a model based on a UTM or UPS grid projection. These models are characterized by true angles and high precision of projected distances. A particular quality of these is also the possibility of easily and precisely accounting for projection distortions.

References:


