The change in impedance of a single-turn coil located above a moving medium

A. A. KOLYSHKIN  
Department of Engineering Mathematics  
Riga Technical University  
1 Meza Street, Riga  
LATVIA  
akoliskins@rbs.lv  

I. VOLODKO  
Department of Engineering Mathematics  
Riga Technical University  
1 Meza Street, Riga  
LATVIA  inta.volodko@cs.rtu.lv

Abstract: - The effect of a moving medium on the change in impedance of a single-turn coil located above the medium is investigated in the present paper. Two cases are considered in detail: (a) a moving half-space and (b) a moving plate. The change in impedance in both cases is expressed in terms of double integrals and is evaluated numerically. Three different norms are used to measure the difference between the change in impedance for fixed and moving medium. Results of numerical calculations are presented.

Key-Words: - change in impedance, moving medium,

1 Introduction
Mathematical models for eddy current testing of fixed conducting multilayer samples are well-known in the literature [1]-[4]. In many engineering applications controlled object is moving with respect to the coil. Examples include protective or decorative coating of a continuously moving metal sheet or a coin movement inside coin validator.

Exact solutions for the magnetic vector potential for an infinite metal plate moving with respect to the system of parallel straight wires with alternating current are obtained in [5]. Similar problems for a single-turn coil located above a moving conducting medium are discussed in [6], [7].

In the present paper a detailed numerical investigation is conducted in order to analyze the conditions where the movement of a conducting medium on the change in impedance of a single-turn coil located above a moving half-space or a plate can be neglected. Two problems are solved in each case: the change in impedance is computed for the case of a fixed and moving medium. Relative errors with respect to different norms describing the difference between fixed and moving medium are computed for a wide range of the parameters of the problems. Recommendations are formulated with respect to a suitable frequency range of the current in the coil when movement of metal objects with respect to the coil can be neglected. The results of this study can be used for design of coin validators.

2 Mathematical Analysis
The analysis of the effect of a moving object on the characteristics of the eddy current probe is quite important in applications. Examples include movement of metal sheet in metallurgical plants or design of coin validators. The following simple model (see Fig. 1) is considered below in order to minimize the number of parameters that can affect the change in impedance of the coil due to the presence of a moving object.
Consider a single-turn coil of radius $R$ located at distance $h$ above a conducting two-layer medium. The medium is assumed to be infinite in the $x$- and $y$- directions. The upper layer is moving in the $x$- direction with constant velocity $v$. Two particular cases of the problem (relevant to coin validators) are considered in detail: (a) moving half-space and (b) moving plate. Note that the solution found for the case of a single-turn coil can be easily generalized for the case of a coil with finite dimensions. However, our objective is to investigate the effect of a moving medium on the change in impedance where the number of other parameters is as small as possible.

### 2.1 Moving half-space

The change in impedance of a single-turn coil located above a conducting half-space (with $v = 0$) is (see [1]):

$$
Z^{(0)}_{\text{ind}} = j \omega \mu_0 R \beta \pi \int_0^\infty \frac{y - \sqrt{y^2 + j} \beta y e^{-\alpha y}}{y + \sqrt{y^2 + j}} \, dy.
$$

(3)

Dividing (2) and (3) by $\omega \mu_0 R$ we obtain the normalized change in impedance of the form

$$
\tilde{Z}^{(v)}_{\text{ind}} = \frac{Z^{(v)}_{\text{ind}}}{\omega \mu_0 R} = j \beta \pi \int_0^\infty \frac{y - \sqrt{y^2 + j} \beta y e^{-\alpha y}}{y + \sqrt{y^2 + j}} \, dy,
$$

(4)

$$
\tilde{Z}^{(0)}_{\text{ind}} = \frac{Z^{(0)}_{\text{ind}}}{\omega \mu_0 R} = j \beta \pi \int_0^\infty \frac{y - \sqrt{y^2 + j} \beta y e^{-\alpha y}}{y + \sqrt{y^2 + j}} \, dy.
$$

(5)

The effect of a moving half-space on the change in impedance of the coil can be evaluated by computing the relative error which can be defined in many different ways. In particular, we computed the following relative errors:
\[ \delta_1 = \frac{|\tilde{Z}_{in}^{(v)} - \tilde{Z}_{in}^{(0)}|}{|\tilde{Z}_{in}^{(v)}|}, \quad \delta_2 = \frac{|\Re(\tilde{Z}_{in}^{(v)}) - \Re(\tilde{Z}_{in}^{(0)})|}{|\Re(\tilde{Z}_{in}^{(v)})|}, \]
\[ \delta_3 = \frac{|\Im(\tilde{Z}_{in}^{(v)}) - \Im(\tilde{Z}_{in}^{(0)})|}{|\Im(\tilde{Z}_{in}^{(v)})|}. \]

Computations are done with Mathematica. The results of calculations are shown in Figs. 2 – 4.

Fig. 2. Relative error \( \delta_1 \) versus \( \beta \) for three values of \( V : 0.9, 0.5 \) and 0.1 (from top to bottom).

Fig. 3. Relative error \( \delta_2 \) versus \( \beta \) for three values of \( V : 0.9, 0.5 \) and 0.1 (from top to bottom).

Fig. 4. Relative error \( \delta_3 \) versus \( \beta \) for three values of \( V : 0.9, 0.5 \) and 0.1 (from top to bottom).

As can be seen from Figs. 2 – 4, the relative error in using fixed half-space instead of a moving one is very small for \( V = 0.1 \) for all values of \( \beta \). Recall that \( \beta = R \sqrt{\omega \sigma \mu_0} \) so that for fixed \( R \) and \( \sigma \) the values of \( \beta \) are proportional to the square root of the frequency. Thus, for \( V = 0.1 \) in a wide frequency range the effect of the moving half-space on the change in impedance of the coil is negligible (all relative errors \( \delta_1, \delta_2 \) and \( \delta_3 \) are much smaller than 1%).

### 2.2 Moving plate

Consider a plate of finite thickness \( d \) moving along the \( x \)-axis with constant velocity \( v \) (see Fig. 1). Region \( R_3 \) is free space \((\sigma_3 = 0)\). The system of the Maxwell’s equations for the \( x \)-component of the vector potential \( (A_x) \) can be written in the following form (see [6]):

\[ \Delta A_x = \mu_1 \mu_0 j_{C Tx}, \]
\[ \Delta A_x + k_2^2 A_x - \tilde{\nu} \frac{\partial A_x}{\partial x} = 0, \]
\[ \Delta A_x + k_3^2 A_x = 0, \]

where \( j_{C Tx} \) is the current density in the coil, \( k_2^2 = -j \omega \mu_2 \mu_0 \sigma_2, \tilde{\nu} = \mu_2 \mu_0 \sigma_3 v \), and
\( k_3^2 = -j \omega \mu_3 \mu_0 \sigma_3 \). A similar system of
equations is obtained for the component \( A_y \). The boundary conditions (for the components \( A_x \) and \( A_y \)) are

\[
A_{x,y} \bigg|_{z=+0} = A_{x,y} \bigg|_{z=-0},
\]

(9)

\[
\frac{1}{\mu_1} \frac{\partial A_{x,y}}{\partial z} \bigg|_{z=+0} = \frac{1}{\mu_2} \frac{\partial A_{x,y}}{\partial z} \bigg|_{z=-0},
\]

(10)

\[
A_{x,y} \bigg|_{z=-d+0} = A_{x,y} \bigg|_{z=-d-0},
\]

(11)

\[
\frac{1}{\mu_3} \frac{\partial A_{x,y}}{\partial z} \bigg|_{z=-d+0} = \frac{1}{\mu_4} \frac{\partial A_{x,y}}{\partial z} \bigg|_{z=-d-0}.
\]

(12)

Applying the Fourier transform to (6) – (8) we obtain

\[
d^2 A - \lambda^2 A = B \delta(h-z),
\]

(13)

\[
d^2 A - q_2^2 A = 0,
\]

(14)

\[
d^2 A - q_3^2 A = 0,
\]

(15)

where \( A \) is the Fourier transform of the function \( A_x \), \( B = 2 j \pi R \mu_1 \mu_0 J_1(\lambda R) \sin \varphi \), \( \delta \) is the Dirac delta-function, \( q_2^2 = \lambda^2 - k_2^2 - j \nu \lambda \cos \varphi \), \( q_3^2 = \lambda^2 - k_3^2 \), \( \lambda \) is the parameter of the Fourier transform, \( I \) is the amplitude of the current in the coil.

Similarly, the boundary conditions (9) – (12) in the transformed space are

\[
A \bigg|_{z=0} = A \bigg|_{z=-0},
\]

(16)

\[
\frac{1}{\mu_1} \frac{dA}{dz} \bigg|_{z=0} = \frac{1}{\mu_2} \frac{dA}{dz} \bigg|_{z=0},
\]

(17)

\[
A \bigg|_{z=-d+0} = A \bigg|_{z=-d-0},
\]

(18)

\[
\frac{1}{\mu_3} \frac{dA}{dz} \bigg|_{z=-d+0} = \frac{1}{\mu_4} \frac{dA}{dz} \bigg|_{z=-d-0},
\]

(19)

where \( A \) denotes the Fourier transform of the function \( A_y \) (similar system of equations can be obtained for the Fourier transform of the function \( A_y \)).

Solving (13) – (19) we obtain the induced change in impedance of the coil \( Z_{ind} \). The normalized change in impedance is computed by the formula

\[
Z_{ind}^{(v)} = \frac{Z_{ind}}{\omega \mu_0 R},
\]

where

\[
Z_{ind}^{(v)} = j \beta \int_0^\infty J_1^2(\beta y) e^{-a y} dy 
\int_0^{2\pi} \frac{E}{D} d\varphi,
\]

(20)

\[
D = y[y + g(y)] e^{2J(y)} - y + g(y),
\]

\[
E = [y + g(y)]^2 e^{2J(y)} - [y - g(y)]^2,
\]

\[
g(y) = \sqrt{y^2 + j - jyV \cos \varphi}.
\]

Here we used the following dimensionless variables: \( \alpha = \frac{2h}{R} \), \( \beta = R \sqrt{\omega \sigma \mu_0} \), \( V = \frac{\nu}{\omega R} \), \( \gamma = \frac{d}{R} \).

For the case of a fixed plate (\( v = 0 \)) formula (20) has the form

\[
Z_{ind}^{(v)} = 2 \pi \beta \int_0^\infty J_1^2(\beta y) e^{-a y} dy.
\]

(21)

In order to estimate the effect of a moving plate on the change in impedance of the coil we compute the following relative errors:

\[
\delta_1 = \frac{\left| Z_{ind}^{(v)} - Z_{ind}^{(0)} \right|}{\left| Z_{ind}^{(0)} \right|},
\]

\[\delta_2 = \frac{\left| \text{Re}\left( Z_{ind}^{(v)} \right) - \text{Re}\left( Z_{ind}^{(0)} \right) \right|}{\left| \text{Re}\left( Z_{ind}^{(0)} \right) \right|} \text{ and}
\]

\[
\delta_3 = \frac{\left| \text{Im}\left( Z_{ind}^{(v)} \right) - \text{Im}\left( Z_{ind}^{(0)} \right) \right|}{\left| \text{Im}\left( Z_{ind}^{(0)} \right) \right|}.
\]

(22)
The results of computations are shown in Figs. 5 – 8.

The relative errors $\delta_1, \delta_2$ and $\delta_3$ versus $\beta$ for different values of $V$ and fixed dimensionless thickness of the plate $\gamma = 0.2$ (recall that $\gamma = d / R$) are shown in Figs. 5 – 7. As in the case of a moving half-space, the effect of a moving plate on the change in impedance is important only for relatively large velocities (the values of the dimensionless parameter $V$ of order one).

The effect of the thickness of the plate on the relative errors $\delta_1, \delta_2$ and $\delta_3$ versus $\beta$ is shown in Figs. 8 – 10 for $V = 0.1$ and three values of $\gamma : 0.2, 0.5$ and 0.8 (from bottom to top where the reference point is the point with the smallest $\beta$).
The graphs in Figs. 8 – 10 show that for all values of β considered (β is proportional to the square root of frequency) and for all values of γ in the interval 0.2 ≤ γ ≤ 0.8 (γ is the dimensionless thickness of the plate) the relative error in using the model for the fixed plate instead of the moving plate with V = 0.1 does not exceed 0.09%.

3 Conclusion

Let us estimate the value of V for any realistic coin validator. The dimensionless parameter V is related to the velocity v of the moving plate by the formula $V = \frac{V}{\alpha R} \beta$. Since $\beta = R \sqrt{\omega \sigma \mu_0}$ we can rewrite v in the form $v = V \sqrt{\frac{\sigma \mu_0}{\omega}}$. The last formula can be further simplified assuming that the conductivity of a coin is of order $\sigma = 10^7$ S/m (in addition, $\omega = 2\pi f$ and $\mu_0 = 4\pi \cdot 10^{-7}$). Thus,

$$v = V \sqrt{\frac{2}{f}}$$

It follows from (23) that $V = 0.1$ if $f = 20$ kHz and $v = 10$m/s. The results of the extensive parametric study presented in this paper show that for any thickness of the plate the relative error in using fixed plate model instead of a moving one in any norm is negligible if $V = 0.1$. Our estimate for the velocity of the moving plate ($v = 10$m/s) is, most likely, too large for any coin validator. It follows from (23) that if v is decreasing then $V$ is decreasing as well. As a result, for smaller values of $V$ the relative error between the two models (moving plate and fixed plate) is even smaller.

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