Simplifying Assessment Complexity with Many Valued Logics

Sylvia Encheva
Stord/Haugesund University College
Bjørnsonsg. 45, 5528 Haugesund, Norway
Telephone: 52702685; Fax: 52702601
Email: sbe@hsh.no

Abstract—Current tendencies in high education world wide include merging of several existing small and medium small educational institutions in considerably bigger entities. Such processes generate both administrative and pedagogical challenges. In this work we address a question related to how to provide better automated assessment of students’ knowledge, obtained skills, and terms understanding if it is possible to use learning materials developed by different educators.

I. INTRODUCTION

Assessment of students knowledge nowadays needs special attention due to many reasons some of which are increased number of people taking education both full and part time, limited amount of resources available for high education, and last but not least constantly improving technologies just waiting to be introduced in educational processes. In this work we address a question related to how to provide better automated assessment of students knowledge, obtained skills, and terms understanding if it is possible to use learning materials developed by different educators.

Many automated tests use Boolean logic in the process of decision making. Thus if the response does not appear to be necessarily true, the system selects false. While Boolean logic appears to be sufficient for most everyday reasoning, it is certainly unable to provide meaningful conclusions in presence of inconsistent and/or incomplete input [1], [2]. This problem can be resolved by applying many-valued logic.

II. BACKGROUND

Grading student projects can be tedious and time consuming, [3]. The tester described in [3] help to automate the testing and grading of circuits built using digital logic simulators. Digital design and computer architecture problems are presented in [4]. The role of computers for creating constructivist environments is discussed in [5]. A level-based instruction model is proposed in [6]. A model for student knowledge diagnosis through adaptive testing is presented in [7]. An approach for integrating intelligent agents, user models, and automatic content categorization in a virtual environment is presented in [8].

The Questionmark system [9] applies multiple response questions where a set of options is presented following a question stem. The final outcome is in a binary form, i.e. correct or incorrect because the system is based on Boolean logic [10], [11].

Let \( P \) be a non-empty ordered set. If \( \sup\{x, y\} \) and \( \inf\{x, y\} \) exist for all \( x, y \in P \), then \( P \) is called a lattice, [12].

In a lattice illustrating partial ordering of knowledge values, the logical conjunction is identified with the meet operation and the logical disjunction with the join operation.

A three-valued logic, known as Kleene’s logic is developed in [13] and has three truth values, truth, unknown and false, where unknown indicates a state of partial vagueness. The semantic characterization of a four-valued logic for expressing practical deductive processes is presented in [14]. The Belnap’s logic has four truth values true (T), false (F), both (B), none (N). The meaning of these values can be described as follows: an atomic sentence is stated to be true only (T), an atomic sentence is stated to be false only (F), an atomic sentence is stated to be both true and false, for instance, by different sources, or in different points of time (B), and an atomic sentences status is unknown. That is, neither true, nor false (N). A logic lattice for the four truth values is shown on Fig. 1. A truth table for the ontological operation \( \land \) is

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Fig. 1. Logic lattice for the truth values in Belnap’s logic.
TABLE II
TRUTH TABLE FOR THE ONTOLOGICAL OPERATION ∨

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Fig. 3. Answer combinations related to the truth value tt

Fig. 4. Answer combinations related to the truth value ww

Fig. 5. Answer combinations related to the truth value pp

Fig. 2. Lattice of the five-valued logic

The committees responses for each question in a test is obtained applying operations for truth values in a five value logic, [16]. All possible responses are grouped in four sets as illustrated on Fig. 7. Each of the four sets contains five of the twenty possible answer combinations. Any five answer combinations within one set are arranged in a lattice like the one shown on Fig. 2. Thus all the twenty possible answer combinations are reduced to four cases that correspond to the truth values in Belnap’s logic. The final outcome is a result of the operation between the outcomes of all committees responses. Corresponding truth values are taken from the Belnap’s logic, [14]. Every response of a committee member concerning correspondence between subjects goals and tests content can be
- yes, it corresponds c,
- no it does not correspond i, or
- it corresponds to a certain degree p.
- The case where no answer is provided is denoted by n.

Thus we obtain the twenty response combinations.
Inner lattices - all answer combinations are grouped in the four lattices with respect to the five truth values, [16].
Any two nodes, in Fig. 3, Fig. 4, Fig. 5, and Fig. 6 connected by an edge differ in one answer only. Going upwards from one level to another in the lattices on Fig. 3, Fig. 4, Fig. 5, and Fig. 6 increases the level of knowledge.
Outer lattice - the lattice with the four truth values of Belnap’s logic from Fig. 1.
The nested lattice - the previously described twenty answer combinations are arranged in a nested lattice, Fig. 7. This lattice visualizes the outcomes for a test. The summarized result for a test after some new questions being introduced or old ones being corrected can move inside one of the four outer circles and implies small changes of its status within one of the four truth values. If it moves from one of the outer circles to another than it implies significant changes of the tests status.

IV. CONCLUSION
The proposed approach can be used to combine work in this case tests development done by different educators and follow the effect of the introduced changes. The advantages are that it can be done in automated way and does not limit the amount of changes. Instead it points to which direction these changes are leading, i.e. improving or not.

REFERENCES