Mathematical Modelling of Slurry Flow with Medium Solid Particles

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Abstract: - The paper deals with a solid-liquid turbulent flow with moderate and high solids concentration, which appears widely in chemical and mining industries. The slurry transported through a straight pipe contains medium solid particles of averaged diameters higher than 0.1 mm and lower than 0.8 mm surrounded by water as a carrier liquid. Mathematical model assumed that that flow is turbulent, fully developed and axially symmetrical with steady viscosity equal to carrier liquid viscosity and density depending on solids concentration. The model uses time averaged momentum equation and the problem of closure is solved by two-equation turbulence model, which includes a new turbulence damping function specially developed for such slurry flows. The mathematical model is suitable to predict slurry flow with medium solid particles and solids concentration from 10% to 40% by volume. Numerical predictions were compared with measurements showing good accuracy. Possible reason of turbulence damping which appears close to the pipe wall is discussed.

Key-Words: - experiments and predictions of slurry flow, modelling of slurry turbulence.

1 Introduction

Solid-liquid turbulent flow appears widely in chemical engineering, power plants, food and mining industries. Mathematical modelling of such flow, named slurry flow, is far away from the knowledge gathered from Newtonian flow and the turbulent slurry flow is still the main challenge of computational fluid dynamics. When considering predictions of solid-liquid turbulent flow the mixture theory models are the most general and are based on rigorous fluid mechanics frameworks, [1].

Settling slurry is formed mainly by coarse particles. However, it can exist as well for fine and medium particles if slurry bulk velocity is sufficiently low. When predicting the frictional head loss of slurry flow with coarse or medium particles, it is reasonable to assume the Newtonian model, as now one can measure rheology in such slurries, [2]. In coarse dispersive slurry flow one should mention basic researches of Bagnold, [3] and recent mathematical models, [4], [5] which include solid-solid and solid-liquid interactions.

Non-settling slurries contain fine particles with diameter below 0.05 mm and can form stable homogeneous mixture exhibiting increased apparent viscosity. They usually exhibit yield stress and require proper rheological model into the mathematical model. Those with very fine particles demonstrate increased viscous sublayer, which means that damping of turbulence appears in the near-wall region. In this case a mathematical model which includes apparent viscosity concept, and a suitable rheological model, and properly defined wall damping function is required, [6-9].

Slurries with medium solid particles of averaged diameters between 0.05 mm and 0.8 mm are usually assumed as Newtonian solid-liquid mixture or as a mixture of two separated phases. If slurry bulk velocity is sufficiently high even slurry with medium solid particles can exhibit non-settling type. It is important to emphasis that slurries with medium solid particles exhibit enhanced damping of turbulence since frictional head loss is almost equal or below that for carrier liquid flow.

It has been the endeavour of researchers around the world to develop accurate models for frictional head loss and velocity distribution in slurry flow. Frictional head loss is one of the most important technical parameters to be evaluated by the designers for designing a pipeline slurry transportation system, and the parameter which dictates the selection of pump capacity. Determining the most efficient and economical way out of pumping any solids in carrier liquid requires careful consideration and analysis of numerous factors, some of which can have a significant impact on performance and costs. Among them, there is an averaged solid particle diameter, solids concentration, particle density, deposition velocity, and properly matched characteristics of a pipeline and characteristics of a pump.
2 Literature Review

Influence of solid particles on turbulence in a slurry flow has been investigated experimentally by several researchers. Sumner et al., [10], Nasr-El-Din et al., [11], and Eskin and Miller, [12] measured solids concentration distribution in slurry flow with medium and coarse particles. They concluded that averaged particles diameter has a crucial influence on distribution of solids concentration across a pipe. If solid particles are coarse the solids concentration decreases at a pipe wall.

Slurry turbulence in the region close to a pipe wall was examined by Nouri and Whitehall, [13], Schreck and Kleis, [14]. Their research concluded that ejection–sweep cycle is affected strongly by particles. They concluded that slip velocity decreases with solids concentration increase. A review of experimental studies on turbulence modification by particles is given by Gore and Crowe, [15].

Sundaresan et al., [16], outlined a number of scientific challenges which represent building blocks for the comprehensive understanding of disperse flows encountered in a variety of technologies and in nature. Researchers concluded that new experiments and/or analyses are needed to cast light on the important phenomena that cause turbulence damping or generation. The authors suggested that the experiments should be conducted in simple turbulent flows such as grid turbulence, fully developed pipe or channel flow, or simple axisymmetrical flows. Regardless of geometry, experiments must include a wide range of particle parameters in a single fixed facility. Their conclusions are still outstanding in mathematical modelling of slurry flow.

The main objective of the paper is to present a mathematical model suitable to predict frictional head loss in a slurry flow with enhanced damping of turbulence.

3 Physical and Mathematical Model

Physical model assumes that the slurry comprises medium solid particles with averaged solid particles diameter \( d = (0.125; 0.240, 0.471, 0.780) \) mm and water as a carrier liquid. All solid particles are rounded and narrowly sized. The solid particle density is 2440 kg/m\(^3\) for Canasphere and 2650 kg/m\(^3\) for Sand. The solids concentration varies from \( C_v=10\% \) to \( C_v=40\% \). The slurry flow takes place in a horizontal pipe with sufficiently high bulk velocity, so the flow can be treated as non-settling and homogeneous. The slurry flow is stationary, turbulent, fully developed and axially symmetrical. It is assumed that slurry viscosity is equal to the carrier liquid viscosity while slurry density depends on solids concentration and is calculated as follows:

\[
\rho_{n} = \rho_c \left[ 1 - C_v \right] + \rho_s C_v
\]

The starting point of building mathematical model is the Navier-Stokes equation. Taking into account the physical model the time-averaged form of the Navier-Stokes equation, written in cylindrical co-ordinates, can be described as follows:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \left( \mu \frac{\partial U}{\partial r} - \bar{\rho}_u u'v' \right) \right) = \frac{\partial p}{\partial x}
\]

The component of the turbulent stress tensor, which appears in equation (2), is designated by Boussinesque hypothesis, as follows:

\[
-\bar{\rho}_u u'v' = \mu_t \frac{\partial U}{\partial r}
\]

The turbulent viscosity (\( \mu_t \)), stated in equation (3), is designated with support of dimensionless analysis, as follows, [17]:

\[
\mu_t = \mu \frac{k^2}{\varepsilon}
\]

The kinetic energy of turbulence (\( k \)) and its dissipation rate (\( \varepsilon \)), which appears in equations (2)-(4), are delivered from the Navier-Stokes equation. For the aforementioned assumptions the final form of \( k \) and \( \varepsilon \) equations, proposed by Launder and Sharma, [17], is the following:

- equation for kinetic energy of turbulence:

\[
\frac{1}{r} \left[ r \left( \mu + \frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial r} \right) + \mu_t \frac{\partial U}{\partial r} \right] = \bar{\rho}_u k + 2\mu_t \left( \frac{\partial U}{\partial r} \right)^2
\]

- equation for dissipation rate of kinetic energy of turbulence:

\[
\frac{1}{r} \left[ r \left( \mu + \frac{\mu_t}{\sigma_k} \frac{\partial \varepsilon}{\partial r} \right) + C_1 \frac{\varepsilon}{k} \left( \frac{\partial U}{\partial r} \right)^2 \right] = C_2 \left[ 1 - 0.3 \exp \left(-Re_f^2 \right) \right] \frac{\bar{\rho}_u k}{k} = 2\mu_t \left( \frac{\partial U}{\partial r} \right)^2
\]

The turbulent Reynolds number, which appears in equation (6), is defined using dimensionless analysis, as follows:

\[
Re_f = \frac{\bar{\rho}_u k^2}{\mu \varepsilon}
\]

The crucial point in the above turbulence model is proper determining of turbulence damping function (\( f_i \)) in equation (4). This function is also
known as the wall damping function. For a Newtonian flow Launder and Sharma, [17] proposed following empirical function:

\[ f_r = 0.09 \exp\left[ \frac{-3.4}{1 + \frac{Re_T}{50}} \right] \]  

(8)

For slurry flow with medium solid particles author proposed modification of such turbulence damping function by taking into account averaged particles diameter (d) and solids concentration (C_v). The new turbulence damping function is following:

\[ f_r = 0.09 \exp\left[ \frac{-3.4\left(1 + A_3\frac{d^3}{(8-88A_4d)C_v^x}\right)^2}{1 + \frac{Re_T}{50}} \right] \]  

(9)

where A_3 is an empirical constant.

The new turbulence damping function was developed on the basis of comparison between numerical predictions and measurements of global parameters, such as dp/dx=f(U_s), in slurry flow for comprehensive range of averaged solid particle diameters d=(0.125; 0.240, 0.471, 0.780) mm and for C_v=(10-40)%. The new empirical function (f_r), described by (9), approaches the standard turbulence damping function, described by (8), if solids concentration or averaged particles diameter goes to zero. This is in accordance with our assumption as the mathematical model should be suitable for single phase flow (d=0 and C_v=0%) and for slurry flow if (0.1<d<1) mm and (10 ≤ C_v ≤ 40) %.

Figure 1 presents the influence of averaged solid particles diameter and the turbulent Reynolds number on the new turbulence damping function (f_r) at C_v=40%.

The mathematical model was solved by finite difference scheme using own computer code. The set of differential equations (2), (5) and (6) is solved by means of TDMA method with iteration procedure using control volume method, [18]. Numerical calculations were carried out for 80 nodal points of the differential grid distributed on the radius of the pipe. The majority of the nodal points were localized in close vicinity of the pipe wall by introducing expansion coefficient. The number of grid points was set experimentally to provide nodally independent computations.

The mathematical model is able to predict frictional head loss and velocity distribution in slurry flow with medium solid particles of averaged diameter 0.1<d<1 mm and 0≤C_v≤40%. It is possible to extend the model to non-isothermal flow if a proper energy equation is used.
4 Numerical Predictions

The lack of measurements of slurry velocity fluctuations components in vicinity of a solid wall is a limiting factor in developing new turbulence models. However, it is possible to suggest modification of standard turbulence models on the basis of comparison between predictions and measurements of global parameters which is demonstrated in the paper.

Experiments of turbulent slurry flow with medium solid particles of averaged diameter from 0.125 mm to 0.780 mm demonstrate that frictional head loss significantly depends on particle size and solids concentration. Experiments showed that with averaged solid particles diameter increase from d=0.125 mm to 0.471 mm the frictional head loss decreases. If averaged solid particles diameter increases from d=0.471 mm to d=0.780 mm the frictional head loss increases too. Experiments proved that minimum value of the frictional head loss appears at d=0.471 mm and the frictional head loss is below that for carrier liquid flow. This is surprising as slurry frictional head loss supposed to be higher compared to pure carrier liquid flow. In such case one can say that enhanced turbulence damping appears. Such phenomenon is properly predicted if the new turbulence damping function is incorporated into the mathematical model.

With a view to proving that the mathematical model works well, the predictions and measurements of slurry frictional head loss are presented for two different types of solid particles, named Canasphere and Sand. Solid particle density of Canasphere and Sand is \( \rho_p=2440 \text{ kg/m}^3 \) and \( \rho_p=2650 \text{ kg/m}^3 \), respectively. The predictions and measurements are performed for turbulent slurry flow and for solids concentration \( C_V=(10-40)\% \) in inner pipe diameter \( D=0.026 \text{ mm} \). In order to demonstrate the importance of turbulence damping function used in the mathematical model, the numerical computations with new and with standard turbulence damping functions were performed.

Figure 2 demonstrates experiments of Shook and Bartosik, [19] and predictions of the frictional head loss using two different turbulence damping functions, named the standard (Std.) and the new (New), for slurry flow with averaged solids particle diameter \( d=0.240 \text{ mm} \) and \( C_V=30\% \). When the new turbulence damping function is taken into account, the numerical predictions and measurements are matching well while for the standard damping function they are too high compared to measurements.

![Fig.2. Prediction and experiments of frictional head loss for slurry (Canasphere) and for water flow. D=0.026 m, d=0.240 mm, \( \rho_p=2440 \text{ kg/m}^3 \), \( \mu_L=0.0008941 \text{ Pa s} \), \( C_V=30\% \).](image)

Fig.2. Prediction and experiments of frictional head loss for slurry (Canasphere) and for water flow. D=0.026 m, d=0.240 mm, \( \rho_p=2440 \text{ kg/m}^3 \), \( \mu_L=0.0008941 \text{ Pa s} \), \( C_V=30\% \).

It is interesting to present frictional head loss for Sand slurry of \( d=0.471 \text{ mm} \) and \( C_V=40\% \) - Fig.3. It is seen that measured frictional head loss is almost the same like for carrier liquid flow although the slurry density is very high and equal \( \rho_m=1659 \text{ kg/m}^3 \). In such case it is evident that using the standard turbulence model is unacceptable. The standard turbulence damping function gives too high wall shear stress in comparison with measurements of Sumner, [20]. In this particular case predicted frictional head loss which uses the standard turbulence damping function is almost 50% higher in comparison with the measurements. However, using the new turbulence damping function in the model, which depends on \( d \) and \( (C_V) \), causes that predictions and measurements match well. It is not shown in the paper but it is worthy to mention that for solids concentration in the range (10-30)\%, the slurry frictional head loss is significantly lower compared to carrier liquid flow.

![Fig.3. Prediction and experiments of frictional head loss for slurry (Sand) and for water flow. D=0.026 m, d=0.471 mm, \( \rho_p=2650 \text{ kg/m}^3 \), \( \mu_L=0.0008941 \text{ Pas} \), \( C_V=40\% \).](image)

Fig.3. Prediction and experiments of frictional head loss for slurry (Sand) and for water flow. D=0.026 m, d=0.471 mm, \( \rho_p=2650 \text{ kg/m}^3 \), \( \mu_L=0.0008941 \text{ Pas} \), \( C_V=40\% \).
Taking into account solid-liquid flow with averaged solid particles diameter equal $d=0.780$ mm and solids concentration equal $C_V=40\%$, it is seen in Fig.4 that turbulence damping process is less pronounced, compared to slurry flow with $d=0.471$ mm. In this case the measurements of the frictional head loss are slightly higher than for water. However, we expect that for slurry flow with density equal to 1659 kg/m$^3$, the frictional head loss should be significantly higher than it is. Therefore, one can conclude that turbulence damping appears for this case again, however, is much lower than for slurry flow with the same flow condition and $d=0.471$ mm. Comparing predictions using standard and new turbulence damping functions show high discrepancies. Figure 5 demonstrates that the prediction using the standard turbulence damping function give too high wall shear stress compared to the measurements of Sumner, [20]. In this particular case the mathematical model with the new turbulence damping function give high accuracy with measurements for moderate and high slurry bulk velocities. However, for low bulk velocity predictions are lower compared to measurements which could be due to laminarisation of the flow. This is not frightening as we avoid low bulk velocity in slurry flow with medium solid particles in order to ensure that sedimentation process will not exist.

### 5 Discussion and Conclusions

This paper demonstrates the influence of averaged solid particles diameter on damping of turbulence in solid-liquid flow with medium solid particles. Additional important role in damping of turbulence is played by the solids concentration.

Such conclusions have been made by analysing experimental data of the frictional head losses for comprehensive range of $d$ and $C_V$. The experiments clearly indicated that for slurry flow with averaged solid particles about 0.5 mm the frictional head loss has a minimum, depending on $C_V$, and is below data for carrier liquid flow. For averaged solid particles diameter below and above 0.5 mm the frictional head loss is higher than for carrier liquid flow but not as much as we expect on the basis of the slurry density.

When a slurry flow with medium solid particles is considered, there are evidences in literature that solids concentration decreases towards the pipe wall, [10], [20]. This is caused by lift forces, which act from the wall toward the symmetry axis. Such forces cause that the contact of solid particles with a pipe wall is less intensive. It is also known that the presence of solid particles in a carrier liquid can reduce the level of turbulence. Experiments of Schreck and Kleis proved that swirls, whose dimensions are lower compared to solid particle diameter, drastically reduce particles shade causing that the level of turbulence decreases, [14]. Unfortunately, there is no simple expression in literature which can resolve whether there is an increase or damping of turbulence.

It was proved in the paper that the mathematical model of slurry flow, which uses the standard turbulence damping function, is not suitable to predict a slurry flow with medium solid particles. However, it was also proved that using the mathematical model with the new turbulence damping function, depending on averaged solid particles diameter and solids concentration, is suitable to predict frictional head loss.

The mathematical model does not include separately: slip velocity between liquid and solid phase, lift forces acting on solid particles in close vicinity of a pipe wall, and bursting phenomena. However, the model includes aforementioned phenomena globally by taking into account the new turbulence damping function depending on $f_\mu=f(d, C_V)$.

### Notation:

- $A_S$ -- constant in turbulence damping function
- $C_i$ -- constant in Launder and Sharma turbulence model, $i=1, 2$
- $C_V$ -- solids concentration (volume fraction of solids averaged in cross section), %
- $d$ -- averaged solid particles diameter, mm
- $D$ -- inner pipe diameter, m
- $f_\mu$ -- turbulence damping function
- $k$ -- kinetic energy of turbulence, m$^2$/s$^2$
\( p \) – static pressure, Pa
\( r \) – distance from symmetry axis, m
\( U \) – velocity component in \( ox \) direction, m/s
\( u'v' \) – component of turbulent stress tensor, m/s
\( x \) – coordinate for \( ox \) direction, m
\( y \) – distance from the pipe wall, m
\( \bar{\cdot} \) – time averaged

**Greek Symbols:**
\( \varepsilon \) – rate of dissipation of kinetic energy of turbulence, \( m^2/s^3 \)
\( \mu \) – dynamic viscosity coefficient, Pa·s
\( \nu \) – kinematic viscosity coefficient, \( m^2/s \)
\( \rho \) – density, \( kg/m^3 \)
\( \sigma_i \) – diffusion coefficients in \( k-\varepsilon \) turbulence model, \( i = k, \varepsilon \)
\( \tau \) – shear stress, Pa

**Indexes:**
\( b \) – bulk (cross-section averaged value)
\( i \) – index, \( i = 1, 2 \)
\( L \) – liquid
\( m \) – slurry (solid-liquid mixture)
\( t \) – turbulent
\( w \) – solid wall

**References:**