Abstract – The paper reacts to recent calls in the simulation industry to replace hydraulic cylinders with electromechanical actuators while keeping the kinematic and dynamic parameters unaffected. This paper looks at the design and optimal control of an electromechanical linear actuator to be used in a six-degrees-of-freedom motion platform application intended for use in simulation technology. The paper provides a comparison of both system types with a description of the design of optimal control for electromechanical actuators.

Keywords – electromechanical linear actuator, Stewart platform, simulation technology, optimal control

I. INTRODUCTION

A. History

Motion platforms with six degrees of freedom, also known as hexapods, are possibly the most popular robotic manipulators used in simulation technology. This parallel mechanism was first described by V. E. Gough [1], who constructed an octahedral hexapod to test the behaviour of tyres subjected to forces created during airplane landings. The first document providing a detailed description of this structure used as an airplane cockpit simulator was published in 1965 by D. Stewart [2] (hence the name of the structure).

The Stewart platform is a closed kinematic system with six degrees of freedom and six adjustable length arms (see Fig. 1). Compared to other similar structures, its main advantage is high rigidity and a high input-power-to-device-weight ratio.

Until the nineties of the last century, the main obstacle hindering more intensive application development was the insufficient computing power of the available hardware. Determining the position of a hexapod is significantly more difficult than that of conventional serial structures. With regard to the device being controlled in real time, the main challenges are the transformation of coordinates and speed of resolving mathematical procedures [3]. Despite the computing power issue having been more or less removed in recent years, direct kinematic transformation (i.e. transforming the length of arms to the position of the frame) still remains a challenge.

B. Use in Simulation Technology

The Stewart platform is frequently used in simulation technology to simulate motion effects in vehicle or airplane simulators. By using this equipment, it is possible to simulate the forces acting upon the pilot (driver) during the flight (journey), thus bringing the simulator even closer to reality. The concept and role of motion effect simulation in training is discussed, for instance, in [4]; apart from describing the structure of simulators, the book expressly underlines the role of this aspect during emergency event training. With motion effects being generally perceived before other kinds of perceptions [5], they provide the first possibility of detecting undesired and dangerous behaviour of the airplane or vehicle.

C. Current Status

Due to the significant weight of the simulator cockpit fitted with the required audiovisual equipment and controls (the
The drawbacks of hydraulic systems include environmental aspects too. In particular, hydraulic oil has to be replaced after a certain number of operation hours and can, in cases of system malfunctions and breakdowns, cause local pollution. Therefore, any malfunctions of hydraulic systems have to be resolved with utmost care and attention.

Hence, in instances where it is envisaged that the device will be transported on a regular basis (such as the light sports aircraft simulator described in [7]) it is often preferable to use an electric motion system, which, being more affordable and requiring significantly shorter installation times, makes the device more attractive to customers.

Compared to hydraulic solutions, electric systems benefit from many advantages – in particular much less noise, higher energy efficacy and more sophisticated control methods.

However, besides their indisputable advantages, electric systems also have several disadvantages. The electromechanical transmission is subject to higher friction, increasing the wear and tear of the actuator. Therefore, the lifetime of electric systems is typically somewhat shorter than that of hydraulic solutions. In addition, electric systems require a procedure to bring the device to a safe halt after unexpected power cuts. In the case of hydraulic systems, the ‘safety landing’ procedure is catered for using oil from an appropriately sized hydraulic accumulator. With regard to UPS units significantly increasing the price of the system, power failure emergencies are typically handled using mechanical locks which, in case of an unexpected power cut, fix the platform in its current position. Nonetheless, emergency descents of platforms with an electric drive still remain problematic.

Due to their many pros, there have been growing calls to replace, in certain applications, hydraulic cylinders with electric linear actuators while keeping their static and dynamic properties. The first certified aircraft simulator in the world using an electric motion system was finished in 2006 and, according to [8], this trend will prevail in future as well.

The following section provides a detailed description of an electric linear actuator intended for the application mentioned above. The main emphasis has been put on the preparation of a mathematical model which will be subsequently used to design the optimal control.

II. ELECTROMECHANICAL LINEAR ACTUATOR

With regard to the forces required, the electromechanical actuator is based on converting rotary motion to linear motion via a ball screw. Benefitting from high efficiency, rigidity and accuracy ball screws are often used in machine tool construction. The design of the whole electromechanical actuator is shown in detail in Fig. 2.

**A. Mathematical Model of the Electromechanical Actuator**

In order to further analyse the electromechanical system and, in particular, with respect to the need to design the optimal control, this section deals with the mathematical model to be used. However, it is not necessary at this point to fine-tune all model parameters, the main aim being to ensure that the model reflects all significant dynamic properties of the electromechanical actuator and that it works with quantities which can be easily derived or directly measured in the system. The following assumptions were made before designing the mathematical model:

- for economical reasons, position or velocity is measured only at one location (motor shaft or ball screw). Therefore, the electromechanical system will be modelled as a system with one degree of freedom;
- with respect to the above, the model will ignore the torsional rigidity effects of the ball screw and the mechanical compliance of the connection between the shaft and the ball screw. As shown later by the control results, these factors have only a minor impact on the quality of control with respect to the criteria used to assess regulation quality.

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The differential equation of motion can be defined immediately after replacing the system with a single virtual body with the generalized mass $m_{red}$ subject to all forces and
moments $F_{red}$. Called the ‘Generalized Forces Method’, the basic equation can be written as:

$$m_{red} \cdot \ddot{x} + \frac{1}{2} \frac{dm_{red}}{dx} \dot{x}^2 = F_{red}.$$  \hfill (1)

In addition, the following properties apply (see, for instance, [9]):

- the kinetic energy of the generalized mass is equal to the sum of all kinetic energies of the elements of the system,
- the virtual work of the generalized force is equal to the sum of the virtual works of all forces and moments in the system.

The kinetic energy of the mechanical system can be expressed as:

$$E_k = \frac{1}{2} m \cdot \dot{x}^2 + \frac{1}{2} I \cdot \dot{\varphi}^2 = \frac{1}{2} m_{red} \cdot \dot{x}^2,$$

where $m$ is the mass of the load, $x$ the displacement of the ball screw, $I$ the moment of inertia of the rotary parts of the system and $\varphi$ the rotation angle.

The relationship between the translational position $x$ and the rotation angle is defined by the conversion constant $k_{mex}$, while:

$$\varphi = k_{mex} \cdot x.$$  \hfill (3)

By substituting into the previous expression, we obtain the following equation:

$$m_{red} = I \cdot k_{mex}^2 + m.$$  \hfill (4)

According to the virtual work principle, the following relationship holds true:

$$F_{red} \cdot \ddot{x} = \sum_{j=1}^{N} \delta W_j = M_h \cdot \ddot{\varphi} - m g \cdot \ddot{x} - F_f \cdot \ddot{x}$$

\hfill (5)

where $F_f = F_{fr} \text{sgn}(\dot{x})$ is the friction force.

The above expression can be used to determine the generalized force $F_{red}$:

$$F_{red} = M_h \cdot k_{mex} - m g - F_{fr} \text{sgn}(\dot{x}),$$  \hfill (6)

where $M_h$ is the motor torque, $m$ the load mass and $g$ the gravitational acceleration.

By substituting into the basic ‘Generalized Forces Method’ equation, we obtain the equation of motion for the mechanical part of the actuator:

$$(I \cdot k_{mex}^2 + m) \ddot{x} = M_h \cdot k_{mex} - m g - F_{fr} \text{sgn}(\dot{x}).$$  \hfill (7)

The driving torque $M_h$ is generated by an AC servomotor controlled by a servo driver. With regard to these devices being shipped by the manufacturer with the optimum current / moment regulator settings, the device shall be modelled as a first-degree dynamic system with the time constant $\tau$. With respect to small time constants, the remaining dynamic properties of the servo drive can be left out of consideration, playing only a negligible role in the overall behaviour of the system and being irrelevant for the control design. Servo drive dynamics can be expressed with the transfer function

$$G_{el}(s) = \frac{k_{el} - \frac{M_h(s)}{u(s)}}{\tau \cdot s + 1},$$  \hfill (8)

where $u$ is the variable corresponding to the required moment and, by the same token, the input signal of the system and $k_{el}$ the electric constant of the motor.

B. Design of the Optimal Control

For the sake of convenience, the mathematical model will be written using matrices of state and transformed into the discrete form. Non-linearity caused by friction forces can be compensated for by adding to or subtracting from the input signal $u$ the value $u_f$, corresponding to the friction force $F_{fr}$, and therefore will be left out of consideration. Below are the equations of state of the system:

$$\dot{x} = Ax + Bu$$

$$y = C \hat{x} + Du,$$  \hfill (9)

where

$$A = \begin{bmatrix} 0 & 0 & k_{mex} \tau & m_{red} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau} & k_{el} \\ 0 & 0 & \frac{1}{\tau} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -mg \\ m_{red} & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \hat{x} = \begin{bmatrix} \dot{x} \\ x \end{bmatrix}, \quad \hat{u} = \begin{bmatrix} u \\ 1 \end{bmatrix}.$$  \hfill (10)

The corresponding discrete form of the system for the sampling period $T$ can be written as:

$$\hat{x}((n+1)T) = A_d \hat{x}(nT) + B_d \hat{u}(nT)$$

$$y(nT) = C \hat{x}(nT) + Du(nT),$$  \hfill (11)

where
\[ A_d = e^{AT} \]
\[ B_d = e^{AT} \int_0^T e^{-At} dt \cdot B. \]  

(12)

In designing the optimal control, the following quality criteria will be considered:

- The regulated variable will be the displacement of the ball screw \( x \).
- In simulation technology (where the device in question is used to reproduce forces), system response time (i.e. transmission bandwidth) is the most important and essential quality parameter.
- With regard to small steady-state deviations from the desired position not being perceptible to persons sitting in the simulator cockpit, regulation accuracy is not critical in this application.
- In spite of this requirement not being as critical as, for instance, in machine tools control, where similar mechanisms involving ball screws are often used, unit step response should result in small overshoots only.
- Control has to be sufficiently robust to react flexibly to changes in the load \( m \), which can be a value from the pre-set interval \( m \in (0, m_{\max}) \).

In terms of control theory, it is advisable to use as much information about the controlled system as possible. Ideally, we should be able to either directly measure or somehow derive all state variables. In this case, the control law equals to (see, for instance, [10]):

\[ u = -K_x \cdot \dot{x} + K_r \cdot ref + u_0, \]  

(13)

where \( K_x \) is the row vector, \( K_r \) the scalar, \( ref \) the reference / desired position and \( u_0 = \frac{mg}{k_{el}k_{max}} \) the constant compensating the effects of the load.

It can be proved that using state space control (state feedback loops) it is theoretically possible to control the behaviour of the system as required [11]. In practice, however, one is limited by the input signal, which amounts to finite values from the range \( u \in (u_{\min}, u_{\max}) \).

The following sections focus on determining the optimal state space control to regulate the position of the electromechanical linear actuator. The optimal control is one which minimises the optimality criterion; let us now define a quadratic criterion based on the control quality requirements set forth hereinabove:

\[
\min_u \ J = \min_u \sum_{n=1}^N J_n = \min_u \sum_{n=1}^N J^y_n + J^u_n + J_{\text{overshoot}}^u
\]

(14)

where, at the optimisation horizon \( N \), \( J^y \) penalises the deviation from the desired position:

\[
J^y_n = q_1 \left[ y(n) - \text{ref}(n) \right]^2,
\]

with \( J^u \) penalising the input signal if it exceeds the allowed limits:

\[
J^u_n = \begin{cases} 
q_2 \left( u - u_{\max} \right)^2 & u > u_{\max} \\
q_2 \left( u - u_{\min} \right)^2 & u < u_{\min} \\
0 & u_{\min} \leq u \leq u_{\max}
\end{cases},
\]

and \( J_{\text{overshoot}} \) penalising the control in case of overshoots during positive unit step responses (i.e. non-monotonous responses):

\[
J_{\text{overshoot}}^u = \begin{cases} 
q_3 \dot{x}^2 & \dot{x} < 0 \\
0 & \dot{x} \geq 0
\end{cases}
\]

(17)

The quantities \( q_1, q_2 \) and \( q_3 \) are the masses and, at the same time, normalisation coefficients of individual elements of the criterion.

C. Minimising the Criterion via a Genetic Algorithm

Following on from the above, the control strategy in equation (13) is completely described by the vector \( K = [K_x \ K_r] \), with control quality being determined by the criterion \( J \). Hence, in determining the optimal control, the goal is to find the vector \( K \) with the minimum value of (the criterion) \( J \). This issue can be approached using a genetic algorithm employing the principles of evolution biology (crossbreeding and mutation) to solve complex problems. Each individual in the population is described by their “chromosome” (in our case the vector \( K \)), with the probability of this genetic information being passed to the next generation being directly proportional to its quality (lower values of the criterion \( J \)). In addition, there is a small chance that this genetic information will mutate (random changes of genes within the chromosome).

The initial population of several randomly chosen individuals will be left to evolve under the simple evolution rules defined above. After several generations, we select the best individual whose “chromosome” contains the ideal solution for our problem (essentially, we are ‘breeding’ the solution for the optimum control problem).

With regard to the nature of this paper, it is not possible to provide here comprehensive information on the properties of genetic algorithms and convergence conditions. However, a detailed description can be found, for instance, in [12].

The following chart (Fig. 3) shows the algorithm convergence when searching for the optimal control value after 400 generations.
Fig. 3: Determining the optimum control.

The criterion $J$ and the whole algorithm used to determine its minimum value can be easily implemented for example in the MATLAB-Simulink environment, as shown in Fig. 4.

Fig. 6 shows the simulation results (response to step changes of the position) of the optimal state regulator controlling the system under the parameters defined in Tab. I. These parameters correspond to the system shown in Fig. 2.

Tab. I: Parameters of the electromechanical actuator.

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{max}$</td>
<td>800π</td>
<td>rad/m</td>
</tr>
<tr>
<td>$k_{el}$</td>
<td>0.2</td>
<td>Nm/V</td>
</tr>
<tr>
<td>$l$</td>
<td>5.5429$\times$10$^4$</td>
<td>kg.m$^2$</td>
</tr>
<tr>
<td>$m$</td>
<td>&lt;0...512&gt;</td>
<td>kg</td>
</tr>
<tr>
<td>$g$</td>
<td>9.81</td>
<td>m/sec$^2$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.01</td>
<td>s</td>
</tr>
<tr>
<td>$T$</td>
<td>0.01</td>
<td>s</td>
</tr>
<tr>
<td>$u_{ref}$</td>
<td>0.4</td>
<td>V</td>
</tr>
<tr>
<td>$F_{ref}$</td>
<td>16g</td>
<td>N</td>
</tr>
</tbody>
</table>

Simulation results show that step responses do not result in overshoots and only small deviations from the desired steady-state position can be observed.

D. The Robustness Condition

The designed control has to ensure that the system remains stable after a load change and for all load values from the relevant interval.

It can be proved by simulation that the system will remain stable if the force exerted by the load does not exceed the maximum force which can be generated by the drive. Therefore:

$$m < \frac{u_{max} k_{el} k_{max}}{g} = m_{max}.$$  \hspace{1cm} (18)

If the above condition is met the system shall remain stable and load changes not compensated for by the input signal $u_0$ (see Equation (13)) will have an impact on the size of the steady-state deviation only.

E. Implementing the Control on a Real System

In order for state space control to be possible, all state variables have to be known in each control step. In practice, however, the electromechanical system is equipped with one sensor only, namely the position sensor fitted on the motor shaft. The remaining state variables (velocity and moment) have to be derived accordingly. Velocity can be determined by differentiating the current position, and the current moment at the shaft of the motor can be ascertained based on the input signal $u$ via relationship (8).

F. Comparing Dynamic Properties of Electromechanical and Hydraulic Systems

In this case, dynamic properties cover, in particular, the transmission bandwidth, in other words the input signal frequency range which can be transmitted by the unchanged / undamped system. In literature, this maximum frequency is defined as the frequency when the amplitude of the output signal drops to -3 dB.

Fig. 5 shows and compares the results of frequency characteristics measurements conducted for hydraulic and
electromechanical actuators. The position of the hydraulic actuator is controlled by a typical PID regulator, whereas the electromechanical actuator uses the results of the optimal state space regulator described here. As given in the Figure, the transmission frequency bandwidth of the electromechanical actuator equals to twice that of the hydraulic system.

III. CONCLUSION

This paper describes the process of designing an electromechanical actuator which can be used as a suitable replacement for hydraulic cylinders in simulation technology applications using six-degree-of-freedom motion platforms. Providing a description of the optimal state space control, the paper compares the operational and dynamic properties of hydraulic and electromechanical systems, proving that the latter can achieve better dynamics results. However, one has to take into consideration a decrease in the lifetime of the device as a consequence of increased wear and tear resulting from mechanical friction.

In addition, ensuring a safe shut-down procedure in case of unexpected power cuts can be quite costly. In spite of these drawbacks, current trends and customer demand show that electromechanical motion platforms will gradually replace current hydraulic systems, the main reasons, apart from better dynamic properties, being significantly lower noise, increased ease of installation and better energy efficiency.

Fig. 5: Measured frequency characteristics of the real system (signal 10 %).

![Frequency and Magnitude Graph](image1)

![Phase and Frequency Graph](image2)

Fig. 6: Simulation results.

REFERENCES