A re-examination of Okun's law in the presence of the shadow economy. An empirical investigation for the case of United States

Adriana AnaMaria Alexandru, Ion Dobre, Cătălin Ghinăraru

Abstract—The paper aims to investigate the relationship between unemployment rate and shadow economy with U.S. data using a re-examination of Okun's law. The shadow economy is estimated as percentage of official GDP, using MIMIC model. The size of the shadow economy as % of official GDP is estimated using a MIMIC model with four causal variables (taxes on corporate income, contributions for government social insurance, unemployment rate and self-employment) and two indicators (index of real GDP and civilian labour force participation rate).The size of the shadow economy (SE) is estimated to be decreasing over the last two decades.

We extend the classical Okun's law, in order to estimate the relationship between growth rate of official economy, unemployment rate and the size of the shadow economy. The results reveal a significant direct relationship between shadow economy and the unemployment rate and an indirect relation between shadow economy and growth of official sector.

***Keywords*—**shadow economy, unemployment rate, MIMIC models, Okun's law, United States.

I. INTRODUCTION

The relationship between the shadow economy and the level of unemployment is one of major interest. People work in the shadow economy because of the increased cost that firms in the formal sector have to pay to hire a worker. The increased cost comes from the tax burden and government regulations on economic activities. In discussing the growth of the shadow economy, the empirical evidence suggests two important factors: (a) reduction in official working hours, (b) the influence of the unemployment rate.

Enste [16] points out that the reduction of the number of working hours below worker's preferences raises the quantity of hours worked in the shadow economy. Early retirement also increases the quantity of hours worked in the shadow economy.

In Italy, Bertola and Garibaldi [6] present the case that an increase in payroll taxation can have effect on the supply of labour and the size of the shadow economy. An increase in tax and social security burdens not only reduces official employment but tends to increase the shadow labour force. This is because an increase in payroll tax can influence the decision to participate in official employment. Also, Boeri and Garibaldi [7] show a strong positive correlation between average unemployment rate and average shadow employment across 20 Italian regions between 1995-1999.

The paper examines the possible relationship between unemployment rate and the size of the shadow economy a re-examination of Okun's law.

II. ESTIMATING THE SIZE OF THE U.S. SHADOW ECONOMY

II.1. Data and Methodology

II.1.1. Data issues

The data series are quarterly, seasonally adjusted covering the period 1980:Q1 to 2009:Q2.

The series in levels or differences have been tested for unit roots using the Augmented-Dickey Fuller (ADF) test and PP tests. All the data has been differentiated for the achievement of the stationarity (appendix, unit root analysis). While all the variables have been identified like integrated on first order, the latent variable is estimated in the same transformation of independent variables (first difference).

II.1.2 Methodology

The size of the U.S. shadow economy is estimated as % of official GDP using a particular type of structural equations models-MIMIC model.

The MIMIC model- Multiple Indicators and Multiple Causes model (MIMIC model), allows to consider the SE as a "latent" variable linked, on the one hand, to a number of observable indicators (reflecting changes in the size of the SE) and on the other, to a set of observed causal variables, which are regarded as some of the most important determinants of the unreported economic activity [10].

The model is composed by two sorts of equations, the structural one and the measurement equations system. The equation that captures the relationships among the latent variable (η) and the causes (X_q) is named "structural model" and the equations that links indicators (Y_p) with the latent

variable (non-observed economy) is called the "measurement model".

A MIMIC model of the hidden economy is formulated mathematically as follows:

$$Y = \lambda\eta + \varepsilon \quad (1)$$

$$\eta = \gamma' X + \xi \quad (2)$$

where:

η is the scalar latent variable(the size of shadow economy);

$Y' = (Y_1,....Y_p)$ is the vector of indicators of the latent variable;

$X' = (X_1,...X_q)$ is the vector of causes of η ;

$\lambda_{(p\times 1)}$ and $\gamma_{(q\times 1)}$ vectors of parameters;

$\varepsilon_{(p\times 1)}$ and $\xi_{(q\times 1)}$ vectors of scalar random errors;

The $\varepsilon' s$ and ξ are assumed to be mutually uncorrelated.

Substituting (2) into (1), the MIMIC model can be written as:

$$\mathrm{Y} = \Pi\mathrm{X} + \mathrm{z} \quad (3)$$

where: $\Pi = \lambda\gamma^{'}, z = \lambda\xi + \varepsilon$.

The estimation of (1) and (2) requires a normalization of the parameters in (1), and a convenient way to achieve this is to constrain one element of λ to some pre-assigned value ([17]-[18]).

The possible causes of shadow economy considered in the model are: tax burden decomposed into personal current taxes (X_1), taxes on production and imports(X_2), taxes on corporate income(X_3), contributions for government social insurance(X_4) and government unemployment insurance(X_5), unemployment rate(X_6), self-employment in civilian labour force (X_7), government employment in civilian labour force (X_8) called bureaucracy index. The indicator variables incorporated in the model are: real gross domestic product index (Y_1), currency ratio M_1/M_2 (Y_2) and civilian labour force participation rate (Y_3).

The variables used into the estimation of the shadow economy are also quarterly and seasonally adjusted covering the period 1980-2009. All the data has been differentiated for the achievement of the stationarity.

In order to estimate the MIMIC model, by Maximum Likelihood, using the LISREL 8.8 package, we normalized the coefficient of the index of real GDP ($\lambda_1 = -1$) to sufficiently identify the model. This indicates an inverse relationship between the official and shadow economy.

In order to identify the best model, we have started with MIMIC model 8-1-3 and we have removed the variables which have not structural parameters statistically significant.

A detailed description and implementation of the MIMIC model for the USA shadow economy is provided in [15].

II.2. Empirical results

In order to estimate the size of the shadow economy, we have identified the best model as MIMIC 4-1-2 with four causal variables (taxes on corporate income, contributions for government social insurance, unemployment rate and self-employment) and two indicators (index of real GDP and civilian labour force participation rate).

Taking into account the reference variable ($Y_1, \frac{\mathrm{Re}\,al\,GDP_t}{\mathrm{Re}\,al\,GDP_{1990}}$) the shadow economy is scaled up to a value in 1990, the base year, and we build an average of several estimates from this year for the U.S.A. shadow economy (table I).

The index of changes of the shadow economy (η) in United States measured as percentage of GDP in the 1990 is linked to the index of changes of real GDP as follow:

Measurement Equation: $$\frac{GDP_t - GDP_{t-1}}{GDP_{1990}} = \frac{\tilde{\eta}_t - \tilde{\eta}_{t-1}}{GDP_{1990}} \quad (4)$$

I: Estimates of the size of U.S.A. shadow economy (1990)

Author	***Method***	***Size of Shadow Economy***
Johnson et. Al(1998)	Currency Demand Approach	13.9%
Lacko(1999)	Physical Input(Electricity)	10.5%
Schneider and Enste(2000)	Currency Demand Approach	7.5%*
Mean 1990		***10.6%***

*means for 1990-1993

The estimates of the structural model are used to obtain an ordinal time series index for latent variable (shadow economy):

Structural Equation:

$$\frac{\tilde{\eta}_t}{GDP_{1990}} = -0.24\Delta X_{3t} + 3.00\Delta X_{4t} + 1.49\Delta X_{6t} + 1.01\Delta X_{7t} \quad (5)$$

The index is scaled to take up to a value of 10.6% in 1990 and further transformed from changes respect to the GDP in the 1990 to the shadow economy as ratio of current GDP:

$$\frac{\tilde{\eta}_t}{GDP_{1990}} \times \frac{\eta^*_{1990}}{GDP_{1990}} \times \frac{GDP_{1990}}{\tilde{\eta}_{1990}} \times \frac{GDP_{1990}}{GDP_t} = \frac{\hat{\eta}_t}{GDP_t} \quad (6)$$

ISBN: 978-1-61804-034-3

I. $\frac{\tilde{\eta}_t}{GDP_{1990}}$ is the index of shadow economy calculated by eq.(2);

II. $\frac{\eta^*_{1990}}{GDP_{1990}} = 10.6\%$ is the exogenous estimate of shadow economy;

III. $\frac{\tilde{\eta}_{1990}}{GDP_{1990}}$ is the value of index estimated by (5);

IV. $\frac{GDP_{1990}}{GDP_t}$ is to convert the index of changes respect to base year in shadow economy respect to current GDP;

V. $\frac{\hat{\eta}_t}{GDP_t}$ is the estimated shadow economy as a percentage of official GDP.

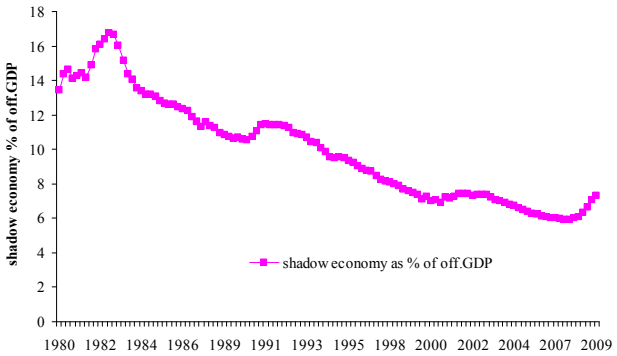

Fig. 1. The size of the shadow economy in United States as % of official GDP

The shadow economy measured as percentage of official GDP records the value of 13.41% in the first trimester of 1980 and follows an ascendant trend reaching the value of 16.77% in the last trimester of 1982. At the beginning of 1983, the dimension of USA shadow economy begins to decrease in intensity, recording the average value of 6% of GDP at the end of 2007. For the last two year 2008 and 2009, the size of the unreported economy it increases slowly, achieving the value of 7.3% in the second quarter of 2009.

The results are not far from the last empirical studies for USA ([16], [26]).Schneider estimates in his last study, the size of USA shadow economy as % of GDP, at the level of 7.9% in 2005, respectively 8% in 2006.

II. A RE-EXAMINATION OF OKUN'S LAW IN THE PRESENCE OF THE SHADOW ECONOMY

The Okun's law relates decreases in the unemployment rate to increases in output growth. We want to test if the shadow economy has any significant effect on this empirical evidence. We go on the hypothesis that a lower growth rate of official GDP from potential output is associated with higher deviations of the unemployment rate from its "natural" level. The increase in unemployment leads to an increase in the number of laborers who work in the unofficial labour market.

The graphical evolution of the shadow economy versus unemployment rate reveal the existence of a strong positive relationship between the two variables, quantified by a value of about 0.80 of correlation coefficient. The unemployment rate is expressed in %, taken from U.S. Bureau of Statistics, Labour Force Statistics from Current Population Survey.

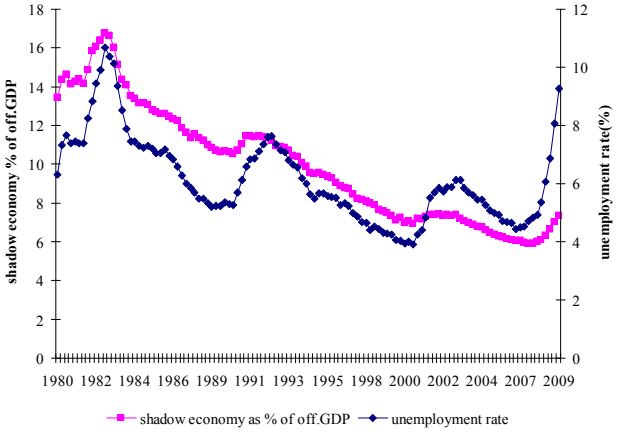

Fig.2. Shadow economy vs. Unemployment rate in United States

Giles ([17], [18]) states that the effect of unemployment on the shadow economy is ambiguous (i.e. both positive and negative). An increase in the number of unemployed increases the number of people who work in the black economy because they have more time. On the other hand, an increase in unemployment implies a decrease in the shadow economy. This is because the unemployment is negatively related to the growth of the official economy (Okun's law) and the shadow economy tends to rise with the growth of the official economy

In fig.1(appendix), we present the significant statistical relationships among growth rate of official GDP, changes in unemployment rate and growth of shadow economy for the case of United States covering the period 1980-2009.

The estimates obtained based on the standard relation given by Okun's law are presented in the following table:

$$g_t^Y = \alpha_0 \Delta u_t + \varepsilon_t \quad (10)$$

where:

$g_t^Y = (g_t^{off} - \bar{g}^Y_{(80-09)})$ indicates the difference of growth rate of the official gross domestic product (g_t^{off}) from it average calculated over the period 1980 to 2009;

$g_t^{\eta} = (g_t^{shad} - \overline{g}^{\eta}_{(80-09)})$ indicates the difference of shadow economy(g_t^{shad}) from it average calculated over the period 1980 to 2009, Δu_t id the first difference of unemployment rate, ε_t are residuals i.i.d.

III. Estimation output of regression:

$$g_t^{Y} = \alpha_0 \Delta u_t + \varepsilon_t$$

Dependent Variable: G_GROWTH
Method: Least Squares
Date: 06/02/11 Time: 12:41
Sample: 1980Q2 2009Q2
Included observations: 117

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DU	-6.872256	0.587002	-11.70738	0.0000

R-squared	0.541616	Mean dependent var	-0.001795
Adjusted R-squared	0.541616	S.D. dependent var	3.167911
S.E. of regression	2.144806	Akaike info criterion	4.372485
Sum squared resid	533.6223	Schwarz criterion	4.396094
Log likelihood	-254.7904	Hannan-Quinn criter.	4.382070
Durbin-Watson stat	2.006326		

The estimates show an inverse relationship between changes in unemployment and the growth rate of official output. Furthermore, we use a modified version of Okun's law by including the shadow economy:

$$g_t^{Y} = \alpha_1 \Delta u_t + \beta g_t^{\eta} + \varepsilon_t \quad (11)$$

IV. Estimation output of regression:

$$g_t^{Y} = \alpha_1 \Delta u_t + \beta g_t^{\eta} + \varepsilon_t$$

Dependent Variable: G_GROWTH
Method: Least Squares
Date: 06/02/11 Time: 12:41
Sample: 1980Q2 2009Q2
Included observations: 117

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DU	1.101578	0.859603	1.281497	0.2026
G_ETA	-1.402706	0.132050	-10.62249	0.0000

R-squared	0.768632	Mean dependent var	-0.001795
Adjusted R-squared	0.766620	S.D. dependent var	3.167911
S.E. of regression	1.530398	Akaike info criterion	3.705879
Sum squared resid	269.3437	Schwarz criterion	3.753096
Log likelihood	-214.7939	Hannan-Quinn criter.	3.725049
Durbin-Watson stat	2.004615		

The econometric results reveal that we have a significant negative relationship between the growth rate of official economy and the size of the shadow economy. Instead, in the presence of shadow economy the coefficient of Okun's law is not statistically significant (prob.>0.05).

In order to investigate the impact of shadow economy on the unemployment rate, we develop a structural relationship, taking into account also the growth rate of official GDP:

$$g_t^{shad} = \gamma g_t^{off} + \lambda \Delta u_t + \varepsilon_t \quad (12)$$

where:
(g_t^{off}) is the first difference of annual growth rate of the official gross domestic product;
g_t^{shad} is the first difference of the shadow economy;
Δu_t is the first difference of unemployment rate; ε_t residuals;

V. Estimation output of regression:

$$g_t^{shad} = c + \gamma g_t^{off} + \lambda \Delta u_t + \varepsilon_t$$

Dependent Variable: G_SHAD
Method: Least Squares
Date: 06/02/11 Time: 12:41
Sample: 1980Q2 2009Q2
Included observations: 117

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.380005	0.119943	3.168216	0.0020
G_OFF	-0.349935	0.033285	-10.51335	0.0000
DU	3.298842	0.311691	10.58369	0.0000

R-squared	0.881661	Mean dependent var	-0.491138
Adjusted R-squared	0.879585	S.D. dependent var	2.208503
S.E. of regression	0.766369	Akaike info criterion	2.331001
Sum squared resid	66.95469	Schwarz criterion	2.401826
Log likelihood	-133.3636	Hannan-Quinn criter.	2.359755
F-statistic	424.6681	Durbin-Watson stat	2.291181
Prob(F-statistic)	0.000000		

The parameter γ of the equation shows an inverse relationship between the growth of the official economy (g_t^{off}) and growth of the shadow economy (g_t^{shad}). On the other-hand, the parameter λ shows a direct relationship between changes in unemployment and the growth of the shadow economy.

The coefficients are statistically significant (prob.<5%) and the degree of determination in the model is high, 88% of the variation of shadow economy is explained by the two exogenous variables unemployment rate and growth rate of official GDP.

Our estimations show that the presence of the shadow economy acts as a buffer as it absorbs some of the unemployed workers from the official economy into the shadow economy.

ISBN: 978-1-61804-034-3

IV. CONCLUSIONS

The main goal of the paper is to investigate the nature of the relationship between unemployment rate and the size of the shadow economy of the USA data using a re-examination of Okun's law. The shadow economy is estimated as percentage of official GDP, using MIMIC model. The results show that the size of the shadow economy varies from thirteen to seventeen percent between 1980 and 1983 and then decreases steadily up to 7 percent of official GDP in 2009.

We extend the classical Okun's law, in order to estimate the relationship between growth rate of official economy, unemployment rate and the size of the shadow economy. The results reveal a significant direct relationship between shadow economy and the unemployment rate and an indirect relation between shadow economy and growth of official sector.

Moreover, we can conclude that employment in the shadow economy constitutes a form of labor market transition between or rather from unemployment back into formal employment.

REFERENCES

[1] A, Alexandru, I., Dobre, C., Ghinararu, "*Revisiting the relationship between Unemployment rate and the size of the shadow economy for United States using Johansen Approach for Cointegration"*, Proceedings of the 11th WSEAS International Conference on Mathematics and Computers in Business and Economics, Iasi, Romania, june 13-15, 2010, pg.199-204, ISBN 1790-2769.

[2] A., Alexandru, I., Dobre, C., Ghinararu, "The relationship between unemployment rate and the size of the shadow economy. The case of United States", Wseas Transactions on Business and Economics, issue 4, vol.7, pg.359-369, october 2010, ISSN: 1109-9526.

[3] A., Alexandru, I., Dobre, C., Ghinararu. "The relationship between shadow economy and unemployment rate: a SVAR approach" Proceedings of 5th WSEAS International Conference on Economy and Management Transformation, Timisoara, Romania, october 23-26, 2010, pg.242-249, ISSN: 1792-5983.

[4] M.E., Andreica, L., Aparaschivei, A., Cristescu, and N., Cataniciu "Models of Minimum Wage Impact upon Employment, Wages and Prices: The Romanian Case", in Proc. of the 11th WSEAS Int. Conf. Mathematics & Computers in Business & Economics, Iasi, Romania, 2010, pp. 104-109.

[5] L., Aparaschivei, M.E., Andreica, A.,, Cristescu, N., Cataniciu, "Effects of the Real Minimum Wage upon Employment and Labour Supply", in Proc. of the 5th WSEAS Int. Conf. on Economy and Management Transformation, Timisoara, Romania, 2010, pp. 213-218

[6] G., Bertola, P., Garibaldi, *The Structure and History of Italian Unemployment*, *CESifo Working Papers*, n.907, 2003.

[7] T., Boeri, P., Garibaldi *Shadow Activity and Unemployment in a Depressed Labor Market*, *CEPR Discussion papers,* n.3433, 2002.

[8] R., Dell'Anno *Estimating the shadow economy in Italy: A structural equation approach*, Working Paper 2003, Department of Economics, University of Aarhus.

[9] R., Dell'Anno, M., Gomez, M., A., Alañón Pardo *Shadow economy in three different Mediterranean countries: France, Spain and Greece. A MIMIC approach*, Empirical Economics 33/2005, pp. 51-84.

[10] R. Dell'Anno, F. Schneider *The Shadow Economy of Italy and other OECD Countries: What do we know?*, Mimeo, 2004.

[11] R., Dell'Anno, O., Solomon Shadow economy and unemployment rate in USA. Is there a structural relationship?, Annual Meeting of the European Public Choice Society, Finland, April 20-23, 2006

[12] D., Dickey, D., W.A., Fuller Likelihood ratio statistics for autoregressive time series with a unit root, Econometrica, Vol. 49, 1981, pp. 1057-72.

[13] I., Dobre, A. Alexandru The impact of unemployment rate on the dimension of shadow economy in Spain: a Structural Equation Approach, European Research Studies Journal, vol. XIII, no. 4/2009, pg.179-197, ISSN: 1108-2976.

[14] I., Dobre, A. Alexandru *Estimating the size of the shadow economy in Japan: A structural model with latent variables*, Economic Computation and Economic Cybernetics Studies and Research, vol.43 no.1/2009, pg.67-82, ISSN 0424 – 267 X.

[15] I., Dobre, A., Alexandru *A nonparametric analysis of the relationship between unemployment rate and shadow economy using local polynomial regression models*, Economic Computation and Economic Cybernetics Studies and Research, vol.44, no.1/2010, pg.21-44, ISSN 0424 – 267 X.

[16] D.H., Enste *Shadow Economy and Institutional Change in Transition Countries* in Boyan Belev (eds.), *The Informal Economy in the EU Assessment Countries: Size, Scope, Trends and Challenges of the Process of EU-enlargement*, Center for Study of Democracy, 2003, Sofia, 81-114.

[17] D.E.A., Giles *Measuring the hidden economy: Implications for econometric modeling*, The Economic Journal, vol.109, no. 456/1998 pp.370-380.

[18] D.E.A., Giles Modeling *the hidden economy in the tax-gap in New Zealand*, Empirical Economics, vol.24, no.4/1999, pp.621-640.

[19] H., Hsu *Network View of Capital Market Integration and Disintegration- An Example by VAR Model*, Proceedings of the 10th WSEAS Int. Conf. on mathematics and computers in business and economics (MCBE'09), Venice, Italy, November 15-17, 2004.

[20] S., Johnson, D., Kaufmann, P., Zoido-Lobatón *Regulatory discretion and the unofficial Economy*, The American Economic Review, vol.88, no.2/1998, pp. 387-392.

[21] K., Jöreskog, A.S., Goldberger *Estimation of a model with multiple indicators and multiple causes of a single latent variable*, *Journal of the American Statistical Association*, 70/1975, pp.631-639.

[22] K., Jöreskog, D., Sörbom *LISREL 8 User's Reference Guide* (Scientific Software International, Chicago), 1993.

[23] M., Lackó *Hidden economy an unknown quantitiy? Comparative analyses of hidden economies in transition countries in 1989-95*, working paper 9905, Department of Economics, University of Linz, 1999.

[24] P.B., Phillips, P., Perron *Testing for a unit root in time series regression*, Biometrica, Vol. 75, 1985, pp. 335-346.

[25] F., Schneider, D.H., Enste *Shadow economies: size, causes and consequences*, Journal of Economic Literature 38, 2000, pp. 77-114.

[26] F., Schneider *Shadow Economies and Corruption all over the world: New estimates for 145 Countries,* Economics, 2009, pp. 1-47.

*** www.bea.gov , U.S. Economic Accounts

*** www.bls.gov , U.S. Department of Labour Statistics

*** Eviews 6.0 software

*** Lisrel 8.8 package

*** R 2.9.1 software

APPENDIX

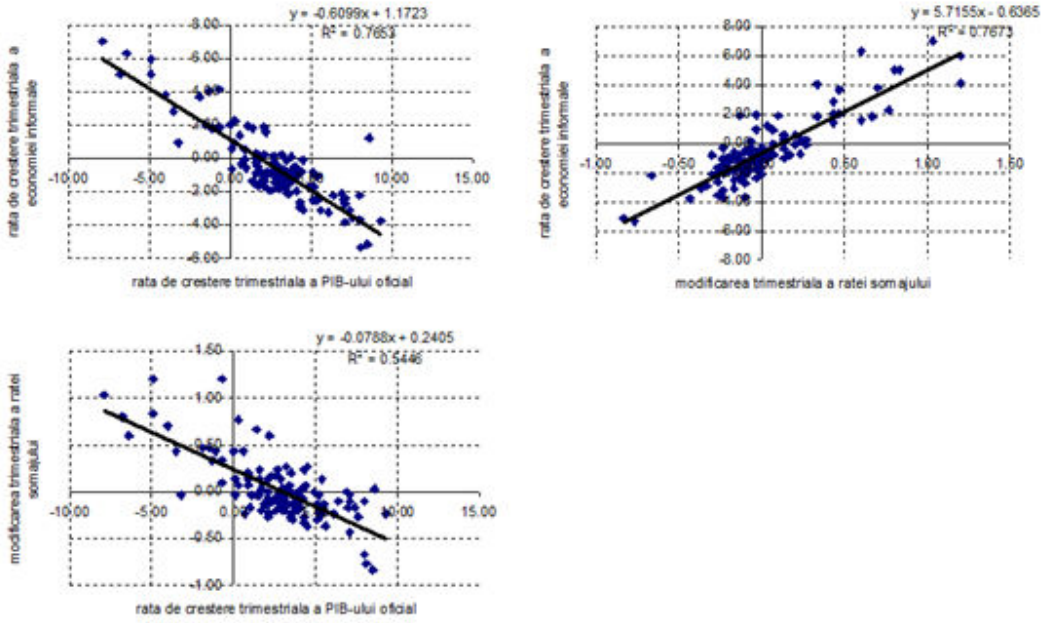

Fig.1. Growth of official GDP, changes in unemployment and growth of shadow economy

ISBN: 978-1-61804-034-3