A Fundamental Conception to Formulate Image Data Hiding Scheme Based on Error Diffusion

Masakazu Higuchi, Shuji Kawasaki, Jonah Gamba, Atsushi Koike and Hitomi Murakami

***Abstract*—In this paper, we consider to formulate an image data hiding scheme based on error diffusion. By using the scheme, visual patterns are embedded in a halftone image without affecting its perceptual quality depending on a halftone key image. The scheme has the feature of visual cryptography with respect to extracting of the embedded patterns, i.e., when we print the two halftone images onto transparencies and stack them together, the embedded patterns are visible by human visual system without any special electronic calculation. We propose a formulation for the scheme in the view of a stochastic analysis. The idea is basic, but theoretical studies by formulating is important trial in this field.**

***Keywords*—Data hiding, Visual cryptography, Halftone image, Error diffusion, Probability theory.**

I. Introduction

THERE are techniques that embed digital data in various multimedia data (music, image, video and so on). The embedded data are visible or invisible, and they can be extracted with some procedure. In this paper, we consider the case that visual patterns are embedded in some images and the embedded patterns are invisible to humans, i.e., this case is image data hiding.

On the other hand, visual cryptography is a technique which encrypts a secret image into plural share images such that when some of the share images are overlaid, the secret image will be revealed. The decryption can be performed by human visual system without any special electronic calculation for decryption. The first visual cryptographic technique was developed by Noar and Shamir in [3]. They have developed the scheme generates share images with not meaningful random dot pattern.

In recent years, data hiding techniques with the feature of visual cryptography with respect to decryption have been studied actively. The techniques embed secret visual patterns in some halftone images such that when the halftone images are overlaid, the embedded patterns can be viewed directly on the halftone images. There have been many reports on the studies in [4]–[13]. Fu and Au have dealt with binary or ternary images like text images as secret visual patterns in [6], while other many researchers have studied about natural gray-scale images like photographs as secret visual patterns in [5], [7]–[13]. Also, Koga and Yamamoto have challenged to handle color secret images in [4].

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In [5]–[13], halftonings have been used in order to embed secret data. Halftoning is a process of converting a gray scale image into a binary image. It is well-known that error diffusion is one of halftoning techniques and generates a halftone image with apparently high quality from a multivalued image [1]. Error diffusion has been used as halftoning in [5]–[7], [9]–[13]. Applying error diffusion makes noise less noticeable which arises by embedding information of secret data into halftone images. Accordingly secret data are embedded in halftone images without affecting their perceptual quality.

In [7], [9]–[13], Myodo et al. and authors have achieved to extract secret data with apparently high quality by modifing Fu's and Au's method in [6] and placing the process of giving appropriate transformations to original input data in the modified method. Myodo et al. have improved Fu's and Au's method and changed intensities of each pixel of input images in the first step of the method by affine transformations in [7]. Authors have applied the histogram equalization to input secret data and adjusted parameters of affine transformation to input images according to the properties of those images based on Myodo's method in [9]–[13].

The main theme of studies in the field are to embed secret visual patterns in halftone images without affecting their perceptual quality such that the embedded patterns can be restored with apparently high quality when the halftone images are overlaid. In order to solve the theme, it is necessary to theoretically investigate behavior of data embedding algorithm. However, it is difficult because of nonlinearity of halftoning, change of pixel values of images according to various conditions, difficulty of representing variations in intensities of images as functions, and so on. In the previous works, parameters affect the performance of data embedding algorithm have been determined experimentally. Authors have tried formulating Fu's and Au's data hiding scheme using a stochastic method in [14]. Fu and Au have dealt with binary or ternary images like text images as secret data. Their method also demonstrates relatively good performance for secret data like natural gray-scale images by giving appropriate transformations to original input data.

In this paper, we review the formulation in [14]. It is difficult to represent variations in intensity of an image as a function, so we represent it as a probability density function based on the relative frequency distribution of intensities of an image. Also we use the similar technique to formulate errors arise in error diffusion. To model the behavior of change of pixel values of images depending on various conditions, it is effective to use probability distributions. Formulating data embedding algorithms would allow us to theoretically

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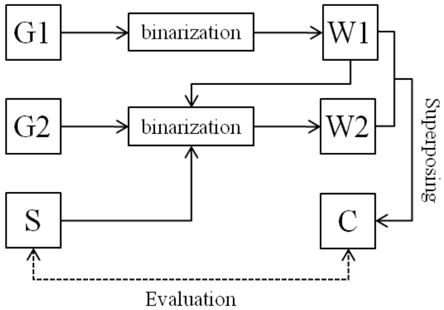

Fig. 1. The flowchart of Fu's and Au's method

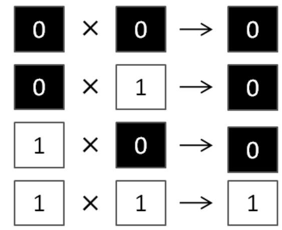

Fig. 2. Superposing two binary pixels (which is the same as logical product of two pixels in Boolean algebra)

investigate behavior of the algorithm. For example, we might be able to find easily pairs of input image such that secret data can be embed in halftone images without affecting their perceptual quality and the embedded data can be restored with apparently higher quality when the halftone images are overlaid. Moreover, we might be able to optimize parameters affect the performance of data embedding algorithm.

II. Image data hiding scheme based on error diffusion

A. The flow of hiding scheme

We focus on Fu's and Au's method in [6]. The method is as follows: It takes three images as input and generates two output halftone images which correspond to two of the three input images. The other one is a secret image. This is reconstructed as a halftone image by stacking (superposing) the two output images together. This operation corresponds to logical product in Boolean algebra.

The flowchart of the method is shown in Fig.1. G1, G2 and S in Fig.1 are input images and have the same size. G1 and G2 are gray-scale images like photographs. S is a secret image with binary or ternary tones, but may be a gray-scale image. W1 and W2 are output halftone images. W1 is produced from G1 by using regular error diffusion with high quality. Then, by binarizing G2 depending on information of W1 and S by using error diffusion, W2 is generated with high quality. It means that pixel arrangements of pixel domains in intermediate image of producing W2 change depending on information of W1 and S. C is obtained by superposing W1 on W2. This is the halftone image corresponding to S The qualities of W2 and C are evaluated by, for example, PSNRs (Peak Signal-to-Noise Ratio) of pairs (G2, W2) and (S, C), respectively. In this method, secret data are embedded in only W2, and W1 is the key image to make the secret data on halftone images.

In this paper, the intensity of a pixel in an image is from 0 to 1. If the intensity is 0, the pixel is black pixel. On the other hand, if the intensity is 1, the pixel is white pixel. Superposing two binary pixels corresponds to calculating logical product of these pixels in Boolean algebra (see in Fig.2).

B. Error diffusion

Error diffusion is a halftoning technique, which can generate a high quality halftone image from a multi-tone, like gray-scale, image. It is a causal single-pass sequential algorithm. Multi-tone images are halftoned line-by-line sequentially. In this algorithm, the past errors are diffused back to the current pixel.

In error diffusion, the process at k-th pixel in an image is as follows:

$$u_k = x_k + ER_k, \quad ER_k = \sum_{i \in J_k} j_i e_i, \tag{1}$$

$$u_k \to w_k = \begin{cases} 0 & (u_k < T) \\ 1 & (u_k \geq T) \end{cases}, \tag{2}$$

$$e_k = u_k - w_k, \tag{3}$$

where x_k denotes the intensity of k-th pixel in an image, and ER_k denotes the accumulative error at k-th pixel in an image, and e_i denotes the error which arises at a pixel before k-th pixel, and j_i denotes a weight of diffusing error, and J_k denotes a pixel domain in an image depending on k-th pixel. Equation (2) expresses the operation of quantization on a threshold T.

C. Data embedding process

In Fu's and Au's method, secret data are embedded in W2 when binarizing G2. If u_k in (1) satisfies the following inequality:

$$T - \Delta u < u_k < T + \Delta u, \tag{4}$$

where Δu denotes a positive constant, then information of k-th pixel in S is embedded at k-th pixel position in W2. Let $w1_k$, $w2_k$ and s_k be intensities of k-th pixel in W1, W2 and S, respectively. The data embedding rule is as follows:

- $w2_k = \overline{w1_k}$ if $s_k^{v3} = 0$ (black),
- $w2_k$ is output of error diffusion at k-th pixel in G2 if $s_k^{v3} = 0.5$ (gray),
- $w2_k = w1_k$ if $s_k^{v3} = 1$ (white),

where s_k^{v3} denotes the value obtained by converting s_k into a ternary value. By this rule, the location in C corresponding to a dark location in S becomes black, and the location in C corresponding to a light location in S becomes gray, and the location in C corresponding to a location like gray in S becomes darker than gray. Therefore, C tends to become a darkish image over all.

D. Example

We show an example of results obtained by using Fu's and Au's method in Fig.3. In this example, S is not an image

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G1 W1 G2 W2 S C

Fig. 3. An example of results by Fu's and Au's method

with binary or ternary tones but a gray-scale image. W1 and W2 are apparently high quality and also C is reconstructed by superposing W1 and W2 with apparently high quality (see in Fig.3). In this case, appropriate transformations are given to input images. The intensities of each pixel in G1 and G2 are converted to around 0.5 by the affine transformation $g' = 0.45g + 0.275$ where g denotes the intensity of a pixel in G1 or G2. The intensity of a pixel in S is transformed into below 0.45 by the affine transformation $s' = 0.45s$ where s denotes the intensity of a pixel in S. The thresholds of quantization to ternary image for S are 0.25 and 0.75. The threshold of quantization to binary image for G1 and G2 is $T = 0.5$. The parameter of embedding data is $\Delta u = 0.067$.

III. Formulation of image data hiding scheme using a stochastic method

A. Preliminary

We formulate Fu's and Au's scheme by using a stochastic method in this section. For the sake of simplicity, we omit intensity transformations for input images. They are not essence of the formulation. If it is necessary to include appropriate transformations in the formulation, replace x with $x' = Tr(x)$ in the formulation, where x denote the intensities of each pixel in input images, and Tr denote appropriate transformations.

We characterize an image by its relative frequency distribution of intensities. We consider intensities of an image to be a random variable. Then the variable has the probability density function obtained from the relative frequency distribution of the intensities of the image corresponding to the variable since the intensities of an image are continuous values on $[0, 1]$. (Theoretically, the relative frequency distribution of the intensities of an image is the same as the probability density function.) Also we use the similar technique to analyze the statistical behavior of errors arise in error diffusion because the errors depend on images. In the paper, a probability density function that a continuous random variable X has is expressed in f_X.

Let U_k be the random variable corresponding to u_k in (1), then, for example, the probability satisfied (4) is as follows:

$$P(T - \Delta u < U_k < T + \Delta u) = \int_{T-\Delta u}^{T+\Delta u} f_{U_k}(u)du. \quad (5)$$

Note that U_k has a probability density function since it is continuous. Let S and S^{v3} be the random variables corresponding to s_k and s_k^{v3}, respectively. Then the probabilities that a pixel in S becomes black, gray and white by ternarizing S are as follows:

$$P_b^S = P(S^{v3} = 0) = P(0 \le S < q_1) = \int_0^{q_1} f_S(x)dx, \quad (6)$$

$$P_g^S = P(S^{v3} = 0.5) = P(q_1 \le S < q_2) = \int_{q_1}^{q_2} f_S(x)dx, \quad (7)$$

$$P_w^S = P(S^{v3} = 1) = P(q_2 \le S \le 1) = \int_{q_2}^{1} f_S(x)dx, \quad (8)$$

respectively, where q_1 and q_2 denote thresholds of quantization to ternary image for S, and have $q_1 < q_2$. Let $W1$ be the random variable corresponding to $w1_k$, then the probability that a pixel in W1 becomes white is as follows:

$$P_w^{W1} = P(W1 = 1) = \frac{\text{(The number of white pixels in W1)}}{\text{(The total number of pixels in W1)}}. \quad (9)$$

B. Formulation of process of embedding data

We consider the intensity distribution of W2. In Fu's and Au's method, W1 is produced from G1 by using regular error diffusion, so its intensity distribution is obtained easily like (9). However, it is difficult to obtain the intensity distribution of W2 because W2 is produced from G2 by using error diffusion with applying the rule of embedding data. Therefore, the intensity distribution of W2 is different from that obtained by regular error diffusion. The distribution depends on S and W1. We analyze the statistical behavior of errors arise in error diffusion by using relative frequency distributions of the errors, and show the stochastic behavior of pixel values of W2.

Let E_k and $W2_k$ be the random variables corresponding to e_k in (3) and $w2_k$, respectively. Then, by (3),

$$E_k = U_k - W2_k. \quad (10)$$

E_k for each k has a probability density function since it is continuous. There are four cases on U_k. The probability density function that E_k has in each case is as follows:

1) The case of $-\infty < U_k \le T - \Delta u$.
   $W2_k$ is 0. By (10), $E_k = U_k$. We obtain $-\infty < E_k \le T - \Delta u$ by rewriting the domain of U_k to it of E_k. In

the domain of E_k, the probability density function of E_k is the same as it of U_k, i.e.,

$$f_{E_k}^{(1)}(e) = f_{U_k}(e), \quad -\infty < e \leq T - \Delta u. \tag{11}$$

2) The case of $T + \Delta u \leq U_k < \infty$.
$W2_k$ is 1. By (10), $E_k = U_k - 1$. We obtain $T + \Delta u - 1 \leq E_k < \infty$ by rewriting the domain of U_k to it of E_k. In the domain of E_k, the probability density function of E_k is the same as it of $U_k - 1$, i.e.,

$$\begin{aligned} f_{E_k}^{(2)}(e) &= f_{U_k-1}(e) \\ &= f_{U_k}(e+1), \quad T + \Delta u - 1 \leq e < \infty. \end{aligned} \tag{12}$$

3) The case of $T - \Delta u < U_k < T$.
$W2_k$ is 0 in regular error diffusion, but it is possible that $W2_k$ is 1 with the rule of embedding data. Therefore, we use the expected value of $W2_k$ instead of $W2_k$. Let $\langle W2_k \rangle$ be the expected value of $W2_k$, then,

$$\begin{aligned} \langle W2_k \rangle = &\left(P_b^S P_w^{W1} + P_g^S + P_w^S P_b^{W1}\right) \cdot 0 \\ &+ \left(P_b^S P_b^{W1} + P_w^S P_w^{W1}\right) \cdot 1. \end{aligned} \tag{13}$$

We obtain $T - \Delta u - \langle W2_k \rangle < E_k < T - \langle W2_k \rangle$ by replacing $W2_k$ in (10) with $\langle W2_k \rangle$ and rewriting the domain of U_k to it of E_k. In the domain of E_k, the probability density function of E_k is the same as it of $U_k - \langle W2_k \rangle$, i.e.,

$$\begin{aligned} f_{E_k}^{(3)}(e) =& f_{U_k - \langle W2_k \rangle}(e) \\ =& f_{U_k}(e + \langle W2_k \rangle), \\ & T - \Delta u - \langle W2_k \rangle < e < T - \langle W2_k \rangle. \end{aligned} \tag{14}$$

4) The case of $T \leq U_k < T + \Delta u$.
We use the expected value of $W2_k$ the same as case 3. Then,

$$\begin{aligned} \langle W2_k \rangle = &\left(P_b^S P_w^{W1} + P_w^S P_b^{W1}\right) \cdot 0 \\ &+ \left(P_b^S P_b^{W1} + P_g^S + P_w^S P_w^{W1}\right) \cdot 1. \end{aligned} \tag{15}$$

We obtain $T - \langle W2_k \rangle \leq E_k < T + \Delta u - \langle W2_k \rangle$ by replacing $W2_k$ in (10) with $\langle W2_k \rangle$ and rewriting the domain of U_k to it of E_k. In the domain of E_k, the probability density function of E_k is the same as it of $U_k - \langle W2_k \rangle$, i.e.,

$$\begin{aligned} f_{E_k}^{(4)}(e) =& f_{U_k - \langle W2_k \rangle}(e) \\ =& f_{U_k}(e + \langle W2_k \rangle), \\ & T - \langle W2_k \rangle \leq e < T + \Delta u - \langle W2_k \rangle. \end{aligned} \tag{16}$$

By (11), (12), (14) and (16), the probability density function that E_k has is

$$f_{E_k}(e) = f_{E_k}^{(1)}(e) + f_{E_k}^{(2)}(e) + f_{E_k}^{(3)}(e) + f_{E_k}^{(4)}(e). \tag{17}$$

Next, we analyze the statistical behavior of u_k in (1) by using relative frequency distributions of u_k. Let $G2$ and ER_k be the random variables corresponding to intensities of G2 and accumulative errors at k-th pixel in G2, respectively. Then, by (1),

$$U_k = G2 + ER_k, \quad ER_k = \sum_{i \in J_k} j_i E_i. \tag{18}$$

Note that ER_k for each $k \geq 2$ has a probability density function since it is continuous, and $ER_1 = 0$. The probability density function of $j_i E_i$ for any $i \in J_k$ in the error term of (18) is

$$f_{j_i E_i}(e) = f_{E_i}(\tfrac{1}{j_i} e). \tag{19}$$

Also for $j_m E_m$ and $j_n E_n$ for any m and $n \in J_k$, let Δy be a small positive constant, then,

$$\begin{aligned} & f_{j_m E_m + j_n E_n}(e) \\ &= \sum_{t=-\infty}^{\infty} f_{j_m E_m + t\Delta y}(e) f_{j_n E_n}(t\Delta y)\Delta y \\ &= \sum_{t=-\infty}^{\infty} f_{j_m E_m}(e - t\Delta y) f_{j_n E_n}(t\Delta y)\Delta y \\ &= \int_{-\infty}^{\infty} f_{j_m E_m}(e - y) f_{j_n E_n}(y) dy, \quad \Delta y \to 0 \\ &= \int_{-\infty}^{\infty} f_{E_m}(\tfrac{1}{j_m}(e - y)) f_{E_n}(\tfrac{1}{j_n} y) dy. \end{aligned} \tag{20}$$

By repeating the operation in (20), we obtain the following probability density function of ER_k, i.e.,

$$\begin{aligned} & f_{ER_k}(e) \\ &= \int_R \int_R \cdots \int_R f_{E_{i'}}(\tfrac{1}{j_{i'}}(e - \sum_{\substack{i \in J_k \\ i \neq i'}} y_i)) \prod_{\substack{i \in J_k \\ i \neq i'}} f_{E_i}(\tfrac{1}{j_i} y_i) dy_i, \end{aligned} \tag{21}$$

where the number of integrals is $|J_k| - 1$. Moreover, by replacing (j_m, E_m) and (j_n, E_n) in (20) with $(1, G2)$ and $(1, ER_k)$, respectively, we obtain the following probability density function of U_k, i.e.,

$$f_{U_k}(u) = \int_{-\infty}^{\infty} f_{G2}(u - x) f_{ER_k}(x) dx. \tag{22}$$

Therefore, the probability that k-th pixel in W2 becomes white is

$$\begin{aligned} P_w^{W2_k} =& P(W2_k = 1) \\ =& P(T - \Delta u < U_k < T + \Delta u) P_b^S P_b^{W1} \\ & + P(T - \Delta u < U_k < T + \Delta u) P_w^S P_w^{W1} \\ & + P(T \leq U_k < T + \Delta u) P_g^S \\ & + P(U_k \geq T + \Delta u) \\ =& \left\{P_b^S P_b^{W1} + P_w^S P_w^{W1}\right\} \int_{T-\Delta u}^{T+\Delta u} f_{U_k}(u) du \\ & + P_g^S \int_T^{T+\Delta u} f_{U_k}(u) du + \int_{T+\Delta u}^{\infty} f_{U_k}(u) du. \end{aligned} \tag{23}$$

C. Formulation of superposing two halftone images

We consider the intensity distribution of C. In Fu's and Au's method, C is produced by superposing W1 on W2. We can obtain the stochastic behavior of pixel values of C by using intensity distributions of W1 and W2. k-th pixel in C becomes white when k-th pixels in W1 and W2 are both white. Let

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C_k be the random variable corresponding to intensities of k-th pixel in C. Then, by the previous results, the probability that k-th pixel in C becomes white is

$$\begin{aligned} P_w^{C_k} =& P(C_k = 1) \\ =& P(T - \Delta u < U_k < T + \Delta u) P_w^S P_w^{W1} \\ &+ P(T \le U_k < T + \Delta u) P_g^S P_w^{W1} \\ &+ P(U_k \ge T + \Delta u) P_w^{W1} \\ =& P_w^{W1} \left\{ P_w^S \int_{T-\Delta u}^{T+\Delta u} f_{U_k}(u) du \right. \\ &\quad + P_g^S \int_{T}^{T+\Delta u} f_{U_k}(u) du \\ &\quad \left. + \int_{T+\Delta u}^{\infty} f_{U_k}(u) du \right\}. \end{aligned} \tag{24}$$

IV. Conclusion

In this paper, we have reviewed a formulation for Fu's and Au's data hiding scheme using a stochastic method. Fu's and Au's data hiding scheme is based on error diffusion, and demonstrates relatively good performance for secret data like natural gray-scale images by giving appropriate transformations to original input data. The formulation is based on relative frequency distributions of intensities of images. As a results, we have shown the stochastic behavior of pixel values of W2 and C. Formulating data embedding algorithms would allow us to theoretically investigate behavior of the algorithm. For example, we might be able to find easily pairs of input image such that secret data can be embed in halftone images without affecting their perceptual quality and the embedded data can be restored with apparently higher quality when the halftone images are overlaid. Also, we might be able to optimize parameters affect the performance of data embedding algorithm.

The concept of the formulation is basic, but theoretical studies by formulating is important trial in this field. In the future works, it will be necessary to perform a verification of the formulation through a simulation.

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