PWL Function Approximation Circuit with Diodes and Current Input and Output

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Abstract: - A passive piecewise-linear function approximation circuit with current input and output is presented. The gain of the circuit is controlled by switching diodes. The breakpoints and slopes of its input-output characteristic are set by resistances. Relations for computation of these resistances are given. As an application of the proposed circuit, a triangle to sine wave shaper is shown. Computer simulation confirmed a good functionality of the circuit even at high frequencies.

Key-Words: - Piecewise-Linear Function Approximation Circuit, Wave Shaping Circuit, Current Mode

1 Introduction
The designers of analog circuits increasingly focus on the current-mode signal processing. It can have several advantages compared to the voltage-mode, such as higher bandwidth, low power consumption, simpler structures etc. Current mode is being used mostly in linear analog circuits (filters, immittance simulators, oscillators, amplifiers), however nonlinear applications can be also found in literature [1] – [15]. Considerable work in this topic has been done with current-mode precision rectifiers [1] – [7] employing various active elements (current conveyors, current followers) and including various improvements to obtain higher bandwidth, lower temperature dependency etc. Moreover, current limiters [13] and controlled resistances as basic elements of nonlinear components or networks have been published.

However, only few contributions describing piecewise-linear (PWL) function approximation circuits that handle current input and output signals can be found. The applications of these circuits can be, for example, linearization of nonlinear transfer characteristics of sensors, in neural networks, in companding A/D and D/A converters, etc. The traditional implementation of the PWL circuit in voltage mode is a closed-loop connection of an operational amplifier with resistors switched by diodes depending on the input voltage [16]. Recently several new approaches to synthesize nonlinear function circuits were reported. They are usually composed of active building blocks e.g. with current mirrors or operational transconductance amplifiers [14], [15]. These blocks provide a basic nonlinear function (with one breakpoint) which is the final PWL characteristic compound of. The number of these building blocks usually equals to the number of segments of the characteristic and thus many active elements are required when many segments are used to approximate.

Our circuit is based on a resistive current divider with variable gain. This gain changes depending on the input current, which is achieved by junction diodes that switch current path in the circuit.

2 Proposed Approximation Circuit
The proposed PWL function approximation circuit is shown in Fig. 1.

Fig. 1. Proposed current-mode PWL function approximation circuit.

Its basic idea is a current divider with variable dividing ratio. When the input current is low, all diodes are switched off. With \( I_{IN} \) increasing, diode voltages also increase depending on the resistances.
When the \( i \)-th diode reaches its threshold voltage \( V_D \), it switches on and leads part of the input current to the \( I_A \) output which increases the slope of the transfer to the \( I_A \) output and decreases the slope of the transfer to the \( I_B \) output.

It is necessary to compute the circuit resistances so that the transfer changes appropriately in the desired breakpoints of the input-output transfer characteristic. If the input current \( (I_{IN}) \) is generated by an ideal current source, the output current \( (I_A \) or \( I_B \)) flows into an ideal current load, and the gain is not required to be higher than one, the proposed circuit can be passive as shown in Fig. 1. Otherwise current-controlled current source(s) (CCCS) must be used for impedance matching at the input and/or output and to set the overall maximum gain of the circuit. The passive circuit characteristic must be then scaled with respect to the CCCS gain(s).

2.1 Analysis for Output B

In this section the relations for computing the circuit resistances will be presented supposing that the output current \( I_B \) is utilized. This is the case when decreasing the characteristic slope with increasing the input current is necessary. An example of such a transfer characteristic is shown in Fig. 2.

2.2 Analysis for Output A

The same relations as in the previous subsection can be used when the circuit output is \( I_A \) and the transfer characteristic slope is increasing with the input current increasing. In this case the desired slopes \( S_{Ai} \) are converted to \( S_{Bi} \) according to the following relation and substituted into (2) – (5). 

\[
S_{Bi} = 1 - S_{Ai}, \quad (6)
\]

where \( i = 1, 2, \ldots, n+1 \) and \( I_{B0} = I_{IN0} = 0 \) A. The \( n \) represents the number of diodes and breakpoints of the characteristic and \( n+1 \) is the number of segments with different slopes. The following relations can be used for computing the element parameters.

\[
R_A = \frac{V_D}{I_{IN1}(1-S_{B1})}, \quad (2)
\]

\[
R_{Bk} = \frac{V_D}{\sum_{j=1}^{k}(I_{INj} - I_{INj-1})S_{Bj}} - \sum_{j=k+1}^{n}R_{Bj}, \quad (3)
\]

where \( k = 1, 2, \ldots, n \). It is necessary to begin to compute the resistances \( R_{Bk} \) from the highest index \( k = n \) and then to decrease the index successively to one.

\[
R'_{Bk} = \frac{R_{Bk} \sum_{j=1}^{k+1}(I_{INj} - I_{INj-1})S_{Bj}}{(S_{Bk} + S_{Bk+1})(I_{INk+1} - I_{INk})}, \quad (4)
\]

\[
R'_{B0} = \frac{R_{B0} \sum_{j=1}^{n+1}(I_{INj} - I_{INj-1})S_{Bj} - V_D}{(S_{B0} + S_{B0+1})(I_{INn+1} - I_{INn})}. \quad (5)
\]

Please note that the circuit and the relations above presume positive non-zero diode threshold voltage \( V_D \).

3 PWL Approximation of Function

The PWL transfer characteristic is often obtained as an approximation of a mathematical function \( f(x) \). It is useful to know how the number of segments depends on the approximated interval \( a \leq x \leq b \) and the desired absolute approximation error \( \varepsilon \). For the case of optimum non-uniform segments, it has been

![Fig. 2. Input-output current transfer characteristic for output \( I_B \) (various scales of axes)](image-url)
shown in [17] that the number of segments is approximately given by
\[ s(\varepsilon) \approx \frac{1}{4\sqrt{\varepsilon}} \int_a^b \sqrt{|f'(x)|} \, dx . \]  
(7)

After determining the number of segments it is necessary to optimally partition the approximated interval into segments. This is not a trivial task even for simple functions and numerical methods are necessary in many cases [18]. Fig. 3 illustrates the situation where an \( i \)-th segment of a function \( f(x) \) is to be linearly approximated.

Fig. 3. Linear approximation of a function segment

The most accurate approximation with maximum error of \( \varepsilon \) is shown here by the dashed secant. Another two commonly used approximations are tangent and chord that have double maximum error compared to the secant approximation. The chord approximation is drawn in Fig. 3 by solid line and it will be used for further analysis. The secant approximation with half error can be obtained from the chord one easily by shifting it by \( \varepsilon \).

The following equations must be valid at the edges of the segment
\[ f(x_{i-1}) = k_i x_{i-1} + q_i , \]  
(8)
\[ f(x_i) = k_i x_i + q_i . \]  
(9)

Here \( k_i \) is the slope of the linear approximation and is equal to \( S_{Ai} \) when the slope of the characteristic is increasing i.e. the function is convex. When the function is concave, \( k_i = S_{Bi} \).

The maximum error \( 2\varepsilon \) occurs at \( x_{e_i} \) and here it is valid
\[ |f(x_{e_i}) - (k_i x_{e_i} + q_i)| = 2\varepsilon , \]  
(10)
\[ f'(x_{e_i}) - k_i = 0 , \]  
(11)

The last equation expresses the differentiation of (10) with respect to \( x \). The error \( \varepsilon \) and the initial point \( x_{i-1} \) are known and the remaining four unknowns \( (x_i, k_i, q_i, x_{e_i}) \) can be calculated from the set of four equations (8) – (11).

4 Computer Simulation

As an example of application of the proposed circuit we have chosen shaping of a triangular waveform to a sinusoid. Output B and diodes 1N4148 will be used in this case. In order to process both polarities of the signal, opposite-oriented diodes were added in parallel to the original diodes in the circuit. No other changes have been made in the circuit and thus it is not necessary to show it again.

The input triangular waveform is assumed to have amplitude of 10 mA; the amplitude of output sinusoid is chosen 0.5 mA which leads to more suitable resistances. The maximum approximation error chosen is \( 2\varepsilon = 10 \mu A \) (2\( \varepsilon \) because of the chord approximation). Evaluating (7) for the sine function with amplitude \( A = 0.5 \text{ mA} \) we get the following relation [17]
\[ s_{\sin}(\varepsilon) \approx \frac{0.2995}{\sqrt{\varepsilon/A}} \approx \frac{0.2995}{\sqrt{0.01}} = 2.995 . \]  
(10)

Thus it is necessary to use three linear segments this case.

After substituting the chosen error, initial point \( x_0 = 0 \text{ A} \) and
\[ f(x) = 0.0005 \sin \left( 100x \frac{\pi}{2} \right) \]  
(11)
into (8) – (11), the positions of breakpoints have been calculated: \( x_1 = 4.36 \text{ mA} \), \( x_2 = 7.23 \text{ mA} \), \( x_3 = 9.82 \text{ mA} \).

Although the third breakpoint \( x_3 \) is at the end of the approximated characteristic, we have used three diodes for one signal polarity. The third diode sets zero slope of the characteristic above the third breakpoint, which rounds the peaks of the sinusoid advantageously. The numerical values of circuit elements are: \( R_A = 161 \Omega \), \( R_{B1} = 621 \Omega \), \( R_{B2} = 133 \Omega \), \( R_{B3} = 1301 \Omega \), \( R_{C1} = 3959 \Omega \), \( R_{C2} = 860 \Omega \), \( R_{C3} = 0 \Omega \).

PWL approximation of the sinusoid and simulated output characteristic are shown for a quarter cycle in Fig. 4.
The input and output waveforms of the triangular to sine shaper are depicted in Fig. 5 for frequency 5 MHz. It is seen that the circuit operates correctly even at relatively high frequency.

Fig. 6 shows the dependency of total harmonic distortion (THD) on frequency for constant input amplitude. The distortion is 3.44% at low frequencies and decreases to 2.82% at 10 MHz. THD again increases and output amplitude decreases above this frequency.

5 Conclusion
Diode piecewise linear approximation circuit with current input and output has been designed and analyzed. The proposed circuit includes only passive elements in case it is connected to an ideal current source and load and when current gain below one is needed. When there is not ideal current source and load or when a gain above one is required, current-controlled current source(s) can be added at input and/or output of the circuit. Relations for computation of element parameters have been derived and presented and computer simulation showed a good functionality of the circuit also at high frequency. The circuit exceeds in simplicity and no need of active elements for each linear segment as another circuits of this kind presented in literature. Thus the circuit is also suitable for an easy and quick implementation by discrete elements.

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