Parametric Analysis of the Mathematical Model of Steam Superheater

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Abstract: One of the aspects of use of mathematical model of steam superheater is determination of how the dynamical behavior of real boiler is affected by hardly measurable or non-measurable signals. Examples of such signals could be the flue gas temperatures of particular heat exchangers. The signal of flue gas temperature is calculated based on mathematical model of the superheater, followed by determination of how this signal affects the output steam temperature for various parameters of the flue gas. Output power plant superheater is described by a set of partial differential equations of the first order. These equations describe heat exchange between media through a wall with unspecified proportions. Equations are assigned into Simulink S-functions. The Simulink models act as mathematical model of the set consisting of the heat exchangers, on which the experiments are carried out.

Key–Words: Loss minimization, Mathematical models, Partial differential equations, Superheater, Verification

1 Introduction

Convectional steam superheater is usually constructed as pipes within pipes or it can be jacketed with tubular exchanger inside [4]. The steam superheater is flow heat exchanger with two media: produced steam and hot flue gases. As for construction, it can be concurrent, counter-flow or combination of these types [3].

Mathematical model of the heat exchanger was tested in [5], where so called the full model was used. That means besides heat transfer between two media and the wall it also includes influences of the pressure and the velocity on pipeline wall and on media itself. So called simplified model of the heat exchanger was described in [6]. The simplified model represents heat transfer between two media without effects of the pressure and the velocity. This paper describes technique of the calculation of the flue gas temperature with the help of the simplified mathematical model of the heat exchanger. Thus calculated signal of the flue gas temperature integrates all the other signals that are hardly measurable or non-measurable in real heat exchanger.

The goal of the paper is to show that if this approach is applied, the changes of the parameters of the heat exchanger regarding flue gas side do not cause relevant changes of the time constants that determine heat transfer between flue gas and the wall.

The whole scheme is created and simulated in MATLAB&Simulink environment. Verification of this model is done by comparison of the real measured operational data with Simulink model.

2 Concurrent steam superheater

The complete mathematical model of concurrent superheater consists of several models of thermal systems. The superheater itself is connected through high-pressure pipeline, omitting the influence of surrounding temperature. This pipeline is described by simplified mathematical model of heat exchange that considers heat transfer among between flowing media and pipeline wall. To simplify the terminology, this mathematical model will be referred to as unheated area. The complete model described in this paper deals with cascade connection of unheated areas and concurrent superheater, as shown in Fig. 1.

Roman numerals I., II., IV. and V. stands for unheated areas. The parts of unheated areas marked as II. and IV. are distribution chambers (collectors) - the places where the pipelines branch off and get connected. The concurrent superheater, marked as III., is exposed to the influence of flue gases. The steam is being heated up to the required operational temperature of a turbine.

A set of partial differential equations (1) - (3) can be derived for concurrent superheater. It assumes that
there is no temperature gradient in the pipeline wall in the radial direction, heat doesn’t conduct along axial direction and heat transfer between media and wall occurs in radial direction only [4]. Concurrent superheater is described by the following equations:

- Steam temperature:
  \[ T_W - T_1 = \tau_1 \left[ u_1 \frac{\partial T_1}{\partial x} + \frac{\partial T_1}{\partial t} \right] \tag{1} \]

- Flue gas temperature:
  \[ T_W - T_2 = \tau_2 \left[ u_2 \frac{\partial T_2}{\partial x} + \frac{\partial T_2}{\partial t} \right] \tag{2} \]

- Wall temperature:
  \[ \frac{T_1 - T_W}{\tau W_1} + \frac{T_2 - T_W}{\tau W_2} = \frac{\partial T_W}{\partial t} \tag{3} \]

where

\[ \tau_1 = \frac{c_1 M_1}{\alpha W_1 O_1 |u_1|} \quad \text{and} \quad \tau W_1 = \frac{c W G}{\alpha W_1 O_1} \tag{4} \]

and

\[ \tau_2 = \frac{c_2 M_2}{\alpha W_2 O_2 |u_2|} \quad \text{and} \quad \tau W_2 = \frac{c W G}{\alpha W_2 O_2} \tag{5} \]

This mathematical model of superheater contains three state variables:

- \( T_1(x, t) \) – temperature of steam
- \( T_2(x, t) \) – temperature of flue gas
- \( T_W(x, t) \) – temperature of wall

It is obvious that state variables depend both on time \( t \) and distance \( x \). This distance determines current position of the slice in heat exchanger. Derivatives in \( x \)-coordinates in equation (1) and (2) are approximated by numeric differences of the second order. A set of PDE is then transformed into set of ODE (ordinary differential equations) by use of FDM (finite difference method), [2], [6]. The superheater is divided into 20 equidistant segments in \( x \)-coordinate, with step \( h = L/(20 - 1) \). These segments (slices) create a set of the nodes where each node represents a single state variable in a given point. In this case, it works out \( 3 \times 20 \) state variables.

### Table 1: Heat exchanger parameters

<table>
<thead>
<tr>
<th>symbol</th>
<th>description</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>heat capacity of steam</td>
<td>[J/kg/K]</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>heat capacity of flue gas</td>
<td>[J/kg/K]</td>
</tr>
<tr>
<td>( c_W )</td>
<td>heat capacity of superheater’s wall material</td>
<td>[J/kg/K]</td>
</tr>
<tr>
<td>( G )</td>
<td>weight of wall per unit of length in ( x ) direction</td>
<td>[kg/m]</td>
</tr>
<tr>
<td>( L )</td>
<td>active length of the wall</td>
<td>[m]</td>
</tr>
<tr>
<td>( M_1 )</td>
<td>steam mass flow rate</td>
<td>[kg/s]</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>flue gas mass flow rate</td>
<td>[kg/s]</td>
</tr>
<tr>
<td>( O_1 )</td>
<td>surface of wall per unit of length in ( x ) direction for steam</td>
<td>[m]</td>
</tr>
<tr>
<td>( O_2 )</td>
<td>surface of wall per unit of length in ( x ) direction for flue gas</td>
<td>[m]</td>
</tr>
<tr>
<td>( u_1 )</td>
<td>velocity of the steam in ( x ) direction</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( u_2 )</td>
<td>velocity of the flue gas in ( x ) direction</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( \alpha W_1 )</td>
<td>heat transfer coefficient between the wall and the steam</td>
<td>[J/m²/s/K]</td>
</tr>
<tr>
<td>( \alpha W_2 )</td>
<td>Heat transfer coefficient between the wall and the flue gas</td>
<td>[J/m²/s/K]</td>
</tr>
</tbody>
</table>

### 3 Testing the mathematical model of the heat exchanger with the real data

From technological point of view, tested steam superheater is located at the output of the boiler. The steam generated by this superheater then drives the power plant turbine. Parameters of the mathematical model of the concurrent superheater and unheated areas were taken over the real heat exchanger operating at coal-fired power plant in Dětmarovice, Czech Republic [1]. Implementation of the mathematical model of the steam superheater was described in [6].

Using Simulink environment, the input of the steam superheater was agitated by steam temperature signal \( T_1(0, t) \) together with constant signal representing flue gas temperature \( T_2(0) \) (see Fig. 2).

As it is obvious from simulation results (see Fig. 3), the trendline representing the output steam temper-
Figure 2: The steam superheater agitated by measured temperature signal $T_1(0, t)$ and by constant temperature signal $T_2(0)$. 

Output steam temperature signal $T_1(L, t)$ calculated by the mathematical model significantly differs from the measured signal. Output steam temperature signal $T_1(L, t)$ copies the trendline of the input steam temperature $T_1(0, t)$ up to a certain point.

Figure 3: Comparison of the temperature signals $T_1(L, t)$ for the model of the heat exchanger without corrected parameters and constant temperature $T_2(0)$.

Calculation of the mean values of the working signals shows the inconsistency between technologically prescribed values and the measured values. The steam temperature at superheater’s input $T_1(0)$ is 457.2 °C according the technological assignment, but the mean value calculated from the measured data works out as $T_1(0) = 416.7$ °C.

The difference between measured signal $T_1(L, t)$ and simulated signal $T_{1sim}(L, t)$ reaches up to 30.2 °C. These differences both in steam temperatures trendlines $T_1(L, t)$ and in their mean values are caused by the fact that flue gas temperature at superheater’s input $T_2(0, t)$ is unknown. Signal representing the flue gas temperature is not measured on a real boiler setup, there’s only a rough technological value of 990 °C.

3.1 Calculation of the flue gas temperature based on assistance of mathematical model

Signal of the flue gas temperature at superheater’s input $T_2(0, t)$ can be calculated based on knowledge of steam temperature at superheater’s input $T_1(0, t)$, steam temperature at superheater’s output $T_1(L, t)$ and mathematical model of the superheater. Thus calculated signal will be referred to as virtual flue gas temperature at the superheater’s input $T_{2v}(0, t)$. This signal integrates all the other hardly measurable or non-measurable signals of the boiler that affect the temperature at the superheater’s output together with flue gas temperature. Virtual temperature signal $T_{2v}(0, t)$ integrates fluctuations of the mass flow rate of the flue gas $M_2$, velocity of the flue gas $u_2$ and the flue gas temperature itself $T_2(0, t)$ that affects the output steam temperature $T_1(L, t)$. Algorithm for calculation of the signal $T_{2v}(0, t)$ is shown in Fig. 4.

Calculation of the virtual temperature of the flue gas is closely related to the calculation of the steady state of temperature distribution in the superheater.

For calculation of the steady state distribution of the temperatures over the superheater’s length it is necessary to modify partial differential equations (1) - (3). All the terms with time derivatives have been set to zero. For mathematical model of concurrent superheater, the modified equations (6), (7) and (8) become a set of differential algebraic equations (referred to as DAE).

For calculation of the flue gas temperature distribution over the length (concurrent steam superheater):

$$
\frac{\partial T_1(x, t_{\infty})}{\partial x} = \frac{T_W(x, t_{\infty})}{\tau_1 u_1} - \frac{T_1(x, t_{\infty})}{\tau_1 u_1}
$$

(6)
• For flue gas temperature distribution over the length (concurrent steam superheater):

\[
\frac{\partial T_2(x, t_\infty)}{\partial x} = \frac{T_W(x, t_\infty)}{\tau_2 u_2} - \frac{T_2(x, t_\infty)}{\tau_2 u_2}
\]  
(7)

• For the wall temperature distribution over the length (concurrent steam superheater):

\[
T_W(x, t_\infty) = \frac{\tau_1 w_1}{\tau_1 w_1 + \tau_2 w_2} \cdot T_1(x, t_\infty) + \frac{\tau_2 w_2}{\tau_1 w_1 + \tau_2 w_2} \cdot T_2(x, t_\infty)
\]  
(8)

• For distribution of the steam temperature over the length (unheated area):

\[
\frac{\partial T_1(x, t_\infty)}{\partial x} = \frac{T_W(x, t_\infty)}{\tau_1 u_1} - \frac{T_1(x, t_\infty)}{\tau_1 u_1} = 0
\]  
(9)

• For distribution of the wall temperature over the length (unheated area):

\[
T_W(x, t_\infty) = T_1(x, t_\infty)
\]  
(10)

Distribution over the superheater’s length of the steam temperature and wall temperature in a steady state for the mathematical model of unheated area work out as constant functions, as shown by equations (9) and (10). These unheated areas thus do not have to be considered for further computations regarding flue gas temperature, the relevant equations for this purposes are (6) - (8).

Calculation of virtual flue gas temperature at the superheater’s input \(T_{2v}(0, t)\) is being performed at each time point \(t_k\) from the entire data file. It is supposed that flue gas temperature at the superheater’s input lies within the range \(\langle T_{2v_{\min}}(0, t_k), T_{2v_{\max}}(0, t_k)\rangle\) (see Fig. 4). Half of the interval is denoted as \(T_{2v_{\text{temp}}}(0, t_k)\). Temperature \(T_{2v_{\text{temp}}}(0, t_k)\) together with measured temperature at the superheater’s input \(T_1(0, t_k)\) are used as initial conditions for computation of the temperature distribution along the length of the superheater. For clear arrangement reasons, Fig. 4 only shows temperature distributions of the steam \(T_1(x, t_k)\) and flue gas \(T_{2v}(x, t_k)\) along the superheater’s length \(x\) over the time \(t_k\). After the temperature distribution over the superheater’s length is computed, the simulated output steam temperature \(T_{1\text{model}}(L, t_k + t_a)\) is compared with the corresponding measured temperature \(T_{1\text{meas}}(L, t_k + t_a)\) and relative error between these values is computed according formula (11).

\[
\delta_{T_1(L)} = \frac{|T_{1\text{model}}(L) - T_{1\text{meas}}(L)|}{T_{1\text{meas}}(L)} \cdot 100 \, [\%]
\]  
(11)

This relative error is compared to its maximum acceptable value. If it is smaller than the prescribed threshold \(\delta_{T_1(L)}^{\text{max}} = 0.01\, \%\), then \(T_{2v_{\text{temp}}}(0, t_k)\) is designated as \(T_{2v}(0, t_k)\). In case it’s higher than the threshold \(\delta_{T_1(L)}^{\text{max}}\) program is divided in two branches.

In case \(T_{1\text{model}}(L, t_k + t_a)\) is higher than \(T_{1\text{meas}}(L, t_k + t_a)\), it means working virtual flue gas temperature \(T_{2v_{\text{temp}}}(0, t_k)\) was too high. The algorithm continues with computing the half of a new interval \(\langle T_{2v_{\min}}(0, t_k), T_{2v_{\text{temp}}}(0, t_k)\rangle\), that replaces previous value \(T_{2v_{\text{temp}}}(0, t_k)\). On the contrary, if \(T_{1\text{model}}(L, t_k + t_a)\) is smaller than \(T_{1\text{meas}}(L, t_k + t_a)\), it means working virtual flue gas temperature \(T_{2v_{\text{temp}}}(0, t_k)\) was small the other way around. Then the half of a new interval \(\langle T_{2v_{\text{temp}}}(0, t_k), T_{2v_{\text{max}}}(0, t_k)\rangle\) is computed, replacing the previous value \(T_{2v_{\text{temp}}}(0, t_k)\). Values \(T_{2v_{\min}}(0, t_k)\) and \(T_{2v_{\max}}(0, t_k)\) depend on number of iterations. After this computation of a new working virtual temperature of the flue gas \(T_{2v_{\text{temp}}}(0, t_k)\) a relative error is computed according (11). Algorithm is repeated until the relative error exceeds a given limit \(\delta_{T_1(L)}^{\text{max}}\). Each passing of the cycle increments the counter that serves as a watchdog to prevent the algorithm from endless loops.

Computed temperature distributions over the superheater’s length in particular time points \(t_k, t_{k+1}, \ldots, t_{N_d}\) represent steady states of the temperatures along the superheater. Therefore it is impossible to have required temperature \(T_1(L, t_k)\) at superheater’s output at the same time point \(t_k\) as when temperatures are \(T_1(0, t_k)\) and \(T_{2v}(0, t_k)\) brought to the superheater’s input. For computation of signal \(T_{2v}(0, t)\) and for comparison of the simulated and measured trendlines representing steam temperatures at the superheater’s output, the algorithm uses the data sample shifted by \(t_a = 63\, \text{s}\) (delay between the model and the measurement). This value is determined by experimental comparison of measured \(T_1(L, t)\) and simulated \(T_{1\text{slim}}(L, t)\), as shown in Fig. 5 and Fig. 6.

Input of the mathematical model is fed by computed signal of virtual flue gas temperature \(T_{2v}(0, t)\), that replaced original constant signal of flue gas \(T_2(0)\) in Fig. 2.

Fig. 5 shows comparison of the steam temperatures at superheater’s output \(T_1(L, t)\) between measurements and mode, with no shift of the steam temperature at the superheater’s output at time \(t_a = 0\, \text{s}\).

Fig. 6 shows the same comparison as Fig. 5, except for the steam temperature is shifted in time by \(t_a = 63\, \text{s}\).

As it shown in Fig. 7, changing mass flow rate of the flue gas \(M_2\) and velocity of the flue gas \(u_2\) also
causes the change of mean value of virtual flue gas temperature $T_{2v}(0)$ by the time. Together with mass flow rate of the flue gas $M_2$ and velocity of the flue gas $u_2$ there’s also a change of heat transfer coefficient between the wall and flue gas $\alpha_{W2}$, whose values can be read from a nomogram in [3].

Mean value of the virtual flue gas temperature calculated from mathematical model with given parameters worked out $\overline{T_{2v}(0)} = 1113.5 \, ^\circ C$. This value is significantly higher than $T_2(0)$ prescribed in the technological assignment of the superheater. As it was mentioned above, virtual flue gas temperature $T_{2v}(0, t)$ includes varying mass flow rate of the flue gas $M_2(t)$ and flue gas velocity $u_2(t)$ over the time. Changing the parameters up to 25% together with appropriate change of heat transfer coefficient on the flue gas side $\alpha_{W2}$ can lead up to $\overline{T_{2v}(0)} = 1028 \, ^\circ C$. Mean value of virtual flue gas temperature $T_{2v}(0, t)$ is mainly affected by heat transfer coefficient $\alpha_{W2}$.

4 Correspondence evaluation for the superheater model

Simulation results of the mathematical model of the superheater were compared based on mean relative error value between measured and simulated signal representing steam temperature at superheater output $T_1(L, t)$. This approach of comparison was determined for a quick evaluation of the functionality and progress of control circuit designed for the superheater.

The most common method of evaluation of correspondence of the mathematical model is usage of mean square error criteria that supposes minimization of loss function.

$$J = \frac{1}{N_d} \sum_{k=1}^{N_d} (T_M[k] - T_S[k])^2,$$  \hspace{1cm} (12)

Where $T_S[k]$ stands for the temperature values computed by the model related to measured values $T_M[k]$ [7].
Table 2 contains calculated values of criteria $J$ for a steam superheater agitated by constant flue gas temperature. For simulation with the constant flue gas temperature of $T_2(0) = 1113.5 \, ^\circ C$ the loss function value has decreased significantly.

<table>
<thead>
<tr>
<th>$T_2(0)$</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1113.5 , ^\circ C</td>
<td>5.487 , ^\circ C^2</td>
</tr>
<tr>
<td>1113.5 , ^\circ C</td>
<td>5.487 , ^\circ C^2</td>
</tr>
</tbody>
</table>

Table 2: Correspondence evaluation for the superheater model for superheater agitated by constant flue gas temperature.

Table 3 sums up the simulation results for the steam superheater with varying parameters on the flue gas side. Steam superheater was agitated by virtual flue gas temperature $T_{2v}(0, t)$ that includes other varying parameters such as mass flow rate of the flue gas $M_2$ and flue gas velocity $u_2$ together with heat transfer coefficient $\alpha_{W2}$. Increasing of these parameters leads to more precise results of the model. Introduction of virtual flue gas temperature at superheater’s input eminently increased accuracy of the mathematical model of the steam superheater.

<table>
<thead>
<tr>
<th>correction</th>
<th>$M_2$ [kg/s]</th>
<th>$u_2$ [m/s]</th>
<th>$\alpha_{W2}$ [W/m²/K]</th>
<th>$J$ [°C²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 %</td>
<td>98.23</td>
<td>4.9</td>
<td>71.8</td>
<td>0.6241</td>
</tr>
<tr>
<td>5 %</td>
<td>103.14</td>
<td>5.145</td>
<td>73.8</td>
<td>0.6159</td>
</tr>
<tr>
<td>10 %</td>
<td>108.05</td>
<td>5.39</td>
<td>75.8</td>
<td>0.6081</td>
</tr>
<tr>
<td>15 %</td>
<td>112.96</td>
<td>5.635</td>
<td>77.8</td>
<td>0.6007</td>
</tr>
<tr>
<td>20 %</td>
<td>117.88</td>
<td>5.88</td>
<td>79.8</td>
<td>0.5938</td>
</tr>
<tr>
<td>25 %</td>
<td>122.79</td>
<td>6.125</td>
<td>81.8</td>
<td>0.5872</td>
</tr>
</tbody>
</table>

Table 3: Evaluation of model correspondence for steam superheater model agitated by virtual flue gas temperature signal $T_{2v}(0, t)$.

5 Conclusion

Presented paper gave a description of a new method of determination of flue gas temperature which is backward computed, using mathematical model of steam superheater and steam temperatures being measured at input and output of the superheater. The approach can be used when parameters of the mathematical model approach constructional parameters of the superheater on which the temperatures are measured.

Determination of the operational parameters of the superheater which are consequently used for its mathematical model is a rather challenging and difficult job. There are certain parameters that are hardly measurable such as mass flow rate of the flue gas $M_2$, flue gas velocity $u_2$ and above all convectional heat transfer coefficient $\alpha_{W2}$. Parameter $\alpha_{W2}$ is affected by a lot of constructional parameters such as diameter and displacement of the pipelines, flue gas velocity and also pipeline surface fouling. This fact gives quite big possibilities to adapt or correct the parameters of the steam superheaters on flue gas side. Table 3 shows that increasing corrections of parameters causes increasing of the mathematical model. The main issue that plays the most important role in decreasing the mean value of the virtual flue gas temperature is above all the heat transfer coefficient $\alpha_{W2}$.

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