The Mathematical Model of a Three-Phase Diode Rectifier with Multi-Converter Power Electronic Loads

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Abstract: Dynamic models of power converters are normally time-varying because of their switching actions. Averaging methods are widely used to eliminate the switching behavior to achieve the time-invariant models. This paper presents how to derive the mathematical model of a three-phase diode rectifier feeding both resistive load and controlled buck converters behaving as constant power loads. The DQ modeling method is used to analyze the dynamic model of a three-phase rectifier including the transmission line on AC side, while the generalized state-space averaging (GSSA) modeling method is applied to derive the dynamic model of a buck converter. Intensive time-domain simulations via the well-known software packages with the exact topology models are used to validate the proposed models. The simulation results show that the proposed mathematical models provide high accuracies in both transient and steady-state responses. The reported models are suitable for the system stability analysis and design.

Key-Words: Three-phase diode rectifier; Controlled buck converter; DQ modeling method; Generalized state-space averaging method; Modeling and simulation.

1 Introduction

The power converter models are time-varying in nature due to their switching behaviors. It is very complicated to use the time-varying model for the system analysis and design. Therefore, there are several approaches commonly used for eliminating the switching actions to achieve a time-invariant model. Then, the classical linear control theory can be easily applied to the model for a system analysis and design.

The first method is the GSSA modeling method. This method has been used to analyze many power converters in DC distribution systems [1]-[3], as well as uncontrolled and controlled rectifiers in single-phase AC distribution systems [4],[5] and 6- and 12- pulse diode rectifiers in three phase systems [6]. The second widely used for AC system analysis is that of DQ-transformation theory [7]-[9], in which power converters can be treated as transformers. The DQ modeling method can also be easily applied for modeling a power system comprising vector-controlled converters where the GSSA and AV models are not easily applicable. The DQ models of three-phase AC-DC power systems have been reported in the previous works for stability studies of the power system including a constant power load (CPL) [10]-[12]. The DQ method for modeling the three-phase uncontrolled and controlled rectifier has been reported in [10] and [13], respectively.

From the literature reviews, this paper presents the combination between the DQ modeling approach and the GSSA modeling method to derive the mathematical model of a three-phase rectifier feeding both resistive load and paralleled buck converters in which it has not been reported in the previous publications. According to the advantages of DQ and GSSA methods, the DQ method is selected to analyze the three-phase diode rectifier including the transmission line components on AC side, while the GSSA method is used to analyze the buck converters with their controls. The proposed model derived from both DQ and GSSA methods is validated by the intensive time-domain simulation via the exact topology model. The results show that the proposed mathematical models provide high accuracies in both transient and steady-state responses. In the future work, the reported models will be used for stability studies of the system due to the effect of a CPL.

The paper is structured as follows. In Section 2, the considered system is described. Deriving the dynamic model of the considered power system is fully explained in Section 3. The model in Section 3 is a nonlinear model derived from both DQ and...
GSSA methods called DQ+GSSA model. Therefore, the linearization technique using the first order term of Taylor's series expansion including the steady-state value calculation is fully explained in Section 4. In Section 5, the model validation using the small-signal simulation is illustrated. Finally, Section 6 concludes and discusses the advantages of the DQ and GSSA modeling methods to derive the model of the AC-DC power system with multi-converter power electronic loads.

2 Considered Power System

The considered system is depicted in Fig.1. It consists of a balanced three-phase voltage source, transmission line, three-phase diode rectifier, and DC-link filters feeding a resistive load \((R_{dc})\) and controlled buck converters. It is assumed that the diode rectifier and the buck converter are operated under a continuous conduction mode (CCM) and the higher harmonics of the fundamental are neglected.

3 Deriving the Dynamic Model

In this paper, the DQ modeling method is firstly selected to derive the dynamic model of a three-phase diode rectifier in which such rectifier can be treated as a transformer [10]. As a result, the equivalent circuit of the power system in Fig. 1 can be represented in the DQ frame as depicted in Fig. 2. Note that the equivalent circuit in Fig. 2 is simplified by fixing the rotating frame on the phase of the diode rectifier switching function \((\phi_i=\phi)\). In Fig. 2, the three-phase diode rectifier including the transmission line on AC side is transformed into the DQ frame via the DQ modeling method. The GSSA modeling method is then used to eliminate the switching action of the controlled buck converter. The PI controllers of the current loop (inner loop) and the voltage loop (outer loop) for each buck converter are represented by \(K_{pv1}, K_{ri1}, K_{pi1}, K_{ii1}, K_{pv2}, K_{ri2}, K_{pi2}, \) and \(K_{ii2}\) respectively. From Fig. 2, \(d_1^*\) and \(d_2^*\) can be derived and given in (1).

\[
\begin{align*}
d_1^* &= -K_{pv1}I_{L1} - K_{ri1}I_{L1} + K_{pi1}X_{d1} + K_{ii1}V_{o1} + K_{pi1}\phi_{1v1} \\
&\quad + K_{pi1}\phi_{i1} + K_{pi1}\phi_{i1}V_{o1} \\
d_2^* &= -K_{pv2}I_{L2} - K_{ri2}I_{L2} + K_{pi2}X_{d2} + K_{ii2}V_{o2} + K_{pi2}\phi_{1v2} \\
&\quad + K_{pi2}\phi_{i2} + K_{pi2}\phi_{i2}V_{o2}
\end{align*}
\]

The dynamic model of the system in Fig. 2 using GSSA modeling method with (1) can be expressed as:

![Fig.1. Considered power system](image)
It can be seen in (2) that the state variables $X_{vi1}$, $X_{vi2}$, $X_{vi3}$, and $X_{vi4}$ are also included. Eq. (2) is the nonlinear differential equations. Therefore, (2) can be linearized using the first order terms of the Taylor expansion so as to achieve a set of linear differential equations around an equilibrium point. The details of the DQ+GSSA linearized model of (2) are given in Section 4

4 DQ+GSSA Linearized Model and Steady-State Value Calculation

As mentioned in Section 3, (2) can be linearized using the first order terms of the Taylor expansion so as to achieve a set of linear differential equations around an equilibrium point. The DQ+GSSA linearized model of (2) is then of the form in (3).

\[
\begin{align*}
\dot{x} &= A(x, u) \delta x + B(x, u) \delta u \\
\dot{y} &= C(x, u) \delta x + D(x, u) \delta u
\end{align*}
\]

(3)

where

\[
\delta x = \begin{bmatrix} \delta I_{d1} & \delta I_{d2} & \delta V_{bus1} & \delta V_{bus2} & \delta I_{dc} & \delta V_{d1} & \delta I_{t1} \\
\delta V_{d1} & \delta X_{d1} & \delta X_{d2} & \delta I_{t2} & \delta V_{d2} & \delta X_{d1} & \delta X_{d2} \end{bmatrix}
\]

\[
\delta u = \begin{bmatrix} \delta V_{d1} & \delta V_{d2} \end{bmatrix}^T
\]

\[
\delta y = \begin{bmatrix} \delta V_{d1} & \delta V_{d2} \end{bmatrix}^T
\]
According to DQ+GSSA linearized model in (3), the model needs to define $V_{dc,o}$, $V_{1,o}$, $V_{2,o}$, $I_{11,o}$, $I_{12,o}$, $X_{11,o}$, $X_{12,o}$, $X_{11,0}$, and $X_{12,0}$. In the paper, the power equation can be first applied to determine the steady state values at the AC side, here are $V_{bus,o}$ and $\lambda_o$. Other steady-state values can be calculated by:

$$V_{dc,o} = \frac{3\sqrt{3}}{\pi} \cdot \frac{(\sqrt{2}V_{bus,o}) - \frac{3Leq\gamma^0}{\pi} I_{dc,o} - r_L I_{dc,o}}{L}$$

$$V_{o1,o} = V_{o1}^*, \quad V_{o2,o} = V_{o2}^*$$

$$I_{11,o} = \frac{V_{o1,o}}{R_1}, \quad I_{12,o} = \frac{V_{o2,o}}{R_2}$$

$$X_{11,o} = \frac{V_{o1}}{K_{pp1}V_{dc,o}}, \quad X_{12,o} = \frac{V_{o2}}{K_{pp2}V_{dc,o}}$$

where

$$I_{dc,o} = \begin{bmatrix} \sqrt{3} \cdot \frac{V_{d^*}e^{j\theta - j\lambda_o}}{2} \cdot \bar{V}_{bus,o}e^{-j\lambda_o} \cdot \bar{Z}_{e_{eq}} \end{bmatrix}, \quad \bar{Z} = \sqrt{\frac{R_eq^2 + (\omega L_eq)^2}{\gamma}}$$

$$\gamma = \tan^{-1}\left(\frac{\omega L_eq}{R_eq}\right)$$

### 5 Small-Signal Simulation

The DQ+GSSA linearized model in (3) is simulated for small-signal transients against a corresponding exact topology model as shown in Fig.3. The set of system parameters is given in Table 1 with the voltage loop controllers $K_{pp1} = 0.05$ and $K_{pp2} = 0.05$.
$K_{v_2} = 50$ ($\omega_{n, voltage} = 64$ Hz, $\zeta_v = 1.0$), and the current loop controllers $K_{i_1} = K_{i_2} = 0.7728$ and $K_{i_1} = K_{i_2} = 11040$ ($\omega_{n, current} = 3200$ Hz, $\zeta_i = 0.7$).

Table 1: Parameters of the Power System in Fig. 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_s$</td>
<td>$220$ $V_{rms/phase}$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$2\pi 50$ rad/s</td>
</tr>
<tr>
<td>$R_{eq}$</td>
<td>$0.1$ $\Omega$</td>
</tr>
<tr>
<td>$L_{eq}$</td>
<td>$24$ $\mu$H</td>
</tr>
<tr>
<td>$C_{eq}$</td>
<td>$2$ nF</td>
</tr>
<tr>
<td>$r_L$</td>
<td>$0.01$ $\Omega$</td>
</tr>
<tr>
<td>$r_c$</td>
<td>$0.4$ $\Omega$</td>
</tr>
<tr>
<td>$L_{dc}$</td>
<td>($\Delta I_{dc} \leq 1.5$ A)</td>
</tr>
<tr>
<td>$C_{dc}$</td>
<td>($\Delta V_{dc} \leq 50$ V)</td>
</tr>
<tr>
<td>$L_f = L_2$</td>
<td>($\Delta I_{f} \leq 0.5$ A)</td>
</tr>
<tr>
<td>$C_f = C_2$</td>
<td>($\Delta V_o \leq 50$ mV)</td>
</tr>
<tr>
<td>$R_f = R_2$</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 3. The exact topology model (SimPowerSystem™ of SIMULINK)

Fig. 4. $V_{dc}$, $V_{o1}$ and $V_{o2}$ responses of the system in Fig. 1 to a step change of $V_{o1}^*$ and $V_{o2}^*$. From the result in Fig. 4, an excellent agreement between both models is achieved under the small-signal simulation. It confirms that the mathematical model of the power system with paralleled controlled buck converters and a resistive load derived from both DQ and GSSA methods provide a good accuracy. The DQ+GSSA linearized model in the paper will be also used for the stability analysis due to the effect of CPL in the future work. In addition, it is well known that simulations of power electronic system using software packages (such as MATLAB, PSIM, and etc.) via the exact topology models consume a huge simulation time due to a switching behavior. It is not easily applicable for simulation of complex systems. Hence, the averaging model of the power electronic based system derived by the proposed modeling method in the paper can be used to reduce the simulation time.
6 Conclusion
This paper presents how to derive the dynamic model of the three-phase diode rectifier feeding multi-converter power electronic loads with their controls. The DQ and GSSA modeling methods are used to eliminate the switching behaviour of the power converter in which the DQ method is used to analyze the three-phase rectifier and the GSSA method is also applied to the buck converter. The proposed models are suitable for the system design and simulation. Moreover, it is well known that when the power converters are regulated, they behave as a CPL. This CPL can significantly degrade power system stability margins. Therefore, the dynamic model of the power system is very important. The DQ+GSSA linearized model in the paper can be also used for stability studies in the future work.

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References: