Fast Correlation Algorithm for an Orthogonal Set of Real-Valued Perfect Sequences from the Huffman Sequences

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Abstract: A perfect sequence has the optimal periodic autocorrelation function where all out-of-phase values are zero, and an orthogonal set has orthogonality that the periodic correlation function for any pair of distinct sequences in the set takes zero at the zero shift. The real-valued perfect sequences of period $N = 2^n$ are derived from a real-valued Huffman sequence of length $2^ν + 1$ with $ν ≥ n$ whose out-of-phase aperiodic autocorrelation function takes zero except at the left and right shift-ends. This paper proposes fast periodic auto- and cross-correlation algorithms for an orthogonal set of real-valued perfect sequences of period $2^n$. As a result, the number of multiplications and additions can be suppressed on the order $N \log_2 N$.

Key–Words: periodic sequence, finite-length sequence, perfect sequence, Huffman sequence, orthogonal set, fast correlation algorithm

1 Introduction

The perfect sequence (also called the periodic orthogonal sequence) [1] has the optimal periodic autocorrelation function where all out-of-phase values are zero, and an orthogonal set has orthogonality that the periodic correlation function for any pair of distinct sequences in the set takes zero at the zero shift. The orthogonal set of perfect sequences is useful in various systems, such as synchronous code division multiple access (CDMA) systems, pulse compression radars and digital watermakings[1].

A binary perfect sequence consisting of elements 1 and −1 is only (1, 1, 1, −1) of period 4. On the other hand, any real-valued perfect sequence consisting of elements of real numbers is generated by its general solution [2]. In addition, the real-valued perfect sequence of period $2^n$ is derived from a real-valued Huffman sequence (also called shift-orthogonal finite-length sequence) [3, 4] of length $2^ν + 1$ with positive integers $n$ and $ν$, and $ν ≥ n$ whose out-of-phase aperiodic auto correlation function takes zero except at the left and right shift-ends [5, 6]. Previously, a fast periodic auto correlation algorithm for a perfect sequence of period $2^n$ derived from a real-valued Huffman sequence of length $2^ν + 1$ has been proposed [6].

In this paper, we propose fast periodic auto- and cross-correlation algorithms for an orthogonal set of real-valued perfect sequences of period $2^n$ derived from the Huffman sequence of length $2^ν + 1$, $ν ≥ n$.

2 The Huffman Sequence

2.1 Definition of the Huffman sequence

Let $\hat{a}_M^\ell$ be a real-valued finite-length sequence of length $M$ consisting of real-elements, written as

$\hat{a}_M^\ell = (\hat{a}_M^0, \ldots, \hat{a}_M^{i}, \ldots, \hat{a}_M^{L-1}), \hat{a}_M^\ell \in \mathbb{R}$,

where $\hat{a}_M^i = 0$, $i < 0$, $i > M - 1$, $i$ denotes the order variable, $\ell$ the sequence number and $R$ the set of real numbers. Let $\hat{A}$ be a set of real-valued finite-length sequences $\hat{a}_M^\ell$, written as

$\hat{A} = \{\hat{a}_M^0, \ldots, \hat{a}_M^\ell, \ldots, \hat{a}_M^{L-1}\}$

$= \{\{\hat{a}_M^{0}, \ldots, \hat{a}_M^{\ell}, \ldots, \hat{a}_M^{L-1}\}\}$,

where $L$ denotes the number of sequences in a sequence set, i.e. family size.

The aperiodic correlation function between sequences $\hat{a}_M^\ell$ and $\hat{a}_M^{i'}$ at shift $i'$ is defined by

$\hat{r}_M^{\ell,i'} = \frac{1}{M} \sum_{i=0}^{M-1} \hat{a}_M^{\ell,i} \hat{a}_M^{i,i'} = \frac{1}{M}(\hat{a}_M^{\ell} \otimes \hat{a}_M^{i,i'})$.

(1)
where $\otimes$ denotes the convolution and $\hat{\rho}_{M,i}^{c,e'} = 0$ for $|i'| > M - 1$. If the aperiodic autocorrelation function satisfies

$$
\hat{\rho}_{M,i}^{c,e'} = \begin{cases} 
1 & : i' = 0, \\
0 & : i' \neq 0, \pm(M - 1), \\
\epsilon_{M-1} & : i' = \pm(M - 1),
\end{cases}
$$

(2)

where $\epsilon_{M-1}$ is called a shift-end value and $|\epsilon_{M-1}| \leq 1/2$, the sequence $\hat{a}_{M}^{c,e'}$ is called a real-valued Huffman sequence [3] or a real-valued shift-orthogonal finite-length sequence [4].

### 2.2 The Huffman sequence of length 2

Let $\hat{a}_{M}^{c,e'}$ be a real-valued Huffman sequence with a negative shift-end value $\epsilon_{M-1} < 0$ and length $M$. Let $\hat{b}_{M}^{c,e'}$ be a real-valued Huffman sequence with a positive shift-end value $\epsilon'_{M-1} > 0$. It is easy to give the Huffman sequence of short length from definition of the sequence. From Eqs. (1) and (2), the Huffman sequence $\hat{a}_{2}^{0}$ of length 2 and a shift-end value $\epsilon_{1} < 0$ is solved as

$$
\frac{\hat{a}_{2,0}^{0}}{\hat{a}_{2,1}^{0}} = \left( \frac{1}{\sqrt{2}} \right) \begin{pmatrix} 1 + 2\epsilon_{1} & 1 - 2\epsilon_{1} \end{pmatrix}.
$$

(3)

Similarly, from Eqs. (1) and (2), the Huffman sequence $\hat{b}_{2}^{0}$ of length 2 and a shift-end value $\epsilon'_{1} > 0$ is solved as

$$
\frac{\hat{b}_{2,0}^{0}}{\hat{b}_{2,1}^{0}} = \left( \frac{1}{\sqrt{2}} \right) \begin{pmatrix} 1 + 2\epsilon'_{1} & 1 - 2\epsilon'_{1} \end{pmatrix}.
$$

(4)

The sequence $\hat{b}_{m}^{0}$ of length $m \geq 3$ and a shift-end value $\epsilon'_{m-1} > 0$ is obtained by insertion of zero values between neighboring values of the sequence $\hat{b}_{2}^{0}$ of length 2 and multiplication by the constant for normalization as follows.

$$
\hat{b}_{m}^{0} = \sqrt{\frac{m}{2}} \left( \hat{b}_{2,0}^{0}, \hat{b}_{2,1}^{0} \right).
$$

(5)

Note that a shift-end value $\epsilon'_{m-1}$ of the sequence $\hat{b}_{m}^{0}$ of length $m \geq 3$ is equal to a shift-end value $\epsilon'_{1}$ of the sequence $\hat{b}_{2}^{0}$ of length 2.

Sequences $\hat{a}_{2}^{0}$ and $\hat{b}_{m}^{0}$, $m \geq 2$ with a sequence number 1 can be replaced by reversed sequences

$$
\hat{a}_{2}^{1} = (\hat{a}_{2,1}^{0}, \hat{a}_{2,0}^{0}), \\
\hat{b}_{m}^{1} = (\hat{b}_{m,0}^{0}, \ldots, \hat{b}_{m,j}^{0}, \ldots, \hat{b}_{m,0}^{0}).
$$

### 2.3 The Huffman sequence of length $2^{\nu + 1}$ derived from the sequence of length 2

The real-valued Huffman sequence $\hat{a}_{M}^{c,e'}$ of length $M = 2^{\nu + 1}, \nu = 1, 2, \ldots$ and a shift-end value $\epsilon_{M-1} < 0$ is given by the $\nu$-multiple convolution of a real-valued Huffman sequence $\hat{a}_{2}^{0}$ of length 2 and a shift-end value $\epsilon_{1} < 0$ and the sequences $\hat{b}_{2}^{0}, \hat{b}_{3}^{0}, \ldots, \hat{b}_{M+1}^{0}$ of length $2, 3, \ldots, M+1 (= 2^{\nu+1} - 1)$ and shift-end values $\epsilon'_{1}, \epsilon'_{2}, \ldots, \epsilon'_{M+1} > 0$ as follows [4].

$$
\hat{a}_{M,i}^{c,e'} = K(\hat{a}_{2,i}^{0} \otimes \hat{b}_{2,i}^{0} \otimes \hat{b}_{3,i}^{0} \otimes \cdots \otimes \hat{b}_{M+1,i}^{0}),
$$

(6)

$$
K_{2m-1} = \frac{\sqrt{2m-1}}{m \sqrt{1 - 2\epsilon^{2}_{m-1}}},
$$

(7)

where $m = 2^{\mu} + 1, \mu = 0, 1, \ldots, \nu - 1$, $\otimes$ denotes the convolution as

$$
\hat{a}_{m,i}^{c,e'} \otimes \hat{b}_{n,j}^{0} = \sum_{k=0}^{m-1} \hat{a}_{m,k}^{c} \hat{b}_{n-j-k}^{0}
$$

and $\ell_{2}, \ell_{2}', \ell_{3}, \ldots, \ell_{M+1}$ denote the sequence numbers. The sequences $\hat{a}_{2}^{0}$ and $\hat{b}_{m}^{0}$, $m = 2^{\mu} + 1, \mu = 0, 1, \ldots, M+1$ are called elementary sequences. From a shift-end value $\epsilon_{M-1}$ of the Huffman sequence $\hat{a}_{M}^{c,e'}$ of length $M$, shift-end values $\epsilon'_{M+1}, \ldots, \epsilon'_{2}, \epsilon'_{1}, \epsilon_{1}$ of elementary sequences $\hat{b}_{M+1}^{0}, \ldots, \hat{b}_{2}^{0}$ and $\hat{a}_{2}^{0}$ are solved as

$$
\left\{\begin{array}{l}
\epsilon'_{M+1} = -\sqrt{-\epsilon_{M-1}} < 0, \\
\epsilon'_{M+1} = -\epsilon_{M+1} > 0,
\end{array}\right.
$$

(8)

These elementary sequences are derived from the Huffman sequence of length 2 shown in Eqs. (3), (4) and (5).

A set of the Huffman sequences $\hat{a}_{M}^{c,e'}$ is constructed by combinations of original elementary sequences with a sequence number 0 and reversed elementary sequences with a sequence number 1. A sequence number $\ell$ of the Huffman sequence $\hat{a}_{M}^{c,e'}$ is expressed in a binary notation as

$$
\ell = (\ell_{M+1}, \ldots, \ell'_{1}, \ell_{2}, \ell_{2}) = \ell_{2} + 2 \sum_{k=0}^{\nu-1} 2^{k} \ell'_{2k+1}.
$$
3 A Perfect Sequence

3.1 Definition of a perfect sequence

Let $a_N^{i,s}$ be a real-valued periodic sequence of period $N$ consisting of real elements, written as

$$a_N^{i,s} = \{a_N^{i,s}, \ldots, a_N^{i,s}, a_N^{i,s}\}, a_N^{i,s} \in \mathbb{R},$$

where $a_N^{i,s+kN} = a_N^{i,s}$ with an integer $k$. $i$ denotes the order variable, $j$ the sequence number, $s$ the set number and $R$ the set of real numbers. Let $A^s$ be a set of real-valued periodic sequences $a_N^{i,s}$, written as

$$A^s = \{a_N^{0,s}, \ldots, a_N^{i,s}, \ldots, a_N^{L-1,s}\},$$

where $L$ is the number of sequences in a sequence set, i.e. family size.

A periodic correlation function between sequences $a_N^{i,s}$ and $a_N^{j,s}$ at a shift $i'$ is defined by

$$\rho_{N,i,j}^{i',s} = \frac{1}{N} \sum_{i=0}^{N-1} a_N^{i,s} a_N^{i',s}(i' \mod N)$$

where $\rho_{N,i,j}^{i',s}$ for any integer $k$. Equation (9) means the autocorrelation function for $j = j'$ and the cross one for $j \neq j'$. If the periodic autocorrelation function satisfies

$$\rho_{N,i,j}^{i',s} = \begin{cases} 1 & ; i' = 0, \\ 0 & ; i' \neq 0, \end{cases}$$

the sequence $a_N^{i,s}$ is called a real-valued perfect sequence [1] or a real-valued periodic orthogonal sequence [2]. In addition, if the periodic correlation function satisfies

$$\rho_{N,i,j}^{i',s} = \begin{cases} 1 & ; i' = 0, j = j', \\ 0 & ; i' \neq 0, j = j', \\ 0 & ; i' = 0, j \neq j, \end{cases}$$

a set $A^s$ of sequences $a_N^{i,s}$ is called an orthogonal set of real-valued perfect sequences.

3.2 Construction of an Orthogonal set of perfect sequences

An orthogonal set of real-valued perfect sequences $a_N^{i,s}$ of period $N = 2^n$ is derived from the Huffman sequences $a_M^{i,s}$ of length $M = 2^n + 1$ with $v \geq n$. A perfect sequence $a_N^{i,s}$ of period $N$ in an orthogonal set is expressed as

$$a_N^{i,s} = \sqrt{\frac{N}{M(1 + 2\varepsilon_{M-1})}} \{a_M^{i,s} \otimes \Delta_{N,i-j}\}$$

$$= K \{\Delta_{N,j} \otimes \hat{a}_2^{i,j} \otimes \hat{b}_2^{i,j} \otimes \cdots \otimes \hat{b}_{N-1}^{i,j}\}$$

$$\otimes \hat{b}_{M+1}^{i,j} \otimes \cdots \hat{b}_{M+N-1}^{i,j}\}$$

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$$= K \{\Delta_{N,j} \otimes \hat{a}_2^{i,j} \otimes \hat{b}_2^{i,j} \otimes \cdots \otimes \hat{b}_{M+1}^{i,j}\}$$

$$\otimes \hat{b}_{M+1}^{i,j} \otimes \cdots \hat{b}_{M+N-1}^{i,j}\},$$

where $\Delta_{N,j}$ denotes the impulse sequence of period $N$ and $\delta_{i,kN}$ the Kronecker delta, and

$$\{\hat{a}_2^{i,j}\} = \{\hat{a}_2^{i,j} \otimes \Delta_{N,j}\} = \{\hat{a}_2^{i,j}, 0, \ldots, 0\},$$

$$\{\hat{b}_m^{i,j}\} = \{\hat{b}_m^{i,j} \otimes \Delta_{N,j}\}$$

and $\alpha_{2+i,kN} = \alpha_{2,i}^{(s)}, \beta_{m+i,kN}^{i,m} = \beta_{m,i}^{i,m}$ for any integer $k$. A shift-end value $\varepsilon_{M-1}$ of the Huffman sequence $a_M^{i,s}$ should be chosen a value to reduce the absolute value of perfect sequences $a_N^{i,s}$ in the set.

A sequence number $j$ and a family number $s$ are expressed by

$$j = (\ell_2^{s+1,1}, \ell_2^{s+1,2}, \cdots, \ell_2^{s+1,1})$$

$$s = \ell_2^r,$$

respectively, where $\ell_2^r \neq \ell_2^r$. Therefore, we can construct two orthogonal sets $A^0$ and $A^1$ of perfect sequences of period $N = M - 1$ or $(M - 1)/2$, family size $L = (M - 1)/2$ and set number $s = \ell_2^r = 0$ and 1 with $\ell_2^r \neq \ell_2^r$. This orthogonal set with $N = (M - 1)/2$ can reach the upper bound $L = N$ on family size [1].

As an example of orthogonal sets, let us construct two orthogonal sets $A^0$ and $A^1$ of 8 perfect sequences $a_N^{i,s}$ of period $N = 8$ derived from the Huffman sequences $a_M^{i,s}$ of length $M = 17$ and a shift-end value $\varepsilon_{16} = -0.00073$. From Eq. (11), a perfect sequence
Table 1: Two orthogonal sets \( A^0 \) and \( A^1 \) of 8 real-valued perfect sequences \( a_{ij}^{ls} \) of period \( N = 8 \) derived from the real-valued Huffman sequences \( \hat{a}_{17}^l \) of length \( M = 17 \) and the shift-end value \( \epsilon_{16} = -0.00073 \).

<table>
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<tr>
<th>( i )</th>
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\( a_{ij}^{ls} \) is generated by convolution of the Huffman sequence \( \hat{a}_{17}^l \) of length 17 and the impulse sequence \( \Delta_{s,j}^l \) of period 8. Table 1 shows two orthogonal sets \( A^0 \) and \( A^1 \) of 8 perfect sequences \( a_{ij}^{ls} \) of period \( N = 8 \).

4 Fast Correlation Algorithm for an Orthogonal Set of Perfect Sequences

4.1 Fast autocorrelation algorithm

Let \( d \) be an input sequence of length \( N \) consisting of real-elements, written as

\[
d = \{d_i\} = \{d_0, \ldots, d_i, \ldots, d_{N-1}\}, \quad d_i \in R,
\]

where \( d_i = 0, i < 0, i > N - 1 \). From Eq. (9), the periodic autocorrelation function between a perfect sequence \( a_{ij}^{ls} \) and an input sequence \( d \) is given by

\[
x_r = \frac{1}{N} \sum_{i=0}^{N-1} d_i a_{N(i-r)}^{ls} \mod N,
\]

(14)

where \( x_r = \{x_r\} = \{x_0, \ldots, x_r, \ldots, x_{N-1}\}, \quad x_r \in R \).

In addition, Eq. (14) is replaced by a determinant of matrix as

\[
X = \frac{1}{N} A_N^{ls} \cdot D,
\]

(15)

where \( X = [x_0, \ldots, x_r, \ldots, x_{N-1}]^T \), \( D = [d_0, \ldots, d_i, \ldots, d_{N-1}]^T \), \( X^T \) denotes a transposed matrix of \( X \), and

\[
A_N^{ls} = \left[ \{a_{N}^{ls}, \{a_{N,i}^{ls} \otimes \Delta_{N,i-1}, \ldots, \{a_{N}^{ls} \otimes \Delta_{N,N-(N-1)} \}\}^T \right]
\]

(15)

Therefore, the number of multiplications \( N_{mult} \) and additions \( N_{add} \) are given by \( N^2 + N \) and \( N(N - 1) \), respectively.

From Eq. (11), Eq. (15) is factorized as

\[
X = \frac{K}{N} R^j \cdot \hat{B}_m^{\ell_1} \ldots \hat{B}_m^{\ell_p} \cdot \hat{B}_2^{\ell_1} \cdot \hat{B}_2^{\ell_2} \cdot \hat{A}_2^{\ell_1} \cdot D,
\]

(16)
where

$$\widehat{A}_2^i = \begin{bmatrix} a_{2,0}^i, & a_{2,1}^i \otimes \Delta_{N,-1}, & \cdots, & a_{2,L-1}^i \otimes \Delta_{N,-(L-1)} \end{bmatrix}^T$$

and

$$\hat{R}^s_m = \begin{bmatrix} [\hat{b}^m_{s,0}], & [\hat{b}^m_{s,1} \otimes \Delta_{N,-1}], & \cdots, & [\hat{b}^m_{s,L-1} \otimes \Delta_{N,-(L-1)}] \end{bmatrix}^T,$$

$$\mathbf{R} = \begin{bmatrix} \{\Delta_{N,-1}\}, & \{\Delta_{N,-2}\}, & \cdots, & \{\Delta_{N,-(L-1)}\}, & \{\Delta_{N,L}\} \end{bmatrix}^T,$$

where

$$\mathbf{R}^0 = 1,$$

$$\mathbf{R}^s = \mathbf{R} \cdots \mathbf{R}.$$

Therefore, the number of multiplications $N_{mul}$ and additions $N_{add}$ are given by $2N(\log_2(N) + 1) + N$ and $N(\log_2(N) + 1)$, respectively.

As an example, Fig. 1 shows a signal flow of fast periodic autocorrelation algorithm for a perfect sequence $a_{2,1}^i$ of period $N = 8$, sequence number $j = 2$ and set number $s = 1$ derived from the real-valued Huffman sequences $\hat{a}_{10}^i$ of length $M = 17$ and sequence length $\ell = (t_1^s, t_2^s, t_3^s, t_4^s, t_5^s, t_6^s, t_7^s, t_8^s) = (010101)_{10} = 10$. Therefore, the numbers of multiplications $N_{mul}$ and additions $N_{add}$ are $72(=2 \times (3+1) + 8) + 32(=8(3+1))$, respectively.

### 4.2 Fast crosscorrelation algorithm

From Eq. (9), the periodic crosscorrelation function between a perfect sequence $a_{N}^{i,s}$ and an input sequence $d$ is given by

$$y_j = \frac{1}{N} \sum_{i=0}^{N-1} d_i a_{N,j}^{i,s}, \quad (17)$$

where $y = \{y_j\} = (y_0, \cdots, y_j, \cdots, y_{L-1})$, $y_j \in R$. In addition, Eq. (17) is replaced by a determinant of matrix as

$$Y = \frac{1}{N} \mathbf{C}_N^s \cdot \mathbf{D}. \quad (18)$$

where $\mathbf{Y} = [y_0, \cdots, y_j, \cdots, y_{L-1}]^T$, $\mathbf{C}_N^s$ is a matrix of size $L \times N$ and

$$\mathbf{C}_N^s = \begin{bmatrix} a_{N,0}^{0,0,} & \cdots & a_{N,0}^{L-1,0} \\ a_{N,1}^{0,0} & \cdots & a_{N,1}^{L-1,0} \\ \vdots & \cdots & \vdots \\ a_{N,L-1}^{0,0} & \cdots & a_{N,L-1}^{L-1,0} \end{bmatrix}$$

Therefore, the number of multiplications $N_{mul}$ and additions $N_{add}$ are given by $LN + L$ and $L(N-1)$, respectively.

From Eq. (11), Eq. (18) is factorized as

$$\mathbf{Y} = \frac{K}{N} \hat{\mathbf{F}}_{M,s+1} \cdot \hat{\mathbf{C}}_{2^{s+1}} \cdots \hat{\mathbf{C}}_{2^{2}+1} \cdot \hat{\mathbf{C}}_{2} \cdot \hat{\mathbf{B}}_{2}^{\ell_2} \cdot \hat{\mathbf{A}}_{2}^{\ell_2} \cdot \mathbf{D}. \quad (19)$$

where $s = \ell_2$, and $\ell_2 \neq \ell_2$, and

$$\hat{\mathbf{C}}_{2^{s+1}} = \begin{bmatrix} [\hat{b}^{0,0}_{2^{s+1},0} \otimes \Delta_{N,-1}], & \cdots, & [\hat{b}^{0,0}_{2^{s+1},L-1} \otimes \Delta_{N,-(L-1)}], \\
[\hat{b}^{0,1}_{2^{s+1},0} \otimes \Delta_{N,-2}], & \cdots, & [\hat{b}^{0,1}_{2^{s+1},L-1} \otimes \Delta_{N,-(L-2)}], \\
[\hat{b}^{0,2}_{2^{s+1},0} \otimes \Delta_{N,-3}], & \cdots, & [\hat{b}^{0,2}_{2^{s+1},L-1} \otimes \Delta_{N,-(L-3)}], \\
\vdots & \cdots & \vdots \\
[\hat{b}^{L-1,0}_{2^{s+1},0} \otimes \Delta_{N,-(L-1)}], & \cdots, & [\hat{b}^{L-1,0}_{2^{s+1},L-1} \otimes \Delta_{N,-(L-1)}] \end{bmatrix}^T,$$
5 Conclusion

In this paper, we have proposed fast periodic auto- and cross-correlation algorithms for an orthogonal set of real-valued perfect sequences of period $N = 2^n$ which are derived from the real-valued Huffman sequences of length $M = 2^n + 1$ with $n \leq \nu$. The sequence is synthesized by multiple convolutions of $\nu + 1$ elementary sequences which are generated by combinations of original and reversed elementary sequences.

As a result, the number of multiplications and additions can be suppressed from the order of $N^2$ to the order of $N \log_2 N$ by proposed algorithms.

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