Relative Frequency Shift Curves Fitting Using FEM Modal Analyses

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Abstract: - The paper presents the good correlation of the relative shift in frequencies for a cantilever beam with a damage with different levels of damage depth obtained with analytic calculus and the FEM modal analyze. The analytic calculus was developed by the authors and it is valid for all Euler-Bernoulli types of damaged beams. The considered damage is an open damage for the entire width of the beam and the width of opening can be variable. The FEM modal analyses was realized for a damaged cantilever beam and the differences of relative shift in frequencies between FEM and analytic calculus are small than 0.3 percent. The biggest values were obtained for the first vibration modes. The results was validated by experiments.

Key-Words: - vibration, natural frequency, finite element method, relative shift in frequency, damage

1 Introduction

The modeling of damage in beam structures and rotating shafts has been a significant research topic. The models fall into three main categories: local stiffness reduction, discrete spring models, and complex models in two or three dimensions [1]. The simplest methods for finite element models reduce the stiffness locally by reducing a complete element stiffness to simulate a small damage. This approach suffers from problems in matching damage severity to crack depth, and is affected by the mesh density. Alternatively two or three dimensional finite element meshes for beam type structures with a crack may be used. Meshless approaches may also be used, but are more suited to crack propagation studies. No element connectivity is required and so the task of remeshing as the crack grows is avoided, and a growing crack is modeled by extending the free surface corresponding to the crack (Belytschko et al., 1995).

One of the major problems in damage location is the reliance on the finite element model. This model is also an important strength because the very incomplete set of measured data requires extra information from the
model to be able to identify damage location [2]. The quality of the damage location assessment is critically dependent upon the updated model being physically meaningful (Friswell et al., 2001; Link and Friswell, 2003). This requires model validation using a control set of data not used for the updating, or using differences between the damaged and undamaged response data in the damage location algorithm (Parloo et al., 2003; Titurus et al., 2003b).

Damage detection using natural frequency shifts is largely presented in literature [3], [4] and [5]. Due to low sensitivity of frequencies shifts to damage very precise measurements are required.

Direct methods use finite elements models, where elements in stiffness, mass or damping matrices are changed in order to tune the models with the real structure. In most cases a large number of elements in the matrices may be changed; this is a major problem for damage location. Sensitivity-based methods use continuous or finite elements models, allowing a wide choice of physically meaningful parameters. The idea is to fit the parameters in order to minimize the difference between modal quantities like natural frequencies or mode shapes of the measured data and model predictions.

The paper try to put in evidence the deviations of the relative shift in frequencies for a damage beam between analytic calculus and results from FEM modal analyses. In the paper an Euler-Bernoulli beam type will be considered as example.

2 The simple cantilever beam

Taking in consideration the equation describing the mode shape \( \phi_i(x) \) for the cantilever beam:

\[
\phi_i(x) = (cosh \alpha L - cos \alpha x) - \frac{(cos \alpha L + cosh \alpha L)}{(sinh \alpha L + sinh \alpha L)} x (sinh \alpha L - sin \alpha x)
\]  

(1)

where, \( L \) is length of the beam, \( x \) distance measured from the clamped end, \( \alpha \) is according to relation (2) and \( \omega \) represents the angular frequency.

\[
\frac{\rho A \omega^2}{EI} = \alpha^4
\]  

(2)

The beam’s geometrical characteristics are: length \( L \), width \( B \) and height \( H \). Consequently, the beam has the cross-section \( A \) and the moment of inertia \( I \). The mechanical characteristics are: mass density \( \rho \), Young’s modulus \( E \) and Poisson’s ratio \( \mu \).

The solutions of the equation (3) permit the calculus of the natural frequencies for the cantilever beam.

\[
I + cos \lambda \cdot cosh \lambda = 0
\]  

(3)

with \( \lambda = \alpha L \), which permits to calculate the \( \lambda_i \) values for \( i \) vibrations modes.

The natural frequencies for the undamaged beam may be determined according to (4):

\[
f_i = \frac{\lambda_i^2}{2 \pi} \sqrt{\frac{EI}{\rho A L}}
\]  

(4)

The analytic calculus was developed by the authors after laborious research and the natural frequencies for the damaged beam are determined with relation (5):

\[
f_{D} = f_i (1 - c_{i,\delta} \cdot \left( \frac{h - \delta}{h} \right)^2 \left( \frac{E_i}{E} \right) \left( \frac{I}{I} \right)^2)
\]  

(5)

where, \( c_{i,\delta} \) is a coefficient depending the vibration mode, level of damage depth and width of the damage, \( \delta \) is the damage depth.

For example, for an undamaged cantilever beam of 1 m length, 50 mm width, 5 mm height, Young’s modulus \( E = 2.0 \cdot 10^{11} \) N/m² and Poisson’s ratio \( \mu = 0.3 \) (made of steel), the first ten natural frequencies are given in table 1 and the \( c_{i,\delta} \) coefficients are presents in table 2.

<table>
<thead>
<tr>
<th>Vibration Mode ( i )</th>
<th>Natural frequency ( f_i ) [Hz]</th>
<th>Vibration Mode ( i )</th>
<th>Natural frequency ( f_i ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.08986</td>
<td>6</td>
<td>347.4518</td>
</tr>
<tr>
<td>2</td>
<td>25.6266</td>
<td>7</td>
<td>485.4578</td>
</tr>
<tr>
<td>3</td>
<td>71.7545</td>
<td>8</td>
<td>646.5624</td>
</tr>
<tr>
<td>4</td>
<td>140.6275</td>
<td>9</td>
<td>830.7827</td>
</tr>
<tr>
<td>5</td>
<td>232.5200</td>
<td>10</td>
<td>1038.1089</td>
</tr>
</tbody>
</table>

Table 1

<table>
<thead>
<tr>
<th>Vibration Mode ( i )</th>
<th>Level of damage depth ( \delta ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>42</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
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<tr>
<td>5</td>
<td>58</td>
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<tr>
<td>6</td>
<td>72</td>
</tr>
<tr>
<td>7</td>
<td>82</td>
</tr>
<tr>
<td>8</td>
<td>106</td>
</tr>
<tr>
<td>9</td>
<td>162</td>
</tr>
<tr>
<td>10</td>
<td>220</td>
</tr>
</tbody>
</table>

Table 2
natural frequencies for the damaged beam with a damage depth of 25% of beam height at first vibration mode and figure 2 presents the distribution of natural frequencies for the damaged beam with a damage depth of 42% of beam height at sixth vibration mode with.

3 Numerical analysis

A similar beam with that presented in the example of section 2 was analyzed using the finite element method (FEM), both in the undamaged and damaged case. The boundary conditions are presented in figure 3.

Fig. 3. Boundary condition for a cantilever beam

The 3D beam was meshed with tetrahedral elements with element size of 2 mm, using smoothing high and transition slow. For the damaged area, the mesh is refinement and after solving the mesh it was obtained 2414576 elements and 3424140 nodes. A detail of the obtained mesh is illustrated in figure 4.

Fig. 4. Detail of the mesh in the damaged area

A series of damages placed separately one after the other on 190 locations along the whole length of the beam were modeled, for each location of damage were considered nine levels of depth. Finally, the static and modal FEM analyses were solved for 190x9=1710 cases of damaged beam and another one case for undamaged beam. The considered damage is an open damage for the entire width of the beam and the opening of the damage is 2 mm.

After solving the static analyze it obtains directional deformation and normal stress (fig. 5).

Fig. 5. Normal stress in the damage area
Figure 6 presents the distribution of natural frequencies for the damaged beam with a damage depth of 58% of beam height at second vibration mode obtained with FEM modal analyses, and figure 7 presents the distribution of natural frequencies for the damaged beam with a damage depth of 17% of beam height at fifth vibration mode.

4 Relative shift in frequency
To evaluate the results of FEM modal analyses and to compare them with relation (5), we introduced the term: relative shift in frequency. The relative shift in frequency is defined with relation (6):

$$\frac{f_{i-D}^{FEM} - f_{i-U}^{FEM}}{f_{i-U}^{FEM}} \cdot 100 \%$$

where, $f_{i-U}^{FEM}$ [Hz] represents the natural frequencies for undamaged beam calculate with FEM modal analyze, $f_{i-D}^{FEM}$ are the natural frequencies for damaged beam for each $D_j$ position of damage and each $\delta_k$ level of depth damage.
For each vibration mode, and each level of damage depth, the relative shift in frequency analytic calculated and numerical obtained will be graphical represented. The differences between two methods are represented and analyzed.

Figure 8 represents the relative shift in frequency fitting curves, both analytic and FEM obtained, for the first mode of vibration of a damaged beam with 50% damage depth and figure 9 shows the FEM curve deviation.

It can be observed that the differences between FEM modal analyses and analytic calculus not exceed 0.3 percent. That means a very good comparison between two methods.

Figure 10 represents the relative shift in frequency fitting curves, both analytic and FEM obtained, for the third mode of vibration of a damaged beam with 33% damage depth and figure 11 shows the FEM curve deviation. The deviation between relative shift in frequency FEM modal analyze and analytic calculus, for tenth vibration mode and damage depth of 17% is shown in the figure 12.

According to figures 9, 11 and 12, the differences between FEM modal analyses and analytic calculus are decreasing at higher modes of vibrations.

4 Conclusion

The method presented in the paper, applicable to beams with open cracks, is based on certain phenomena characteristic to the dynamic behaviour of beams, highlighted as a result of several analytical, numerical and experimental studies developed by the authors.

Computers’ software and hardware development consent to a deeper and much more precise analysis of acquired signals, leading to a deep understanding of material behaviour, both in static and dynamic regime. Regarding the damaged cantilever beams it can be pointed:

- the changes in natural frequencies of beams, due to damages, are significantly influenced by the damage’s location, while its depth just amplifies this effect;
- for each \(i\) vibration mode there are \(i\) locations of the damage for which the corresponding natural frequency remains unchanged, irrespective of the damage depth (see figure 1, 2, 6 and 7), respectively the relative shift in frequencies are zero (see figure 8 and 10);
- for each \(i\) vibration mode there are \(i\) locations of the damage for which the corresponding natural frequency exhibits local minima, amplified by the depth of the damage (see Figure 1, 2, 6 and 7), respectively the relative shift in frequencies are local maxima (see figure 8 and 10);
- the \(2i\) positions of the defect corresponding to the \(i\) vibration modes are closely connected to the critical points of the mode shape (see figure 13).

In this case, for the eighth vibration mode, there are eight inflexion points in the mode shape which represents local maxima in frequencies changes and there are other eight points for local maxima/minima in the mode shape which represents local minima in frequencies changes.

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