How to Correlate Vibration Measurements with FEM Results to Locate Damages in Beams

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Abstract: - The paper presents a new method to detect damages in beams and assess their location and severity by means of vibration measurements. Typically, changes in natural frequency of beams are relatively small, even for an important reduction of their cross-section; this is one of the major difficulties in locating damages in this way. Our research takes into account the particular manner in which the natural frequencies of the first ten bending modes change due to the occurrence of damage. It is a pattern recognition problem, the measured frequency changes being compared with values contained in a database, determined using the FEM. By comparing the shift in frequency obtained through measurements with those determined using the FEM it was possible to locate damages and estimate their geometry with high accuracy, which leads to the conclusion that the method can be successfully used. The method was validated by experiments.

Key-Words: - vibration, natural frequency, finite element method, damage detection, beam

1 Introduction
Regular inspection and control of engineering structures is necessary to detect damages in real time and determine the safety and reliability of the structure. Early damage identification allows properly programmed maintenance with impact on exploitation costs diminution, or the putting out of operation and replacement to avoid accidents. A large series of non-destructive testing methods can be used to detect damages. The conventional ones permit a precise characterization of the damage, but their local nature requires good preliminary knowledge about the position of the area where the damage is located. Among conventional methods we can list visual inspection, magnetic particles, ultrasonic analysis, X- and gamma-ray control etc.
On the other hand, dynamic methods, due to their global perspective, are able to indicate the appearance of possible damages even in large structures, locate the damaged area, but provide little information regarding the characteristics of the damage(s). An important advantage of the dynamic methods is linked to the fact that these techniques do not request access to the damaged area. The two methods do not exclude each other; they can be used complementarily [1].

The process of implementing damage detection strategies implies the definition of the healthy system and that of potential damage scenarios, periodical examination using measurements, the extraction of features from these measurements, and the analysis of these features to determine the current state of health of the system [2]. The result of this process is periodically updated with information regarding the capability of the system to fulfill its designed function under operational loads and environmental actions.

Damages influence the dynamic behavior of structures, changing their mechanical and dynamic characteristics such as natural frequencies, mode shapes, damping ratio, and stiffness or flexibility. These most common features that are used in damage detection are identified from measured response time-histories (most often accelerations or strains) or spectra of these time-histories.

Damage detection using natural frequency shifts is largely presented in literature [3], [4] and [5]. The methods based on frequency change can be classified in two categories: methods limited to damage detection and methods destined to detect, locate and quantify damages. Literature reviews, [4] and [6], affirm that all methods based on natural frequency changes belong to the second category are model-based, typically relying on the use of finite element models. Due to low sensitivity of changes in frequencies to damage, very precise measurements are required. Often the changes in the frequency caused by structural damage are smaller than those observed between the undamaged structure and the mathematical model. This makes almost impossible to discern between inadequate modeling and changes due to damage, consequently the use of models is difficult to be used, [7] and [8]. Other problems in using natural frequencies shifts to detect damages reside in the fact that natural frequencies are sensitive to changes in temperature and loads applied on the structure.

Methods based on change of mode shapes compare differences between the measured modal shapes before and after damage. A single-number measure of mode shape changes, used to detect damages, is the Modal Assurance Criterion (MAC) [9], which compares a mode shape in the undamaged and damaged states, respectively. The Co-ordinate Modal Assurance Criterion (COMAC) combines information from different modes and is able to indicate the probable location of damage [10]. Curvatures, as second order derivatives of mode shapes in respect to position, are sensitive to damage and can also be used [11]. The use of change in modal strain energy (MSE) to detect structural damages, introduced in [12], is based on the decrease in modal strain energy caused by damage and is known in the literature as the damage index method. In all cases the mode shape vectors must be one-to-one associated to the measurement points coordinates. The main disadvantage of this class of methods is the large number of measurement locations required to accurately characterize mode shape vectors and to provide sufficient resolution for determining the damage location. Additionally, because COMAC and MSE use finite elements models with multiple degrees of freedom, the sensor locations must be carefully matched to these finite element degrees of freedom.

Methods based on flexibility of the structure constitute another class of damage identification methods. They make use of dynamically measured flexibility matrix to estimate changes in the static behavior of a structure, in most cases completed with static measurements. Several variants of this method are known, like the change in flexibility method, the change in uniform flexibility curvature method and other methods having the same principle but different procedures, largely described in [13]. Another class of damage identification methods is based on fitting the behavior of analytical models as closely as possible with that of the real structure by adjusting some elements of the model. Both direct methods, one of the first of this class used in damage detection, as well as sensitivity-based methods are discussed in detail in [14]. Direct methods use finite elements models, where elements in stiffness, mass or damping matrices are changed in order to tune the models with the real structure. In most cases a large number of elements in the matrices may be changed; this is a major problem for damage location. Sensitivity-based methods use continuous or finite elements models, allowing a wide choice of physically meaningful parameters. The idea is to fit the parameters in order to minimize the difference between modal quantities like natural frequencies or mode shapes of the measured data and model predictions. One of the problems in model-based vibration-based damage detection is the need for a very accurate mathematical model, differences...
between the real structure and the model should be significantly lower than changes occurring due to damages in the structure. Other methods are of course available; a comprehensive classification and description is made in [4].

Each method, due to its specific advantages, fits a particular application, although no method meets all the requirements imposed by the high variety of analyzed structures and the diverse conditions imposed.

In this paper a new method to detect, locate and evaluate damages in a beam is presented. The method is based on a new approach regarding the interpretation of natural frequency changes for multiple modes.

2 Theoretical background
Since the aim of the research was to develop a simple method to detect, localize and quantify the severity of damages with the least equipment possible, weak-axis bending vibrations were considered and the theory has been developed for this case. In our research we have studied the natural frequency changes on cantilever, simple supported and double clamped beams. For this paper we have chosen to present the cantilever beam, which, due to its asymmetry, is more complex, but uniquely defines the damage localization. The other two types of beams, due to their symmetry, can be treated more easily, but two possible damage locations are found, symmetrical to the beam centre. Future investigations have to eliminate one of them.

We can write the equation describing the bending movement for a prismatic beam like:

\[
\frac{\partial^4 v}{\partial x^4} + \frac{\rho A}{EI} \frac{\partial^2 v}{\partial t^2} = 0
\]

(1)

where \( v \) is the vertical displacement of the beam at distance \( x \) measured from the clamped end. The beam’s geometrical characteristics are: length \( L \), width \( B \) and height \( H \). Consequently, the beam has the cross-section \( A \) and the moment of inertia \( I \). The mechanical characteristics are: mass density \( \rho \), Young’s modulus \( E \) and Poisson’s ratio \( \mu \). Considering that \( v \) depends on distance \( x \) and time \( t \), and the evolution in time is harmonic, the expression of displacement is:

\[
v(x,t) = \phi(x) \cdot y(t) = \phi(x) \cdot \sin(\omega t + \varphi)
\]

(2)

After derivation of relation (2) and substitution in relation (1), one obtains:

\[
\phi''''(x) \sin(\alpha t + \varphi) - \frac{\rho A \omega^2}{EI} \phi(x) \sin(\alpha t + \varphi) = 0
\]

(3)

or, after simplifying by \( \sin(\omega t + \varphi) \)

\[
\phi''''(x) - \frac{\rho A \omega^2}{EI} \phi(x) = 0
\]

(4)

with the solution:

\[
\phi(x) = \tilde{A} \sin x + \tilde{B} \cos x + \tilde{C} \sinh x + \tilde{D} \cosh x
\]

(5)

where we noted

\[
\frac{\rho A \omega^2}{EI} = \alpha^4
\]

(6)

After three derivations of the solution (5), it results the system:

\[
\begin{align*}
\phi(x) &= \tilde{A} \sin x + \tilde{B} \cos x + \tilde{C} \sinh x + \tilde{D} \cosh x \\
\phi'(x) &= \tilde{A} \cos x - \tilde{B} \sin x + \tilde{C} \cosh x + \tilde{D} \sinh x \\
\phi''(x) &= \tilde{A} \sin x - \tilde{B} \cos x + \tilde{C} \sinh x + \tilde{D} \cosh x \\
\phi''''(x) &= \tilde{A} \cos x - \tilde{B} \sin x - \tilde{C} \cosh x - \tilde{D} \sinh x
\end{align*}
\]

These derivatives are proportional with the vertical displacement \( v(x) \), rotation \( \theta(x) \), bending moment \( M(x) \) and shear force \( T(x) \). Putting the boundary condition for the clamped cantilever beam, given by mechanical reasons

\[
\phi(0) = \phi'(0) = \phi''(1) = \phi''''(1) = 0
\]

(7)

to find the constants \( \tilde{A}, \tilde{B}, \tilde{C} \) and \( \tilde{D} \) it obtains the equation

\[
1 + \cos \lambda \cdot \cosh \lambda = 0
\]

(8)

with \( \lambda = \alpha L \), which permits to calculate the \( \lambda_i \) values for \( i \) vibrations modes. Multiplying the relation (6) by \( L^4 \) and substituting the values of \( \lambda_i \), we obtained the analytic calculated angular frequencies \( \omega_i \), and consequently the natural frequencies:

\[
f_i = \frac{\lambda_i^2}{2\pi} \sqrt{\frac{EI}{\rho A L^4}}
\]

(9)

The first ten natural frequencies analytically calculated are given in table 1.

<table>
<thead>
<tr>
<th>Mode ( i )</th>
<th>Natural frequency ( f_i ) [Hz]</th>
<th>Mode ( i )</th>
<th>Natural frequency ( f_i ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.076904</td>
<td>6</td>
<td>346.182256</td>
</tr>
<tr>
<td>2</td>
<td>25.549518</td>
<td>7</td>
<td>483.510758</td>
</tr>
<tr>
<td>3</td>
<td>71.539391</td>
<td>8</td>
<td>643.72734</td>
</tr>
<tr>
<td>4</td>
<td>140.188654</td>
<td>9</td>
<td>826.832006</td>
</tr>
<tr>
<td>5</td>
<td>231.74189</td>
<td>10</td>
<td>1032.82475</td>
</tr>
</tbody>
</table>
Regarding the mode shapes, it is obvious that for every angular frequency \( \omega \), there is a corresponding mode shape \( \phi_i(x) \), deductible from the above presented system, considering the boundary conditions (7). Finally, the mode shape \( \phi_i(x) \) is proportional with the function:

\[
\phi_i(x) = (\cosh \alpha x - \cos \alpha x) - \\
\frac{(\cos \alpha L + \cosh \alpha L)}{(\sin \alpha L + \sinh \alpha L)} (\sinh \alpha x - \sin \alpha x)
\] (10)

However, the mode shape indicates only the shape for each mode, without providing information about the amplitude of the displacement.

This leads to the conclusion that the position of the points along the Euler-Bernoulli beam where important or less changes occur, for each vibration mode independently, are not influenced by the beam’s cross-section shape and area. Consequently for every location of a damage result a unique series of values, representing the change in frequency. This phenomenon can be used to locate damages and evaluate their severity [15].

### 3 FEM investigation

To find out the distribution of the frequency changes for cantilever beam, a similar beam with that presented in section 2 was analyzed using the finite element method (FEM), both in the undamaged and damaged case.
The beam was meshed by 2 mm elements for the undamaged case, like presented in figure 3. The first ten natural frequencies of the weak-axis bending modes for the undamaged beam were determined; the values are presented in table 2. One can remark the similarity to the results presented in table 1, which were analytically obtained.

Table 2

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural frequency ( f_i ) [Hz]</th>
<th>Mode</th>
<th>Natural frequency ( f_i ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.097</td>
<td>6</td>
<td>347.46</td>
</tr>
<tr>
<td>2</td>
<td>25.647</td>
<td>7</td>
<td>485.47</td>
</tr>
<tr>
<td>3</td>
<td>71.757</td>
<td>8</td>
<td>646.59</td>
</tr>
<tr>
<td>4</td>
<td>140.63</td>
<td>9</td>
<td>830.81</td>
</tr>
<tr>
<td>5</td>
<td>232.53</td>
<td>10</td>
<td>1038.1</td>
</tr>
</tbody>
</table>

Afterwards, a series of damages placed separately one after the other on 190 locations along the whole length of the beam were modeled. To model the damaged beam also was meshed by 2 mm elements where generally used, but with finer elements in the vicinity of damage. We selected an uncomplicated damage geometry, easy to reproduce on the real structure by saw cuts, with the constant width of 2 mm and 9 levels of depth, reducing the cross-section by 8, 17, 25, 33, 42, 50, 58, 67 and 75% respectively. For all the resulting 1,710 damage cases the first ten natural frequencies were determined. Figure 4 present the extrapolated results, in form of network, for the first three modes.

The frequency changes due to a damage with depth 2.5 mm, width 2 mm and located by a distance \( 0.57L \) from de clamped end, for modes 2 and 5, is presented in figure 5. The values of these changes for the first ten modes, divided by the natural frequency for the corresponding mode, are shown in figure 6. The manner how the values are placed in the diagram is presented with the strong lines at mode 2 and 5 respectively. The diagram contains also the values for the same damage, but with a depth of 3 mm. All values belonging to the depth of 3 mm are unified by the superior dashed line, while the values belonging to the depth of 3 mm are unified by the inferior dashed line. It has to be mentioned that the dashed lines have no physical meaning; they are used just to frame easier a certain damage.

The 1,710 families of curves, determined using the FEM analysis, representing patterns for the same number of damages (with variable depth and location), form a database which is the reference for damage location and evaluation. After determining a series of values for the relative frequency shifts by measurements, for a damaged beam, we can locate and evaluate the damage by comparing these values with that of the database.
The process can be done in two steps, first to find curves with similar allure (which means we locate the damage), afterwards to frame the curve obtained by measurement between similar curves but representing different depths (where we approximate the damage depth). In figure 6, the continuous line with the same allure like the dashed lines represents the measures values. The viability of the method was confirmed by numerous experiments.

4 Conclusion
This paper, as a result of a large study intending to provide a new method to detect, locate and estimate the severity of damages in beams, present the theoretical background of the method and the way how it is used. The method is new, developed by the authors, and base on a new approach in interpreting frequency changes. It permits the location of the damage with high accuracy (max. 3% error) and a good estimation of the depth.

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