Existence and Uniqueness of a Blow-up Solution in Sense of Semigroup Theory of a Degenerate Semilinear Parabolic Problem

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Abstract: We prove under appropriate assumptions the existence and uniqueness in the sense of semigroup theory of a blow-up solution for a degenerate semilinear parabolic problem: $u_t - (a(x)u_x)_x = f(u)$ in $(0,1) \times (0,\infty)$ where f is a given function and $a(0) = 0$, $a(x) > 0$ on $(0,1]$ together with the Dirichlet boundary condition and the suitable initial condition.

Key–Words: Blow-up problems, Semilinear parabolic problems, Semigroup theory, Semilinear evolution problems, Blow-up in finite time, Degenerate parabolic problems

1 Introduction

The subject of blow-up was posed in the 1940's and 50's in the context of Semenov's chain reaction theory, adiabatic explosion and combustion theory. There has been a tremendous amount of recent activities due to the subjects of solutions to various partial differential equations blowing up in finite time. Finite time blow-up occurs in situations in mechanics and other areas of applied mathematics. Studies of these phenomena have very recently been gaining momentum. In the following, we give examples of blow-up problems in the way of blow-up mathematical theory. In 1985, C.E. Mueller and F. B. Weissler [7] studied the semilinear heat equation:

$$\left.\begin{array}{c} u_t = \triangle u - \lambda u + f(u),\ (x,t) \in \Omega \times (0,\infty), \\ u(x,t) = 0,\ x \in \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x),\ x \in \Omega, \end{array}\right\} \quad (1)$$

where Ω is $\mathbb{R}^n$ or Ω is a smooth bounded subset of $\mathbb{R}^n$, $\partial\Omega$ denotes the smooth boundary of Ω, $\triangle = \sum_{i=1}^{n} \partial_i^2$, $\lambda \geq 0$ and f and u_0 are specified functions. Under suitable assumptions, they showed that the solution of (1) blows up in finite time and the blow-up set of blow-up solution consists of only one point. Further, in 2009, J. P. Pinasco [8] established the blow-up positive solutions of problems (2) with reaction terms of local and nonlocal type involving a variable exponent,

$$\left.\begin{array}{c} u_t = \triangle u + f(u),\ (x,t) \in \Omega \times (0,\infty), \\ u(x,t) = 0,\ (x,t) \in \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x),\ x \in \overline{\Omega}, \end{array}\right\} \quad (2)$$

where Ω is a smooth bounded subset of $\mathbb{R}^n$ with smooth boundary $\partial\Omega$ and the source term is of the form $f(u) = a(x)u^{p(x)}$ or $f(u) = a(x)\int_\Omega u^{q(y)}(y,t)dy$ where a, p and q are given functions. For blow-up problems of the degenerate semilinear parabolic type, in 1999, C.Y. Chan and W. Y. Chan [3] studied the existence of a blow-up solution of the degenerate semilinear parabolic initial-boundary value problem

$$\left.\begin{array}{c} x^q u_t - u_{xx} = f(u),\ (x,t) \in \Omega \times (0,\infty), \\ u(0,t = 0 = u(1,t),\ t > 0, \\ u(x,0) = u_0(x),\ x \in [0,1], \end{array}\right\} \quad (3)$$

where $q \geq 0$, f and u_0 are given functions. They proved existence and uniqueness of a blow-up solution of problem (3) by transforming problem (3) into the equivalent integral equation in terms of its associated Green's function. Furthermore, in 2006, C. Y. Chan and W.Y. Chan [4] showed that under certain condition on functions f and u_0, a solution u of problem (3) blows up at every point in $[0,1]$. After paper [3] published, in 2004, Y.P. Chen and C.H. Xie [6] considered the degenerate parabolic problem with the nonlocal term : for any $(x,t) \in (0,1)\times(0,\infty)$,

$$\left.\begin{array}{c} u_t - (x^\alpha u_x)_x = \int\limits_0^1 f(u)dx, \\ u(0,t) = 0 = u(1,t),\ t>0, \\ u(x,0) = u_0(x),\ x\in[0,1], \end{array}\right\} \quad (4)$$

with $\alpha \in [0,1)$ and f and u_0 are given functions. They proved the local existence and uniqueness of a classical solution. Under appropriate hypotheses, they obtained the sufficient conditions for the global existence and for blow-up of a positive solution of problem (4). Additionally, in 2004, Y.P. Chen, Q. Liu and C.H. Xie [5] studied the degenerate nonlinear reaction-diffusion equation with nonlocal source: for any $(x,t) \in (0,1)\times(0,\infty)$,

$$\left.\begin{array}{c} x^q u_t - (x^\alpha u_x)_x = \int\limits_0^1 u^p dx, \\ u(0,t) = 0 = u(1,t),\ t>0, \\ u(x,0) = u_0(x),\ x\in[0,1], \end{array}\right\} \quad (5)$$

They established the local existence and uniqueness of a classical solution of problem (5). Under appropriate hypotheses, they gave the sufficient conditions for a global existence and for blow-up of a positive solution. Furthermore, under certain conditions, they proved that the blow-up set of such a solution of problem (5) is the whole domain. In 2010, P. Sawangtong and W. Jumpen[10] showed, under certain condition, the existence of a blow-up solution of the degenerate parabolic problem: for any $(x,t) \in (0,1)\times(0,\infty)$,

$$\left.\begin{array}{c} x^q u_t - (x^\alpha u_x)_x = x^q f(u), \\ u(0,t) = 0 = u(1,t),\ t>0, \\ u(x,0) = u_0(x),\ x\in[0,1], \end{array}\right\} \quad (6)$$

where $q \geq 0,\ \alpha \in [0,1)$ and f and u_0 are suitable functions. Furthermore the sufficient condition to blow-up in finite time and the blow-up of such a solution of problem (6) are shown. Furthermore, in 2010, P. Sawangtong and W. Jumpen [11] extended problem (6) to more general form: for any $(x,t) \in (0,1)\times(0,\infty)$,

$$\left.\begin{array}{c} k(x)u_t - (a(x)u_x)_x = k(x)f(u), \\ u(0,t) = 0 = u(1,t),\ t>0, \\ u(x,0) = u_0(x),\ x\in[0,1], \end{array}\right\} \quad (7)$$

where $k(0) = 0 = a(0)$, $k, a > 0$ on $(0,1]$ and f and u_0 are given functions. They showed the existence and uniqueness of a blow-up solution of problem (7) by classical method, i.e., Greens'function method. As shown in [11], there are many conditions on functions k and a to obtain the existence of corresponding eigenvalues and eigenfunctions to problem (7) to use their properties in the part of existence of solution of problem (7).

In this paper, we study the following degenerate semilinear parabolic problem closed to problem (7) via semigroup theory:

$$\left.\begin{array}{c} u_t - (a(x)u_x)_x = f(u),\ (x,t)\in(0,1)\times(0,\infty), \\ u(0,t) = 0 = u(1,t),\ t>0, \\ u(x,0) = u_0(x),\ x\in[0,1], \end{array}\right\} \quad (8)$$

where a, f and u_0 are given functions.

The objective of this article is to show the existence of a unique blow-up solution of the problem (8) before blow-up occurs by semigroup theory.

2 Setting out a degenerate problem

We next give the definition of blow-up in finite time.

Definition 1 *A solution u of the problem (8) is said to* **blow-up** *at the point b in finite time T_b if there exists a sequence $\{(x_n,t_n)\}$ with $(x_n,t_n) \in (0,1)\times(0,T)$ and $(x_n,t_n) \to (b,T_b)$ as $n\to\infty$ and $\lim\limits_{n\to\infty} u(x_n,t_n) = +\infty$.*

Because of the function a which expresses the degeneracy we need to introduce a variant of the classical Sobolev space $H^1(0,1)$, namely $H^{1,a}(0,1)$. Throughout this paper, we make the following assumptions on a:

(A) $a \in C^0[0,1]\cap C^1(0,1],\ a > 0$ in $(0,1]$ and $a(0) = 0$;

(B) $\exists K \in [0,1)$ such that $xa'(x) \leq Ka(x)$ for all $x\in[0,1]$.

We note that

1. an example of functions satisfies the conditions (A) and (B) is x^α with $\alpha\in[0,1)$,
2. the condition (B) implies that $\int\limits_0^1 \frac{1}{a(x)}dx$ is finite which is a sufficient condition to obtain that the space $H^{1,a}(0,1)$ is compactly embedded in $L^2(0,1)$.

If u_x denote the derivative in the sense of distribution of the distribution u in $\mathcal{D}'(0,1)$, then

$$H^{1,a}(0,1) = \left\{u\in L^2(0,1) \text{ possessing an absolutely continuous representative on [0,1] and} \sqrt{a}u_x \in L^2(0,1)\right\}.$$

It is known that equipped with the following inner product and norm

$$\langle u,v\rangle_{H^{1,a}(0,1)} = \int_0^1 \left[u(x)v(x) + a(x)u_x(x)v_x(x)\right] dx$$

and

$$\|u\|_{H^{1,a}(0,1)} = \langle u,u\rangle^{1/2}_{H^{1,a}(0,1)}.$$

respectively. The space $H^{1,a}(0,1)$ is a Hilbert space. By due account of the fact that $\int_0^1 \frac{1}{a(x)}dx$ is finite,

$$H^{1,a}_0(0,1) = \left\{u \in H^{1,a}(0,1) \text{ s.t. } u(0) = 0 = u(1)\right\},$$

is a closed subspace of $H^{1,a}(0,1)$ with equivalent norm

$$\|u\|_{H^{1,a}_0(0,1)} = \left\|\sqrt{a}u_x\right\|_{L^2(0,1)},$$

and the injection of $H^{1,a}(0,1)$ and $C^0[0,1]$ is continuous. Eventually we will consider

$$H^{2,a}(0,1) = \left\{u \in H^{1,a}(0,1) \text{ s.t. } au_x \in H^{1,a}(0,1)\right\}$$

with its norm:

$$\|u\|^2_{H^{2,a}(0,1)} = \|u\|^2_{H^{1,a}(0,1)} + \|(au_x)_x\|^2_{L^2(0,1)}.$$

In order to obtain the existence of a blow-up solution u of problem (8), we also make some hypothesis on functions u_0 and f:

(C) $u_0 \in H^{2,a}(0,1) \cap H^{1,a}_0(0,1)$, $u_0 \geq 0$ on $[0,1]$ and $u_0(0) = 0 = u_0(1)$.

(D) f is locally Lipschitz: $\forall M > 0, \exists C_M$ such that $|f(a) - f(b)| \leq C_M |a-b| \, \forall a,b$ with $|a|, |b| \leq M$.

To apply a useful result in the semigroup theory [13], we transform problem (8) into the equivalent semilinear evolution problem:

$$\left.\begin{array}{c} u_t - Au(t) = F(u),\ t > 0, \\ u(0) = u_0, \end{array}\right\} \tag{9}$$

where A is an operator mapping from $D(A)$, the domain of A, into $L^2(0,1)$ with

$$\begin{aligned} & D(A) \\ = & \Big\{u \in H^{1,a}_0(0,1) \text{ s.t. } \exists! w \in L^2(0,1) \text{ satisfies that} \\ & \int_0^1 w(x)\varphi(x)dx = -\int_0^1 a(x)u_x(x)\varphi_x(x)dx, \\ & \text{for all } \varphi \in H^{1,a}_0(0,1)\Big\} \end{aligned} \tag{10}$$

and

$$Au = (au_x)_x = w \text{ for all } u \in D(A) \tag{11}$$

and where F is an operator mapping from $D(A)$ into $L^2(0,1)$ defined by

$$F(u) = f(u) \text{ for all } u \in D(A). \tag{12}$$

3 The main result

Here, we prove that problem (8) has a unique blow-up solution in the sense of semigroup theory.

Theorem 2 *There exists a positive constant T such that the equivalent evolution problem (9) has a unique solution $u \in C([0,T], D(A)) \cap C^1([0,T], L^2(0,1))$ defined by*

$$u(t) = S(t)u_0 + \int_0^1 S(t-\tau)F(u(\tau))d\tau$$

where $S(t)$ is an analytic semigroup generated by the operator A.

Theorem 3 *Let $[0, T_{max})$ be the maximal time interval in which a solution u of problem (9) exists. If f is increasing, then $\lim\limits_{t\to T_{\max}} \max\limits_{x,\in[0,1]} |u(x,t)|$ is unbounded.*

4 The proof of main results

In this section we will give the proof of our main theorems by starting from the proof of theorem 2.

4.1 The proof of theorem 2

In this section, we will first consider some properties of operators A and F defined by (11) and (12), respectively.

4.1.1 Properties of A

Let us state important properties of A:

Proposition 4 *The operator A defined by* (11) *is maximal dissipative and self-adjoint on $L^2(0,1)$ which, consequently, generate an analytic semigroup on $L^2(0,1)$.*

Proof: To prove the maximal dissipative property of A, we have to show two conditions:

1. $\langle Au, u\rangle_{L^2(0,1)} \leq 0$ for all $u \in D(A)$ and

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2. $R(I - \lambda A) = L^2(0,1)$ for any $\lambda > 0$ where $R(I-\lambda A)$ and I denote the range of $I - \lambda A$ and the identity operator on $L^2(0,1)$, respectively.

Condition 1 follows directly from (10), the definition of A. Let $h \in L^2(0,1)$ and λ be any positive constant. For verifying condition 2, we have to show that there exists a unique $u \in D(A)$ such that $u - \lambda Au = h$ which equivalent to show that there exists a unique $u \in D(A)$ such that the following equation holds:

$$\frac{1}{\lambda}\int_0^1 u(x)\varphi(x)dx + \int_0^1 a(x)u_x(x)\varphi_x(x)dx = \int_0^1 h(x)\varphi(x)dx \text{ for all } \varphi \in H_a^1(0,1).$$

Such the existence is guaranteed by Lax-Milgram theorem. Hence, the operator A is maximal dissipative on $L^2(0,1)$. Hence to show that A is self-adjoint it suffices to prove that A is symmetric: let $u, v \in D(A)$. We consider that, by (10),

$$\begin{aligned}\langle Au, v\rangle_{L^2(0,1)} &= -\int_0^1 a(x)u_x(x)v_x(x)dx \\ &= \langle u, Av\rangle_{L^2(0,1)}.\end{aligned}$$

□

The proof of next lemma is not difficult. We can prove directly and then we have:

Lemma 5 $D(A) = H^{2,a}(0,1) \cap H_0^{1,a}(0,1)$.

The next lemma is used to guarantee the existence of corresponding eigenvalues and eigenfunctions of $-A$ refereed to [1].

Lemma 6 *The space $H_0^{1,a}(0,1)$ is compactly imbedded in $L^2(0,1)$.*

Proof: See [1] □

Since the operator $(-A)^{-1}$ is a bounded well-defined operator on $L^2(0,1)$ with values in $H_0^{1,a}(0,1)$, lemma 6 implies that $(-A)^{-1}$ is compact operator on $L^2(0,1)$. The next lemma is the well-known results about the spectral theory of self-adjoint compact operator referred from [2].

Lemma 7 *There exists a sequence $(\lambda_n, \phi_n) \subset (0,+\infty) \times H_0^{1,a}(0,1)$ such that*

1. $A\phi_n = -\lambda_n\phi_n$ *for all* $n \geq 1$,
2. $\int_0^1 \phi_n(x)\phi_m(x)dx = \begin{cases} 0, & n \neq m, \\ 1, & n = m, \end{cases}$
3. $\int_0^1 a(x)\phi_n'(x)\phi_m'(x)dx = \begin{cases} 0, & n \neq m, \\ \lambda_n, & n = m, \end{cases}$
4. $v(x) = \sum_{n=1}^{\infty} \langle v, \phi_n\rangle_{L^2(0,1)} \phi_n(x)$ *for any* $v \in L^2(0,1)$,
5. $\|v\|_{L^2(0,1)}^2 = \sum_{n=1}^{\infty} \langle v, \phi_n\rangle_{L^2(0,1)}^2$ *for any* $v \in L^2(0,1)$,
6. $Av = -\sum_{n=1}^{\infty} \lambda_n \langle v, \phi_n\rangle_{L^2(0,1)} \phi_n(x)$ *for any* $v \in D(A)$ *with* $D(A) = \{v \in L^2(0,1)$ *such that* $\sum_{n=1}^{\infty} \lambda_n^2 \langle v, \phi_n\rangle_{L^2(0,1)}^2 < +\infty\}$.
7. $S(t)v = \sum_{n=1}^{\infty} e^{-\lambda_n t} \langle v, \phi_n\rangle \phi_n$ *for all* $(v,t) \in L^2(0,1) \times [0,\infty)$.

We now can define the domain of $(-A)^{1/2}$ by

$$D((-A)^{1/2}) = \left\{ v \in L^2(0,1) \text{ s.t. } \sum_{n=1}^{\infty} \lambda_n \langle v, \phi_n\rangle^2 < \infty \right\} \tag{13}$$

and the unbounded self-adjoint operator $(-A)^{1/2}$ in $L^2(0,1)$ by

$$(-A)^{1/2}v = \sum_{n=1}^{\infty} \lambda_n^{1/2} \langle v, \phi_n\rangle \phi_n \tag{14}$$

for any $v \in D((-A)^{1/2})$. We then have the following:

Lemma 8 $D((-A)^{1/2}) = H_0^{1,a}(0,1)$ *and* $\|v\|_{D((-A)^{1/2})} = \left\|(-A)^{1/2}v\right\|_{L^2(0,1)} = \|v\|_{H_0^{1,a}(0,1)}$ *and consequently* $D((-A)^{1/2}) \hookrightarrow C^0[0,1]$.

4.1.2 Properties of F

In order to prove lemma 10, we have to use a fact referred to [1]:

Lemma 9 *The space $D(A)$ is completely imbedded in $D((-A)^{1/2})$.*

Proof: See [1] □

Now we state and prove some properties of F.

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Lemma 10 *The operator F defined by (12) is local Lipschitz.*

Proof: Let $u, v \in D(A)$. It follows form lemmas 9 and 8 that there exists a positive constant M such that $|u| \leq M$ and $|v| \leq M$. Locally Lipschitz condition of f and lemma 8 imply that there exists a positive constant L_M depending on M such that

$$\begin{aligned}
&\|F(u) - F(v)\|^2_{L^2(0,1)} \\
= &\int_0^1 |F(u)(x) - F(v)(x)|^2 \, dx \\
= &\int_0^1 |f(u) - f(v)|^2 \, dx \\
\leq & \; L_M^2 \int_0^1 |u(x) - v(x)|^2 \, dx \\
\leq & \; L_M^2 \, \|u - v\|^2_{C^0[0,1]} \\
\leq & \; C_0^2 L_M^2 \, \|u - v\|^2_{D((-A)^{1/2})} \\
\leq & \; C_1^2 L_M^2 \, \|u - v\|^2_{D(A}
\end{aligned}$$

where C_0 and C_1 are the constants involved in the Sobolev embedding $H_0^{1,a}(0,1) \hookrightarrow C^0[0,1]$. and $D((-A)^{1/2}) \hookrightarrow D(A)$, respectively. ☐

Moreover, we show that the operator F defined by (12) is Hölder continuous of exponent $\alpha \in (0,1)$. Before going to that point, we give the definition of mild solution of the equivalent semilinear evolution problem (9).

Definition 11 *A solution u is said to be a* **mild solution** *of the equivalent semilinear evolution problem (9) if there exists $u \in C([0,\infty), H_a^1(0,1))$ such that*

$$u(t) = S(t)u_0 + \int_0^t S(t-\tau)F(u(\tau))d\tau$$

with $u_0 \in H_a^1(0,1)$.

Based on the proof of theorem 2.5.1 of [14], we have the following.

Lemma 12 *The equivalent semilinear evolution problem (9) has a unique mild solution u on the time interval $[0,T]$ for some positive constant T. Moreover, let $u(t)$ and $\widetilde{u}(t)$ be mild solutions corresponding to u_0 and $\widetilde{u}_0$, respectively. Then for all, $t \in [0,T]$, the following estimate holds*

$$\|u(t) - \widetilde{u}(t)\|_{H_a^1(0,1)} \leq \|u_0 - \widetilde{u}_0\|_{H_a^1(0,1)} \, e^{C_1 T^{1/2}},$$

for some positive constant C_1.

By modifying the proof of corollary 2.5.1 of [14], we establish the following lemma.

Lemma 13 *The mild solution u of the equivalent semilinear evolution problem (9) is Hölder continuous of exponent $\alpha = (1/2)$ in t for any $u_0 \in D(A)$.*

Proposition 14 *The operator F defined by (12)is Hölder continuous of exponent $\alpha = (1/2)$ in t.*

Proof: Since F satisfies the locally Lipschitz condition and u is Hölder continuous of exponent $\alpha = (1/2)$ in t, F is also Hölder continuous of exponent $\alpha = (1/2)$ in t. ☐

Now we are in a position to prove theorem 2.

Proof of theorem 2: It follows directly from proposition 4 and 14. ☐

4.2 The proof of theorem 3

Let us modify the proof of theorem 2.5.5 of [14] to obtain the following result.

Lemma 15 *Let $[0, T_{max})$ be the maximal time interval in which the mild solution u of the equivalent semilinear evolution problem (9) exists.*

If T_{max} is finite, then the solution u of the semilinear parabolic problem (8) blows up in finite time T_{max}, i.e.,

$$\lim_{t \to T_{\max}} \|u(t)\|_{H_0^{1,a}(0,1)} = +\infty$$

Before proving theorem 3, we have to find some useful properties of the analytic semigroup $S(t)$ generated by operator A. By modifying the proof of proposition 2.3.1.4 and 2.3.1.5 in [9], we obtain two results

Lemma 16 *If $v \in D((-A)^{1/2})$, then $S(t)v \in D((-A)^{1/2})$ and $\left\|(-A)^{1/2}S(t)v\right\|_{L^2(0,1)} = \left\|S(t)(-A)^{1/2}v\right\|_{L^2(0,1)} \leq \left\|(-A)^{1/2}v\right\|_{L^2(0,1)}$.*

Lemma 17 *There exists a position C_2 such that $\left\|(-A)^{1/2}S(t)v\right\|_{L^2(0,1)} = \|S(t)v\|_{H_0^{1,a}(0,1)} \leq \frac{C_2}{t^{1/2}} \|v\|_{L^2(0,1)}$ for any $(v,t) \in L^2(0,1) \times (0,+\infty)$.*

We next prove theorem 3.

Proof of theorem 3: We will prove theorem 3 by contradiction argument. Suppose that there exists a positive constant M such that $\max_{x \in [0,1]} |u(x,t)| \leq M$ as

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$t \to T_{\max}$. It follows from $u(t) = S(t)u_0 + \int\limits_0^t S(t-\tau)F(u(\tau))d\tau$ that

$$\|u(t)\|_{H_0^{1,a}} \leq \|S(t)u_0\|_{H_0^{1,a}} + \int\limits_0^t \|S(t-\tau)F(u(\tau))\|_{H_0^{1,a}}\, d\tau.$$

By lemmas 16 and 17, we obtain

$$\begin{aligned}\|u(t)\|_{H_0^{1,a}} &\leq \|u_0\|_{H_0^{1,a}} + C\int\limits_0^t \frac{\|F(u(\tau))\|_{L^2(0,1)}}{(t-\tau)^{1/2}}d\tau \\ &\leq \|u_0\|_{H_0^{1,a}} + Cf(M)\int\limits_0^t \frac{1}{(t-\tau)^{1/2}}d\tau \\ &= \|u_0\|_{H_0^{1,a}} + 2Cf(M)t^{1/2},\end{aligned}$$

for some positive constant C. So, as $t \to T_{\max}$, $\|u(t)\|_{H_0^{1,a}(0,1)}$ is bounded which contradicts to lemma 15. Hence the proof of this theorem is complete. □

5 Conclusion

As shown in [11], if we would like to prove the existence and uniqueness of a blow-up solution by Green's function method, we have to make many assumptions on functions k and a to guarantee the existence of eigenvalues and eigenfunctions of such a problem which contrast to method in semigroup theory. But the difficulty of applying semigroup theory is to construct the suitable Banach spaces.

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