Numerical Simulation of Blood Flow in the System of Human Coronary Arteries with Stenosis

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Abstract: This paper aims to present three-dimensional simulation of blood behavior in the system of human coronary arteries. The mathematical model is a set of partial differential equations including continuity equation and Navier-Stokes equations. The pulsatile conditions due to the heart pump during a cardiac cycle is imposed on the boundaries. Computational domain consists of the base of aorta, the left and the right coronary arteries. Finite element method is applied for the solution of the mathematical model Blood flow and temperature distribution in coronary system with normal arteries and stenosed arteries are computed. The results show that the appearance of stenosis reduces blood flow rate in the stenosed artery.

Key–Words: numerical simulation, stenosis, coronary artery, image reconstruction, blood flow.

1 Introduction

A system of human coronary arteries consists of the aorta, the right coronary artery and the left coronary artery. The left coronary artery (LCA) divides into left anterior descending (LAD) and circumflex branches (LCX). The right and the left coronary arteries provide blood supply to the heart muscle. When blood flow in coronary arteries is subjected to severe stenosis, critical conditions occur such as high blood pressure and inadequate blood supply to the heart. To understand the blood flow problem in the stenosed coronary arteries, extensive researches have been carried out to study blood flow problems in either the right coronary artery or the left coronary artery, including experimental, analytical and numerical studies over the last two [2, 4, 5, 8, 9, 10, 12, 13, 14, 15].

It is well established that the fluid-structure interaction determines the behavior of blood flow through arteries [11]. Recently, various studies have focused on the coupled fluid flow and arterial wall deformation problem. Basombrio et al. (2002) constructed numerical experiments for non-trivial flow, close to realistic situations in hemodynamics [7]. Boutsianis et al. (2004) studied non-Newtonian blood flow in human right coronary arteries with transient simulation [6]. The model was constructed from a CT scan. Wiwatanapataphee (2008) investigated the effect of stenosis on non-Newtonian blood flow through a stenosed tube [3]. The results obtained from the domains having 25%, 50% and 75%-area severity show that blood pressure drops dramatically around the stenosis site and creates a jet flow at the throat of the stenosis. Chaniotis et al. studied computational blood flow in curve and bifurcation geometries [1]. Comparison with the Newtonian model, their result showed the significance of the non-Newtonian model on the shear stress distribution.

Due to a difficult task to construct realistic geometry of the coronary artery, the studies as mentioned above used unrealistic geometry of blood vessel such as a straight tube or curve tube with branches and with no branch. It has been recognized that the results obtained from unrealistic domain may not be applicable for clinical use.

In this study, we simulate non-Newtonian pulsatile blood flow in a system of human coronary arteries with stenosed left coronary artery. Influence of stenosis occurring in the middle part of the LCA on blood flow problem is investigated.

2 Governing equations and Boundary conditions

In this study, blood is modeled as an incompressible non-Newtonian fluid. Based on the Carreau model, blood viscosity is a function of shear rate and is determined by

$$\eta(\dot{\gamma}) = \eta_\infty + (\eta_0 - \eta_\infty)[1 + (\lambda\dot{\gamma})^2]^{(n-1)/2}, \qquad (1)$$

where $\eta_\infty, \eta_0, \lambda$ and n are parameters. The governing equations of blood flow including the continuity equation and Navier-Stokes equations are as follows:

$$\begin{aligned} u_{i,i} &= 0, & (2)\\ \rho(\frac{\partial u_i}{\partial t} + u_j u_{i,j}) &= -p_{,i} + (\eta\dot{\gamma}(u_{i,j} + u_{j,i}))_{,j}, & (3) \end{aligned}$$

where u_i is blood velocity in the i-direction for $i = 1, 2, 3$; p denotes blood pressure and ρ is blood density of 1.06 g/cm^3. The shear rate $\dot{\gamma}$ in equation (1) is defined by

$$\dot{\gamma} = \sqrt{2tr(\frac{1}{2}(u_{i,j} + u_{j,i}))^2}. \qquad (4)$$

To specify the boundary conditions for the blood flow, we consider the blood flow mechanism due to the heart pump. The cyclic nature of the heart pump creates pulsatile conditions in the coronary arteries. In this study, we ignore the variation in different cardiac cycle, the pulsatile pressure and flow rate can be expressed by $p(t) = p(t + nT)$ and $Q(t) = Q(t + nT)$ for $n = 0, 1, 2, 3,$ and T is the cardiac period. Mathematically, the pulsatile pressure and flow rate can be written in the form of the truncated Fourier series:

$$p(t) = \bar{p} + \sum_{k=1}^{4} \alpha_k^p \cos(k\omega t) + \beta_k^p \sin(k\omega t), \qquad (5)$$

$$Q(t) = \bar{Q} + \sum_{k=1}^{4} \alpha_k^Q \cos(k\omega t) + \beta_k^Q \sin(k\omega t), \qquad (6)$$

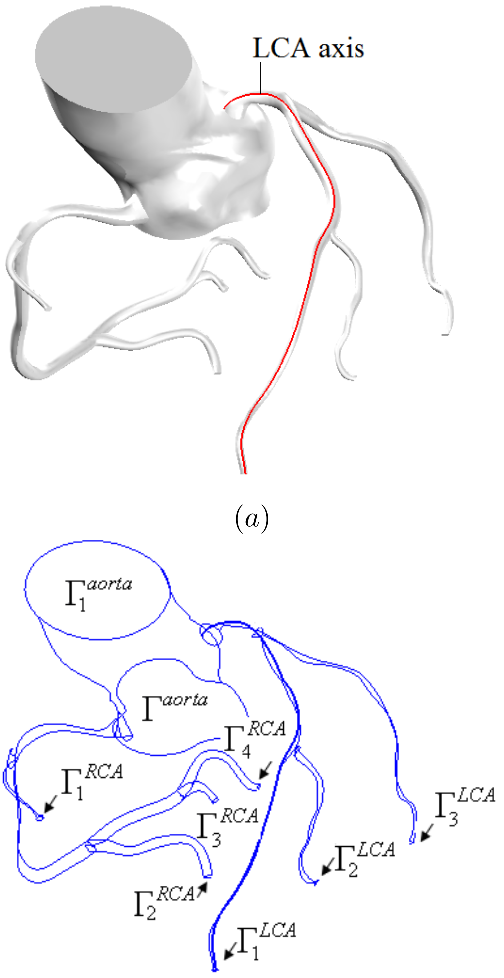
where $\bar{Q}$ denotes the mean flo rate and $\bar{p}$ is the mean presure, $\omega = 2\pi/T$ is the angular frequency with period T.

On the inflow surface of the aorta Γ^{aorta}, we impose the pulsatile velocity

$$u(t) = \frac{Q(t)}{A}, \qquad (7)$$

where A is the cross-sectional area of the inflow surface of the aorta which is 6.7287 cm^2. On the outflow surfaces $\{\Gamma_1^{aorta}, \Gamma_1^{RCA}, \Gamma_2^{RCA}, \Gamma_3^{RCA}, \Gamma_4^{RCA}, \Gamma_1^{LCA}, \Gamma_2^{LCA}, \Gamma_3^{LCA}\}$, we impose corresponding pulsatile pressure condition, i.e., for $i, j = 1, 2, 3$

$$p = p_0(t), \quad (\eta(u_{i,j} + u_{j,i})) \cdot \mathbf{n} = 0. \qquad (8)$$



(a)

(b)

Figure 1: The computational domain with (a) LCA axis, (b) inflow and outflow surfaces of the system of coronary arteries

Figure 2 presents the flow rate waveform and corresponding pressure in the aorta. Values of the mean flow rate on inflow surface of the aorta and mean pressure on outflow surface of the aorta are 5.7222 l/min and 97.2222 $mmHg$, respectively. Other values of in (5) and (6) are given in Table 1.

Table 1: Values of theparameters α_k^Q, β_k^Q, α_k^p and β_k^p

η	α_k^Q	β_k^Q	α_k^p	β_k^p
1	1.7048	-7.5836	8.1269	-12.4156
2	-6.7035	-2.1714	-6.1510	-1.1072
3	-2.6389	2.6462	-1.3330	-0.3849
4	0.7198	0.2687	-2.9473	1.1603

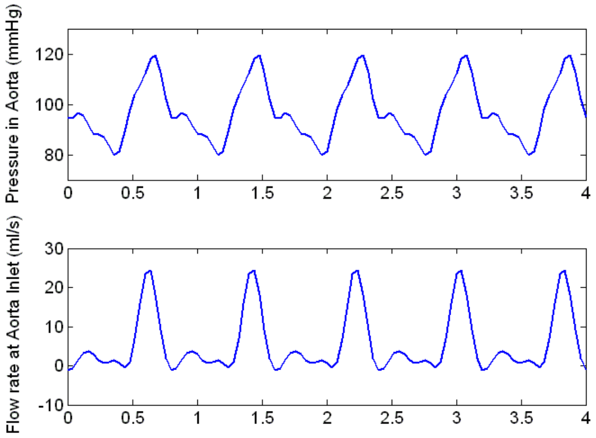


Figure 2: The flow rate wave form and corresponding, pressure in the aorta

In summary, the blood flow in the coronary artery system is governed by the following boundary value problem (BVP):

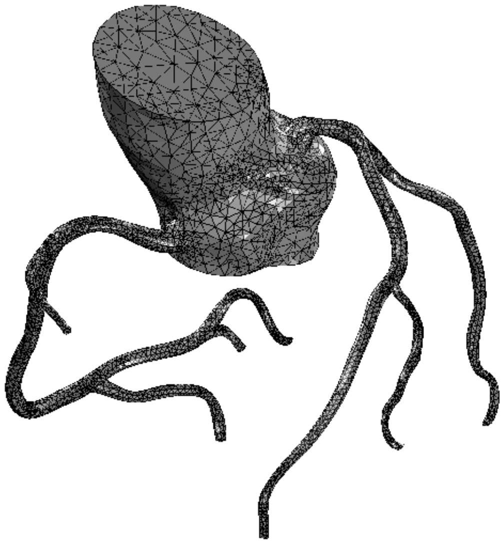
BVP : Find u and p such that equations (1) - (4) and all boundary conditions are satisfied.

3 Numerical Example

We have simulated the 3-D blood flow through the system of coronary arteries with normal arteries and with stenosis at the middle part of left coronary artery (LCA) using COMSOL multipysics. The computation domain is shown in Figure 3

We constructed two domain finite element meshes of the system with normal arteries and with 75% stenosed LCA consisting of 20,192 tetrahedral elements with 115,514 degrees of freedom and 21,446 tetrahedral elements with 121,194 degrees of freedom, respectively.

To investigate the effect of stenosis on the blood flow behavior in the system of coronary arteries, we simulate the blood flow from the models with normal arteries and with 75% stenosed LCA. The pulsatile pressure along the LCA axis (see Figure $1(a)$) and velocity field on the outflow surface Γ_1^{LAD} of the end of LAD are investigated. The results as shown in Figures 4 indicate the stenosis has a significant effect on the blood flow problem. Compared with the results obtained from the model with normal arteries, the pressure drop significantly in the model with 75% stenosed LCA. Figure 5 illustrates the surface plot of the velocity field on the outflow Γ_1^{LAD} of the LAD. It is noted that the maximum velocity at the peak of systole and diastole in the system with 75% stenosed LCA are respectively 22.811 cm/s and 15.840 cm/s while the normal system has more maximum velocity: 29.179 cm/s at the peak of systole; 20.015 cm/s at the peak of diastole. This indicates that when stenosis is present, it reduces the flow rate.


(a)

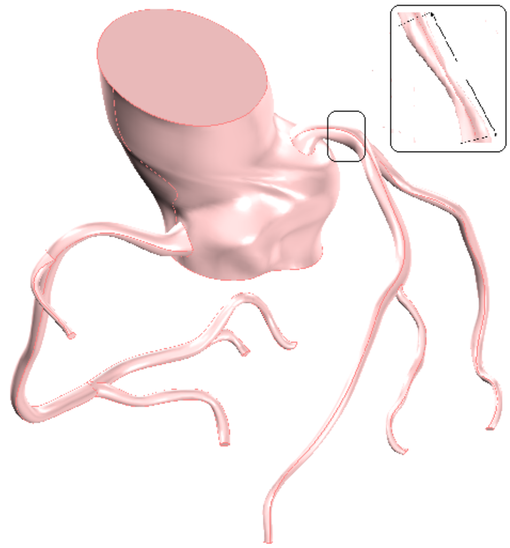

(b)

Figure 3: The element mesh of the computational domain with stenosed LCA

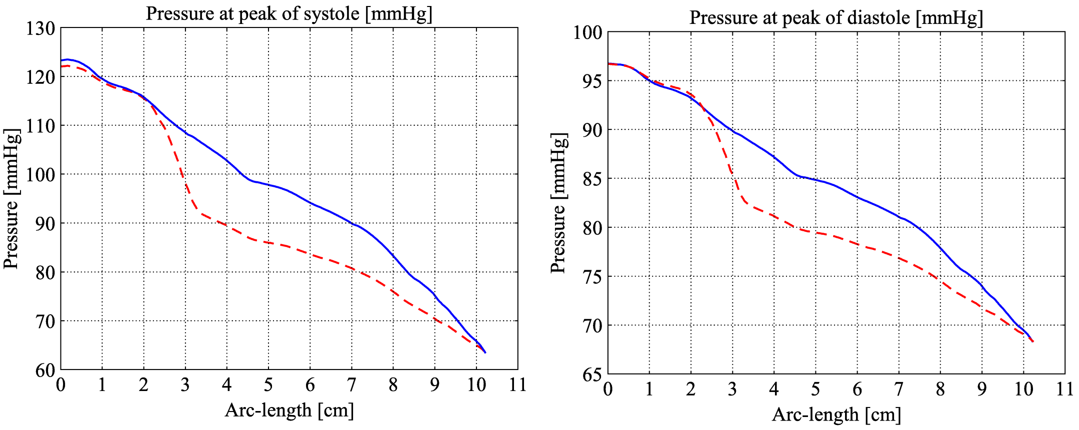
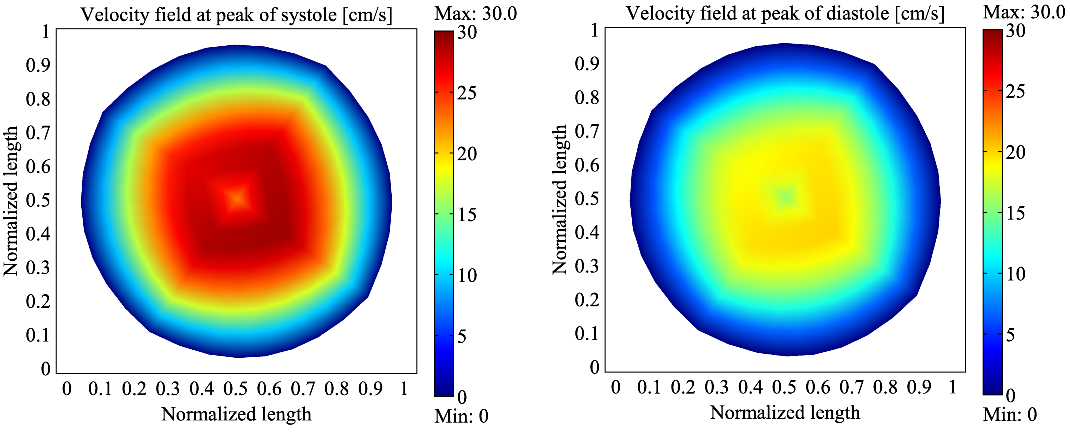
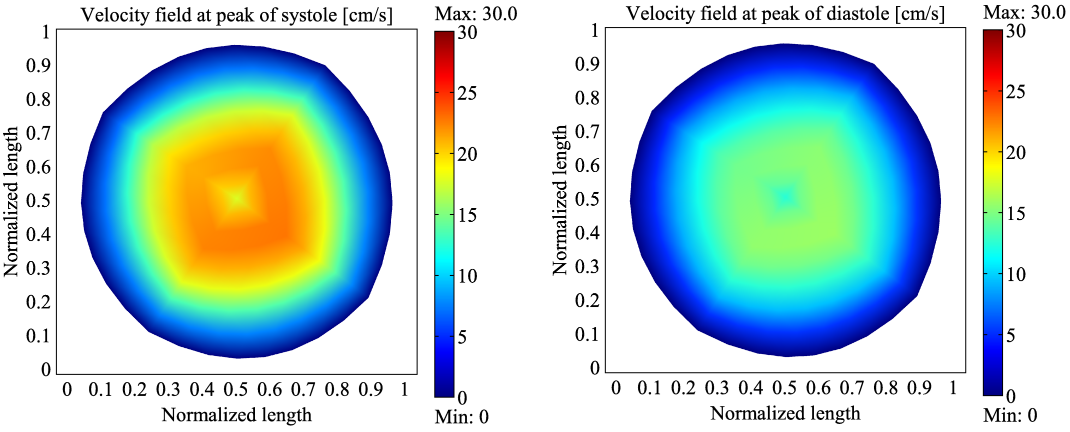


Figure 4: Pressure along the LCA axis at the peak of systole and the peak of diastole obtain from the normal system (solid line) and the system with 75 % stenosed LCA (dash line).



(a) A system with normal arteries



(b) A system with 75% stenosed LCA

Figure 5: The surface plot of velocity field on outflow Γ_1^{LAD} of LAD at the peak of systole and the peak of diastole in two systems: (a) a system with normal arteries; (b) a system with 75% stenosed LCA.

4 Conclusion

In this work, we developed three-dimensional mathematical model of blood flow taking into account the pulsatile boundary condition. The computational domains are two coronary systems with normal arteries and stenosed arteries. The solutions of the problem were solved by finite element method using COMSOL multiphysics. Effect of the system with the stenosed arteries on the blood flow problem was investigated. The result showed that the system with stenosed arteries provided lower blood pressure around the stenosed region and lower blood speed on the outflow of stenosed vessel than those in the system with normal arteries. This implies that the stenosed arteries cause an inadequate blood supply to the heart.

Acknowledgements : The authors are grateful to the Thailand Research Fund (TRF) for support of this research and also wish to thank the Development and Promotion of Science and Technology Talents Project (DPST).

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