The dynamic control of non-marketable systems

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Abstract: In this paper, we propose a very simple method to coordinate evolutive systems which produce results that do not point towards quantitative, but qualitative aspects, such as the education systems which ensure the educational, professional and scientific instruction function.

Key-Words: Control of dynamic systems, non-marketable evolutive systems, the Student-Fisher Test.

1 Introduction

A modern indicator that evaluates the quality of higher education institutions refers to relating the number of graduates that find a job in the field they have been trained for or take a job similar to the field they have studied and specialized in, to the total number of graduates. Another indicator, which is more general and has been used for quite some time, refers to relating the number of graduates of a certain form of training (at any level: secondary or higher education, professional development or reconversion programmes, etc.) to the total numbers of individuals who have enrolled in this kind of programmes.

According to our point of view expressed in papers like [2 - 5], the institutions that provide professional and/or intellectual training for a community can be mathematically modeled by the abstract notion of dynamic or evolutive system. Bearing in mind the fact that the outputs of these systems, or put it differently, the results produced by such systems (given the fact that the input is represented by students), are qualitative (theoretical knowledge, skills and abilities) and not quantitative, authors like R. Färe, S. Grosskopf, F. R. Forsund, K. Hayes, A. Heshmati [1] have included them in the category of non-marketable systems.

In order to fulfill their mission, educational systems, professional reconversion programs or advanced trainings need to be adapted (or controlled, if we used a concept specific to the field of dynamic systems) to both the requirements of the labour market, as well as the latest scientific and technical achievements.

Nowadays, under the auspices of the knowledge based society, this goal turns into a challenge. Besides the inherent difficulties linked with any kind of adaptation (the permanent renewal of the infrastructure, the continuous professional development of the teaching staff / instructors engaged in the teaching activities, the passivity regarding the adaptation to or acceptance of progress when one learns new things or gives up habits and prejudices), the systems we refer to, face one more challenge, the one that the control of such systems need to be made without interrupting their functioning mechanism. This last challenge overcomes all the others, since the need to modernize the infrastructure of the education or training providing institutions or of the professional development has to be identified and remedied in due time.

In its turn, the issue referring to identifying the appropriate moment when one has to intervene upon the system in order to optimize its activity, besides the fact that it needs to be solved in due time, faces the constraint that there is no general valid and theoretically well-grounded method to help solve it. Indeed, due to the complexity of external factors and the way these interact with non-marketable systems (most of the time, the interaction occurs indirectly), despite the fact that basically any kind of modernization brings benefits and also, that it demands efforts and costs, the choice regarding the moment when one has to intervene and on what scale constitutes the key question we shall try to answer within this paper. More precisely, our intention relies in the fact that for those monitoring the scientific-educational processes we provide a
flexible and easy-to-use mathematical instrument to help them in their current activity. This instrument relies on mathematical statistics, namely the sampling theory.

2 Presentation of the theoretical apparatus on which the solving of the enunciated optimal control problem is based

Let \( r \) be the ratio between the number of participants at a training programme and the number of all participants taking part in the programme. This ratio can be expressed as a percentage \( p = r \times 100 \), as well. Certainly, an institution providing instructional-educational services is the better ranked in its field, the bigger ratio \( r \) (or the closer to 100% percentage \( p \) lies) is.

From the analysis of the social reality, economic trends or history of the institution, the managers or decision makers can set a target (expressed as a ratio, or as a percentage) based on which reported performances can be compared. One should notice, that, besides a subjective nature (any education institution aims at achieving a higher and higher such indicator), this target also has an objective nature which conditions its value. Indeed, it represents the equilibrium position of several opposite trends: the social or economic demand for a certain type of activity, and the effort required for the specialization process in a certain field, or the deprivations linked with carrying out that activity. A low social reward for a certain field of activity leads to a diminished interest in taking jobs in a collateral field and vice-versa, the absence of specialists in a certain field leads to an increased interest in promoting jobs in that field, resulting thus, in increased attractiveness and desire to specialize in that field.

Concluding, the interest in holding a job, quantified, in our case, by the graduation rate represents the point of equilibrium for more contrary tendencies: the demand or social importance granted to a specific activity regulated by market saturation, on the one hand, and the difficulties related to getting to exert this profession, on the other. Setting the graduation target in an education institution depends on the manager’s or managers’ skills.

We shall further on assume that this evaluation has been done as accurately as possibly. Once this indicator set, the manager has to only compare the results obtained at the end of each study cycle (whose main outcome is the graduation). If the graduation rate is higher or equal the target, then, the education process will continue its course similarly with the previous one; perhaps one should take measures in ensuring the fact that the results has not been caused by excessive indulgence in the evaluation system regarding graduates’ acquired knowledge. Contrary, one will have to dispose the review of the previous education process.

The method by which the education process is monitored relies, on the one hand, on the use of a reference indicator for which there are no precise / accurate methods to determine, on the other hand, on the outcomes obtained by participants at the final graduation tests, outcomes which can be conjuncture based or influenced by subjective factors. In order to avoid such situation, we can use a statistical method with the following content:

Let \( r_0 \) (or \( p_0 \)) be the reference indicator for a certain type of didactical activity. We denote by \( r_1, \ldots, r_n \) (or by \( p_1, \ldots, p_n \)) the results obtained by the participants at training courses provided in \( n \geq 2 \) previous cycles within an accredited education institution.

**Observation:** When establishing the number \( n \) of outcomes used to realize the test, one has to pay attention to the following rules:

1) The bigger the value of \( n \), the more precise the test is;

2) The value of \( n \) must not exceed the time frame for which the value of the reference indicator is constant or almost constant, knowing the fact that the community’s socio-economic interest (command) can change from one phase to another.

By means of variables \( r_1, \ldots, r_n \) (respectively, \( p_1, \ldots, p_n \)) we build the statistics

\[
t = \frac{r - r_0}{s_r / \sqrt{n}} \quad \text{(respectively } t = \frac{p - p_0}{s_p / \sqrt{n}} \text{)},
\]

where

\[
r = \frac{r_1 + \ldots + r_n}{n}, \quad \text{(respectively } p = \frac{p_1 + \ldots + p_n}{n} \text{),}
\]

and

\[
s_r = \sqrt{\frac{1}{n-1} \left[ (r_1 - r)^2 + \ldots + (r_n - r)^2 \right]},
\]

respectively

\[
s_p = \sqrt{\frac{1}{n-1} \left[ (p_1 - p)^2 + \ldots + (p_n - p)^2 \right]}.
\]

Then, we formulate the following hypotheses:
1) The initial hypothesis $H_0: r = r_0$ ($\geq$), (respectively $H_0: p = p_0$ ($\geq$)) which shows that the real reference ratio $r$ of the respective education unit (respectively, the real graduation percentage $p$ of the education unit) is bigger or equal at least with the target that the education unit is aiming to reach.

2) The alternative hypothesis $H_a: r < r_0$, (respectively $H_a: p < p_0$) which expresses the fact that the performances of the education unit under analysis are inferior to expectations. In order to statistically verify hypothesis $H_0$ with the alternative $H_a$ we set a significance level of $\alpha > 0$ and apply the Student-Fisher test. This test relies in comparing the computed value $t^*$ of statistics $t$ (in $t$’s formula the real values of the sample $\{r_1, ..., r_n\}$, or after case, of the sample $\{p_1, ..., p_n\}$, are being introduced), with the opposite critical value of the Student-Fisher repartition with $df = n - 1$ degrees of freedom corresponding to the significance level $\alpha$, namely with $-t(df, \alpha)$.

Observation: The critical value $t(df, \alpha)$ of the Student-Fisher test has the following probabilistic meaning

$$P(t < -t(df, \alpha)) = \alpha,$$

$$P(t < -t(df, \alpha))$$ in the expression above represents the probability that the random variable $t$ takes smaller values than the number $-t(df, \alpha)$.

The Student-Fisher test based result can be interpreted as it follows:

If $t^* > -t(df, \alpha)$, then we can not reject hypothesis $H_0$, thus it remains valid for a significance level equal with $\alpha$, while if $t^* < -t(df, \alpha)$, then hypothesis $H_0$ is rejected in favour of the alternative hypothesis $H_a$, with the same significance level $\alpha$.

Observations: Accepting hypothesis $H_0$ equates, in our case, with continuing the educational process as it is with no significant changes, while accepting hypothesis $H_a$ instead of $H_0$, equates with the need to reorganize the educational process.

3 Comments upon the method proposed

The use of the Student-Fisher test as a decision factor in the dynamic control regarding the evolution of non-marketable systems bears both advantages and disadvantages. The advantages consist in the fact that the volumes of the samples used to compute statistics $t$ do not compulsorily have to be large (though a larger volume of these samples does not affect the accuracy of the test, on the contrary), as well as in the fact that this test does not require previous knowledge on none of the parameters describing the statistical population being studied.

The disadvantages of the method presented consist in the fact that characteristics of the investigated statistical population, such as: the graduation percentage rate of the educational units, have to be governed by probabilistic laws close to the normal distribution law. This disadvantage is not difficult to overcome, since the volume of the samples used in building statistics $t$ has to be increased, when these distribution laws are not normal.

The calculus of probabilities guarantees that when the number of degrees of freedom $df$ is high (namely $df > 30$), the random variable $t$ acts more and more like a normal variable of average $m = 0$ and standard deviation $\sigma = 1$, namely test $t$ turns into test $z$. Moreover, it is a fact that most of nature’s phenomena and processes including those in the field studied by us, obey the normal law.

The results obtained by using the method presented are not undoubtedly valid, as we would be inclined to believe when mathematics is being applied, but they are only guaranteed with a certain probability of achievement.

4 Case study

In order to exemplify we shall analyze the quality of the instructive and formative process within an educational institution. The educational program stretches over a month and the normal graduation rate is 75%. After having provided the program for one year, the manager of the institution aims to find out whether or not for the next year changes regarding the educational process are being required. To do so, the manager analyzes the results obtained during the past twelve months. The situation is presented in table 1.

Regarding the issue formulated, we make the hypothesis:

$H_0$: the graduation rate is $\geq 75\%$,

with the alternative hypothesis

$H_a$: the graduation rate is $< 75\%$,
and in order to validate one of these hypotheses we choose a significance level of $\alpha = 5\%$.

<table>
<thead>
<tr>
<th>Month</th>
<th>No. of graduates at 100 participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>70</td>
</tr>
<tr>
<td>February</td>
<td>73</td>
</tr>
<tr>
<td>March</td>
<td>59</td>
</tr>
<tr>
<td>Aprilie</td>
<td>77</td>
</tr>
<tr>
<td>May</td>
<td>75</td>
</tr>
<tr>
<td>June</td>
<td>90</td>
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<tr>
<td>July</td>
<td>80</td>
</tr>
<tr>
<td>August</td>
<td>71</td>
</tr>
<tr>
<td>September</td>
<td>58</td>
</tr>
<tr>
<td>October</td>
<td>57</td>
</tr>
<tr>
<td>November</td>
<td>82</td>
</tr>
<tr>
<td>December</td>
<td>79</td>
</tr>
</tbody>
</table>

Table 1.

Based on the data in the table, we calculate:

\[
p = \frac{70 + 73 + \ldots + 79}{12} \approx 72.5833,
\]

\[
s_p = \sqrt{\frac{(70 - \bar{p})^2 + \ldots + (79 - \bar{p})^2}{11}} \approx 10.2820,
\]

\[
t^* = \frac{\bar{p} - 75}{s_p / \sqrt{12}} \approx -0.8142.
\]

From the table of the Student-Fisher distribution we determine the critical value of the $t$ test for $df = 11$ and $\alpha = 0.05$. We obtain $t(11, 0.05) = 1.7958$. Since $t^* > -t(11, 0.05)$ there are not enough reasons to reject hypothesis $H_0$, thus we accept it, given the limit of a statistical significance of 0.05 ($\Leftrightarrow 5\%$). Consequently, for the education institution to function optimally, there is no immediate sense of urgency in improving the educational process of the previous year.

5 Conclusions

The method presented in this paper enables managers to take control based decisions in regard to developing non-marketable systems. The act of controlling the evolution of dynamic systems in due time constitutes a difficult task, partly because it relies on a general approach, partly because the internal mechanisms that cause the responses (reactions) of these systems are vaguely known. The method presented in this paper can be successfully applied in solving these issues. Moreover, it can be applied by anyone. Its importance resides in the fact that it can be used for solving problems with a high degree of uncertainty.

References:


