Brief Notes on Vortex Identification

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Abstract: An update on vortex identification, general vortex-identification requirements, and related vorticity and circulation aspects is presented.

Key-Words: Vortex; Vortex identification; Vortex-identification methods; Vortical structures; Vorticity

1 Introduction

Fluid-mechanical problems are often dominated by vortical structures. Though the intuitive idea of a vortex is of fundamental importance for fluid mechanics, there is still no consensus on the generally acceptable and rigorous definition of this distinct flow phenomenon. A large number of vortex-identification methods, vortex definitions, and vortex-core visualization techniques have been proposed in the literature during last three decades [1-22]. Various vortex-identification schemes are stated in Table 1 where their basic characteristics and/or criteria are pointed out (some well-established symbols used in Table 1 are explained in the Appendix). In Table 1, the region-type definitions of a vortex are distinguished from the line-type definitions of a vortex core. In practice, these methods may be effectively combined [23]. Though there is no final consensus on what is a vortex, fluid vortices have been mostly somehow related to a quite mathematically rigorous and physically well-established quantity expressing an average angular velocity of fluid elements, the so-called vorticity. However, in vortex identification, this quantity plays its role less directly than expected, usually through the whole velocity-gradient tensor $\nabla \mathbf{u}$ needed especially in popular local methods.

In the two following sections, (i) general vortex-identification requirements, and (ii) new vorticity aspects of vortex identification are discussed. It is shown that from the physical viewpoint the general requirements, vorticity and circulation aspects pose challenges for progress in vortex identification.

2 Vortex-Identification Requirements

The requirements for vortex identification, some of them discussed below, are summarized as follows:

- validity for compressible flows
- validity for variable-density flows
- determination of the local intensity of swirling motion (to describe inner vortex structure)
- determination of the swirl orientation
- determination of the integral vortex strength
- vortex-axis identification
- specific vortex-axis requirements: existence and uniqueness for each connected vortex region
- avoidance of the subjective choice of threshold in the vortex-boundary identification
- allowance for an arbitrary axial strain vs. orbital (spiralling) compactness
- non-local properties of the vortex phenomenon
- ability to provide the same results in different rotating frames

The widely used local criteria mentioned in the Appendix (Q, Δ, λ_2, and λ_{ci}) hold for incompressible flows only. For example, considering the approach of the λ_2-method, additional terms occur in the course of derivation for compressible fluids, as analyzed by Cucitore et al. [13].

It has been recently shown by Kolář [24] that from the most popular region-type identification schemes (Q, Δ, λ_2, and λ_{ci}) only the Δ-criterion and the closely associated λ_{ci}-criterion are directly extendable to compressible flows.

There are at least two controversial requirements appearing in the literature on vortex identification. The allowance for an arbitrary axial strain clearly stands against the requirement of orbital (spiralling) compactness. According to [19], rapid radial spreading out (or, similarly, axial stretching out) of instantaneous streamlines may not appear to qualify the region as a vortex, as depicted in a simplified manner in Fig. 1.

Table 1 Vortex-identification methods.

Author/s & Year of publication	Basic characteristics/criteria, R/L: Region/Line-type method
Dallmann (1983) [1]	discriminant Δ-criterion: complex eigenvalues of $\nabla\mathbf{u}$, R
Vollmers et al. (1983) [2]	discriminant Δ-criterion: complex eigenvalues of $\nabla\mathbf{u}$, R
Hunt et al. (1988) [3]	Q-criterion: second invariant of $\nabla\mathbf{u}$, R
Chong et al. (1990) [4]	discriminant Δ-criterion: complex eigenvalues of $\nabla\mathbf{u}$, R
Levy et al. (1990) [5]	extrema of normalized helicity density, L
Berdahl & Thompson (1993) [6]	swirl parameter, similar to λ_{ci}-criterion, R
Banks & Singer (1995) [7]	vorticity-predictor & pressure-corrector scheme, L
Jeong & Hussain (1995) [8]	λ_2-criterion: second eigenvalue of $\mathbf{S}^2+\mathbf{\Omega}^2$, R
Sujudi & Haimes (1995) [9]	eigenvectors of $\nabla\mathbf{u}$, tetrahedral cells, L
Portela (1997) [10]	scheme based on set theory, kinematic set concept, R
Kida & Miura (1998) [11]	sectional-swirl & pressure-minimum scheme, L
Roth & Peikert (1998) [12]	parallel-vectors higher-order method, L
Cucitore et al. (1999) [13]	non-local particle-trajectory method, R
Strawn et al. (1999) [14]	lines of maximum vorticity, L
Zhou et al. (1999) [15]	swirling-strength λ_{ci}-criterion: complex eigenvalues of $\nabla\mathbf{u}$, R
Roth (2000) [16]	generalization of earlier line-type methods, L
Sadarjoen & Post (2000) [17]	advanced streamline method, R
Jiang et al. (2002) [18]	scheme based on combinatorial topology, R
Chakraborty et al. (2005) [19]	enhanced swirling-strength λ_{ci}-criterion, R
Haller (2005) [20]	objective frame-independent vortex definition, R
Zhang & Choudhury (2006) [21]	Galilean-invariant eigen helicity density, R
Kolář (2007) [22]	triple decomposition of $\nabla\mathbf{u}$: residual vorticity, R

The allowance for an arbitrary axial strain of Wu et al. [25] has become a subject of recent debate (Chakraborty et al. [26]; Wu et al. [27]) as this requirement, basically, does not conform to the orbital compactness proposed in [19]. Note that for an incompressible flow the axial strain is directly related to the spiralling compactness [19, 26]. According to [19, 26], the orbital (spiralling) compactness requires for vortex-identification purpose an appropriate threshold dictated by the length and time scales of the given problem. However, following [27], adding a threshold value to the local axial strain or to the orbital compactness is subjective and cannot be rationalized.

The local intensity of swirling motion is not only necessary for the description of the inner vortex structure but also for determining the integral strength of a vortex. According to the widely used concept, the strength of a vortex is calculated as the circulation along the vortex boundary, or equivalently due to Green’s theorem, as the surface integral of vorticity over the planar vortex cross section. However, vorticity absorbs the local effect of an arbitrary "superimposed shear" what makes the circulation a shear-biased vortex characteristic as discussed by Kolář [22, 28]. The vortex cross section and vortex boundary can be generally defined in terms of the local vortex intensity as shown in Fig. 2.

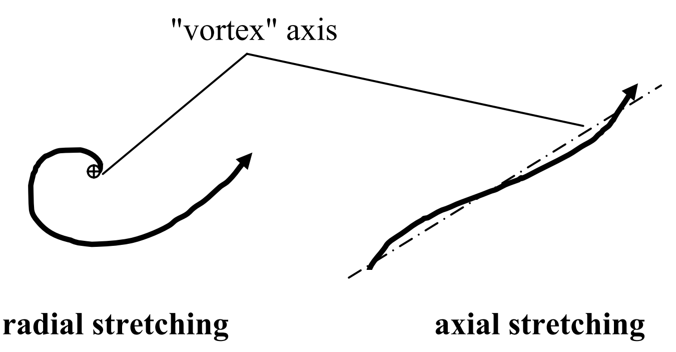

Fig. 1 Vortex stretching.

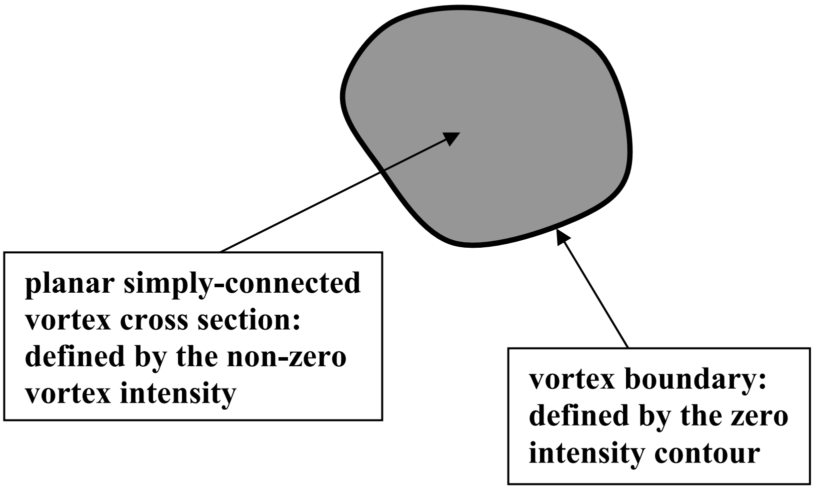

Fig. 2 Generally defined vortex cross section.

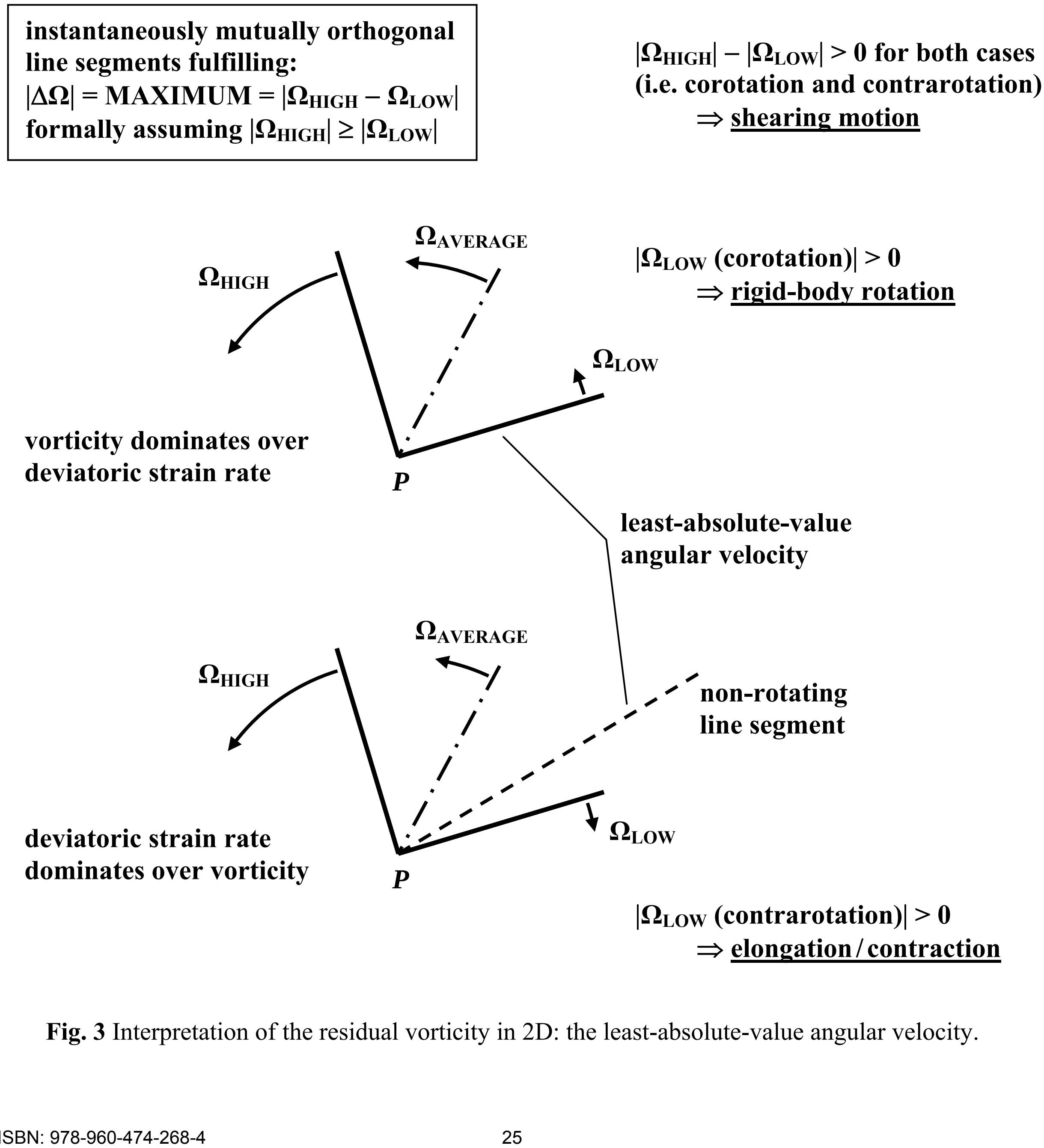

Fig. 3 Interpretation of the residual vorticity in 2D: the least-absolute-value angular velocity.

The vortex region, vortex boundary and, consequently, the integral vortex strength are directly dependent on the choice of the local vortex intensity (see [28] where the unsteady Taylor vortex is examined in detail). Kolář [22] proposed to use in vortex identification instead of vorticity its residual portion, labelled residual vorticity, associated with the local residual rigid-body rotation near a point (to be discussed in the next section). It is interesting that the integral quantity based on the residual vorticity labelled "residual circulation" can be calculated in a similar manner as the conventional circulation as the surface integral over the planar vortex cross section. Residual circulation has already proved its usefulness for the description of single and twin jets in crossflow [29]: for different nozzle arrangements an almost universal behaviour of the residual circulation has been revealed for the resulting secondary-flow counter-rotating vortex pair. For an arbitrary threshold level the region of the residual vorticity forms a subdomain of the vorticity region.

The ability to provide the same results of vortex identification in different rotating frames has to do with the so-called material objectivity or frame indifference (i.e. both translational and rotational independence [20, 30]) important in situations where there is an unclear choice of a reference frame.

3 Vorticity and Vortex Identification

Due to its shear-absorbing nature, vorticity – an average angular velocity of fluid elements – plays its role in vortex identification less directly than expected, usually through the complete information contained in the velocity-gradient tensor $\nabla\mathbf{u}$, as needed especially in popular local methods (Q, Δ, λ_2, and λ_{ci}), see the Appendix. However, all these methods are not entirely free from the shear-based bias in the determination of vortex geometry [22]. The novel decomposition technique of Kolář [22] dealing with the triple decomposition of $\nabla\mathbf{u}$ results in two additive vorticity parts (and in two additive strain-rate parts) of distinct nature, the *shear* component and the *residual* one. The triple decomposition of $\nabla\mathbf{u}$ has to do explicitly with three types of local motion near a point: rigid-body rotation, elongation (contraction), and shearing motion. The residual vorticity obtained after the extraction of an "effective" shearing motion represents a sought local intensity of the swirling motion of a vortex.

There is a straightforward interpretation of the residual vorticity in 2D in terms of the least-absolute-value angular velocity of all line segments, within the flow plane, going through the given point, Fig. 3 (here Ω is angular velocity of a line segment near a point, $\Omega_{\mathrm{AVERAGE}}$ corresponds to vorticity). This approach "reestablishes" the relationship between a vortex and vorticity, namely its specific portion, the residual vorticity. The method is applicable to compressible and variable-density flows.

Though Galilean-invariant quantity, vorticity is not objective and provides different results in different rotating frames, that is, the property of frame indifference, Leigh [30], is not fulfilled. Similarly, the residual vorticity is not objective and depends on the angular velocity of an observer's reference frame. All the most popular $\nabla\mathbf{u}$-based vortex-identification schemes (Q, Δ, λ_2, and λ_{ci}) are not objective. This fact has motivated Haller [20] to his sophisticated vortex definition which is objective relative to an arbitrarily rotating reference frame.

Considering new techniques of vorticity decomposition, it is claimed by Wedgewood [31], who proposed a new vorticity decomposition, that "there is no question that objective information is contained in the vorticity tensor." His results clearly indicates that the objective portion of vorticity is strongly related to the objective strain-rate tensor and the deformational aspects of the flow.

Let us consider a simple flow example with rotationally unclear choice of reference frame: the two-dimensional merging of two identical co-rotating vortices, schematically shown in Fig. 4.

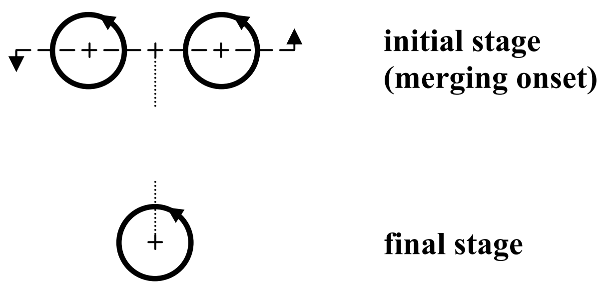

Fig. 4 Corotating vortices: an unclear choice of reference frame.

In this case, one should cautiously use not only vorticity, but also the residual vorticity or any other $\nabla\mathbf{u}$-based identification scheme. The vortex-axis uniqueness for each connected vortex region may be easily broken. The disappearance of originally single vortices separated by a vortex boundary is correctly viewed in the rotating reference frame stuck on the center-to-center connecting line while the evolution of the resulting vortex of the merging process is correctly viewed in the non-rotating reference frame.

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4 Conclusions

Vortex identification represents a challenging field of modern fluid mechanics. An update on vortex identification, vortex-identification requirements, and related vorticity aspects is presented.

The physical reasoning for vortex-identification methods and their application to vortical flows go hand in hand with progress in data acquisition: flow modelling and numerical simulation of transitional and turbulent flows, especially direct numerical simulation (DNS) and large-eddy simulation (LES).

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Appendix: Local vortex-identification methods

The most popular schemes (Q, $\varDelta$, λ_2, and λ_{ci}) are region-type criteria sharing a basis in $\nabla\mathbf{u}$ which are local in character. However, the significance of line-type methods (e.g. Roth [16]), cannot be ignored.

Q-criterion [3]: Vortices of an incompressible flow are identified as connected fluid regions with a positive second invariant of the velocity-gradient tensor $\nabla\mathbf{u}$, $\nabla\mathbf{u}=\mathbf{S}+\mathbf{\Omega}$, $\mathbf{S}$ is the strain-rate tensor, $\mathbf{\Omega}$ is the vorticity tensor (in tensor notation below the subscript comma denotes differentiation),

$$Q \equiv \frac{1}{2}\left(u_{i,i}^2 - u_{i,j}u_{j,i}\right) = -\frac{1}{2}u_{i,j}u_{j,i} = \frac{1}{2}\left(\|\mathbf{\Omega}\|^2 - \|\mathbf{S}\|^2\right) > 0 \tag{1}$$

that is, as the regions where the vorticity magnitude prevails over the strain-rate magnitude.

$\varDelta$-criterion [1, 2, 4]: Vortices are defined as the regions in which the eigenvalues of $\nabla\mathbf{u}$ are complex and the streamline pattern is spiraling or closed in a local reference frame moving with the point. For incompressible flows, the characteristic equation for the eigenvalues λ of $\nabla\mathbf{u}$ reads

$$\lambda^3 + Q\lambda - R = 0 \tag{2}$$

where Q and R are the second and third invariants of $\nabla\mathbf{u}$, Q is given by (1), $R \equiv \mathrm{Det}\left(u_{i,j}\right)$. To guarantee complex eigenvalues of $\nabla\mathbf{u}$ the discriminant $\varDelta$ of Eq. (2) should be positive

$$\varDelta = \left(\frac{Q}{3}\right)^3 + \left(\frac{R}{2}\right)^2 > 0\,. \tag{3}$$

The vortex-identification criterion (3) is valid for incompressible flows only. The Q-criterion is clearly more restrictive than $\varDelta$-criterion (cf. (1) and (3)).

λ_2-criterion [8]: This criterion is formulated on dynamic considerations, namely on the search for a pressure minimum across the vortex. The strain-rate transport equation reads

$$\frac{\mathrm{D}S_{ij}}{\mathrm{D}t} - \nu S_{ij,kk} + \Omega_{ik}\Omega_{kj} + S_{ik}S_{kj} = -\frac{1}{\rho}p_{,ij} \tag{4}$$

where the pressure Hessian $p_{,ij}$ contains information on local pressure extrema. The occurrence of a local pressure minimum in a plane across the vortex requires two positive eigenvalues of the tensor $p_{,ij}$. By removing the unsteady irrotational straining and viscous effects from the strain-rate transport equation (4) one yields the vortex-identification criterion for incompressible fluids in terms of two negative eigenvalues of $\mathbf{S}^2+\mathbf{\Omega}^2$. Vortex region is defined as a connected fluid region with two negative eigenvalues of $\mathbf{S}^2+\mathbf{\Omega}^2$ (that is, if these eigenvalues are ordered, $\lambda_1 \ge \lambda_2 \ge \lambda_3$, by the condition $\lambda_2 < 0$).

λ_{ci}-criterion [15, 19]: The $\varDelta$-criterion has been further developed into the so-called swirling-strength criterion denoted as λ_{ci}-criterion. The time period for completing one revolution of the streamline is given by $2\pi/\lambda_{ci}$ [19]. The two criteria are equivalent only for zero thresholds ($\varDelta$=0 and λ_{ci}=0).

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