Langevin Equations for Pedestrian Motion Modeling

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Abstract: - In the paper a motion of pedestrians in different inside spaces is modeled using the set of N modified Langevin equations. The mental component present in the motion of each pedestrian is taken into account in the model as the additional term - Social Force. , where also the terms describing interactions of pedestrian with surroundings are included. Numerical solutions of these equations for the set of N pedestrians with boundary and initial conditions corresponding to the investigated inside space (e.g. room) and initial locations of pedestrians, respectively, make it possible to observe their trajectories and average velocity of motion. In particular, investigations of the effectiveness of evacuation in the cases of emergency are possible. The level of panic in the evacuation is measured with the value of desired velocity $v_D$, which is one of the parameters in the modified Langevin equations. Some examples of evacuations are presented and factors influencing on evacuation processes are discussed.

Key-Word: Langevin equations, pedestrian motion modeling, evacuation processes.

1 Introduction

Langevin equation were proposed for the description of Brownian motion of particles in a liquid medium [1]. Next, it was used for the description of different dynamical processes in nature. One of the cases of its applications is modeling of pedestrian motion. Such modeling can be important in the cases of evacuation, because due to a high level of panic, motion of pedestrians’ flows can be disturbed, which may results in the injuries and sometimes death of pedestrians. Numerical simulations based on such models can be helpful for the increase of effectiveness of evacuation processes [2].

Also other approaches are used for pedestrian modeling e.g. application of 2-dimensional probabilistic cellular automata, in which spatial distribution of cells corresponds to the internal geometry of rooms in a building. Cellular automaton for modeling pedestrians flow in the room has transitions rules defining shift of a pedestrian to the neighboring cells. They can take into consideration specific dynamic phenomena in pedestrian motion (see e.g. [3-5]). Another type of models are based on the Navier-Stokes equations in which pedestrian motion is treated as a flow of a fluid [1].

A fruitful method of pedestrian motion modeling, is application of Langevin equations with an additional terms describing mental component in pedestrian motion and interactions with surroundings [6-9]. It should be stressed that in pedestrian motion conservation laws for energy and momentum are not fulfill because each pedestrian is able to change suddenly its velocity and its direction of motion, without interactions with other particles or obstacles – pedestrian is an active particle. This is due to the mental processes resulting from the complex interactions with its surrounding and general estimation of situation (e.g. hazard level in the building). Mental component can be introduces to the Langevin equation in the form of an additional term called Social Force (SF), where also interaction terms are included [6-9]. In the present work modified Langevin equations with SF term are used. As an example results of numerical simulations of evacuation for different types of rooms and different level of panic are presented.

2 Modified Langevin Equations

Langevin equation can be adopted to the description of pedestrian motion in different internal spaces using the SF - term, which was proposed by Helbing and
Modified Langevin equations are as follows [1]:

\[
\begin{align*}
\frac{d\bar{v}_i(t)}{dt} & = -\frac{1}{\tau_i} \bar{v}_i(t) + \bar{f}_i(t) + \frac{2\varepsilon_i}{\tau_i} \bar{\xi}_i(t) \\
\frac{d\bar{r}_i(t)}{dt} & = \bar{v}_i(t)
\end{align*}
\]  

(1)

where, \(r_i(t)\) and \(v_i(t)\) – are its position and velocity of \(i\)-th particle, respectively, \(1/\tau_i\) – is a damping coefficient; \(\bar{f}_i(t)\) – is the social force; \(\bar{\xi}_i(t)\) – is the stochastic force with the amplitude proportional to \(\varepsilon_i\).

The social force term is a result of psychological and mental processes e.g., perception of the current situation (including the recognition of an external hazard) and personal aims and interests. After information processing and utility maximization, pedestrian defines the parameters of its further motion (desired velocity, direction, etc.) [7]. The form of the social force is [9]:

\[
\bar{f}_i(t) = \bar{f}_i(v_o) + \sum_{j \neq i} \bar{f}_j(R_i - R_j) + \sum_{B \in \mathcal{B}} \bar{f}_B(R_i - \bar{r}_B)
\]

(2)

where the first term \(\bar{f}_i(v_o) = v_o \bar{e}_i \varepsilon_i^{-1}\) defines the tendency of pedestrian to the motion in a desired direction \(\bar{e}_i\), with a desired velocity \(v_o\); the second term is the territorial effect – it is connected with the tendency of pedestrian to avoid strong approaching to other pedestrians or obstacles. The territorial effect was described using the repulsion force:

\[
\bar{f}_j(R_i - R_j) = A \exp \left( \frac{-E_{gj} v_o \bar{e}_j^n}{C} \right) \bar{e}_j^n
\]

(3)

where \(E_{gj} = (R_i - R_j) - (R_i + R_j)\), \(A\) and \(C\) are scaling constants; \(R_i, R_j\) and \(R_i, R_j\) are the positions of \(i\)-th and \(j\)-th pedestrians and width of the pedestrian’s shoulders, respectively [1].

The third term is the sum of forces coming from granular interactions with other pedestrians and obstacles (located in a position \(\bar{r}_B\)) in the surrounding of \(i\)-th pedestrian; and has following form [9]:

\[
\bar{f}_{ia} = \left( -E_{ia} k_a \bar{e}_a^n - \gamma_{ia}^n \bar{e}_a^n + \varepsilon_{ia}^n k_i \bar{e}_i^n \right) g(e_{ia})
\]

(4)

where \(E_{ia}\) is a distance of \(i\)-th pedestrian to the surface of obstacle \(B\) minus the width \(R_i\) of the shoulder of this pedestrian (in the case of an other pedestrian \(E_{ib} = E_{ib}\)); \(k_a\) and \(k_i\) - are the normal and tangent components of quasielastic force, respectively; \(v_{ia}^n\) and \(v_{ia}^t\) are the normal and tangent components of relative velocity of \(i\)-the pedestrian, respectively; \(\bar{e}_a^n\) and \(\bar{e}_a^t\) - are the normal and tangent versors of the segment connecting \(i\)-th pedestrian with the obstacle \(B\), respectively; \(g(\varepsilon_{ia}) = 1, \) if \(\varepsilon_{ia} < 0\), otherwise \(g(\varepsilon_{ia}) = 0\) [9].

Boundary conditions for the solution of the set of \(N\) equations (1) are constructed according to the geometry of inside spaces (e.g., room) where these pedestrians move. Initial conditions correspond to the positions of pedestrians in \(t = 0\) and may be connected with the positions of their working places or may be random.

The present model takes into considerations different levels of panic which influence on pedestrian motion. It is complex problem which was mainly studied from the perspective of a social psychology. Investigations indicates that pedestrians in the high level of panic shows uncoordinated motion which results in jamming and life-threatening overcrowding [6]. In the present paper as measure of a level of panic we use the desired velocity \(v_o\) (see eq. 2). The value of this parameter equals the velocity the pedestrian wants to reach. It is higher than the real velocity \(v_i\) of the pedestrian, because of its interactions with other pedestrians and obstacles and the grater the value of \(v_o\) the greater is the level of panic. Investigations of evacuation processes indicate that optimal velocity of evacuation is about 1.3 m/sec [8,10]. In our paper it was assumed that no hazard situation is percept by pedestrians in their surrounding when \(v_o\) does not exceeds 2 m/sec. Then, only social repulsion (Eq.3) is present and no granular interactions (Eq.4) between pedestrians are observed [8-10].

The numerical program solving the set of \(N\) Langevin equations (1) contains also the part for designing the internal architecture of the rooms and building in which the evacuation process of \(N\) pedestrians will be investigated. The values of parameters used in the computations were: \(\tau_i = 0.5\) sec; \(k_a = 1.2 \times 10^5\) N; \(k_i = 2.4 \times 10^5\) N; \(\gamma = 100\) kg/s; \(A = 1000\) N; \(C = 0.08\) m; The length of the grid edge shown in the figures equals 1 m.

### 3 Pedestrian motion in a chosen types of rooms

As an example of application of Langevin equations the evacuations from office rooms and classroom were investigated. Geometry and initial positions of pedestrians in these rooms are shown in Fig.1. Trajectories of pedestrians for \(v_o = 1\) m/sec are shown in Fig.2. It can be seen that for the office rooms (Fig.2.a) the trajectory of each pedestrian is smooth and not perturbed, which means that pedestrians motion in the rooms and in these conditions is laminar. In the case of classroom (Fig.2.b), however, the trajectories near
the door are thicken. It is caused by the relative large number of pedestrians (N=31) in the classroom and the presence of number of desks. Also the increase of the number of pedestrians in the office rooms can change the character of trajectories. In the fig. 2.c. the trajectories of N = 50 pedestrians initially present in the office rooms are shown. Comparing the course of trajectories in this case – were the rooms are overcrowded - with the previous cases (see fig. 2.a,c) it can be seen that significant perturbations of motion and clogging of pedestrians near the door are observed.

The relations between the time of evacuation T and the desired velocity \( v_D \) (i.e. the level of panic) are shown in Fig. 3. It can be seen (Fig.3.a), that in the case of the office rooms with normal number of pedestrians inside, the increase of the values of desired velocities causes decreasing of evacuation times. This is an evidence of the laminar character of pedestrian

In a special conditions and for the large values of desired velocity (which means the increased level of panic) laminar motion of pedestrians disappears and turbulent motion can emerge in an evacuation process. Such type of pedestrian motion is observed in the case of overcrowded office room (Fig.2.c). Due to the large clogging of pedestrians near the door, the relation \( T(v_D) \) became non-monotonic and for \( v_D > 3.8 \text{ m/sec} \), the time of evacuation increases with the values of \( v_D \) (Fig. 3.b).

Similar situations appears also for the classroom, where the clogging near the doors became very massive for larger level of panic.

It is interesting to observe the relation between the reduced real velocity \( v_i/v_D \) as a function of time. This relation can reveal the details of a dynamics of a single pedestrian. It is shown in Fig.4 for the case of a simple rectangular room, \( v_D = 3 \text{ m/sec} \) and for the pedestrian which is marked with black color. At the beginning of motion, it can be seen rapid acceleration of pedestrian in lower, almost empty part of room – real velocity quickly approaches to desired velocity. In the next part of motion the marked pedestrian is located closely to the number its of neighbors. All have very similar velocity therefore the velocity of the marked pedestrian saturates. In the nearest neighborhood of the door large clogging leads to a sudden decreasing of the real pedestrian velocity – in the moment of passing the door \( v_i \) equals almost zero. After passing the door pedestrian rapidly accelerate, but fluctuation of his
velocity are visible. They come from the random interactions with other pedestrian running in the same direction.

4 Conclusions

In conclusion it can be noticed that mathematical modeling of pedestrian motion using Langevin equations with the social force is very effective. In particular, such models enable to investigate evacuation processes in the different kind of rooms and to compare the effectiveness of evacuation for different internal geometry. Using this model also evacuation of buildings is possible. Then a proper set of rooms (including staircases, where the motion of pedestrian is modified) can be constructed in the form of the corresponding boundary conditions. It should be noticed that numerical simulations based on this mathematical model make possible to observe dynamical phenomena which have essential influence on the evacuation, like emergence of clogging occurring e.g. near the doors. Such observations can be useful for planning the evacuation strategies in the buildings as well as for the training of services active in the crises situations.

References: