Fuzzy Goal Programming Approach to Multiobjective Linear Plus Linear Fractional Programming Problem

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Abstract: This paper presents a fuzzy goal programming (GP) procedure for solving a multiobjective linear plus linear fractional programming problem. In the proposed approach GP model for achievement of the highest membership value of each of fuzzy goal defined for the linear plus linear fractional objectives is formulated. In the solution process, the method of variable change on the under- and over-deviational variables of the membership goals associated with the fuzzy goals of the model is introduced to solve the problem efficiently by using goal programming (GP) methodology and method of approximation (MAP). A numerical examples is given to illustrate this algorithm. The examples is solved by optimization software TORA@ 2.0 version, 2006.

Key-Words: Multiobjective Programming, Fractional programming, Fuzzy multiobjective linear fractional pro- gramming, Goal programming, Fuzzy goal programming.

1 Introduction

A general linear plus linear fractional programming (LLFP) problem is defined as the following way:

Maximize $F(x) = (p^T x + \theta) + \frac{c^T x + \alpha}{d^T x + \beta}$

subject to

$Ax = b$  
$x \geq 0,$

where $x, c, d, p \in R^n, b \in R^m, \alpha, \beta, \theta \in R.$

For some values of $x,$ $d^T x + \beta$ may be equal to zero but here we take only the case $d^T x + \beta > 0.$ If we take more than one objectives in general linear plus fractional programming problem, then the problem is known as multiobjective linear plus linear fractional programming problem, mathematically it can be written as:

Maximize $F(x) = [F_1(x), F_2(x), \ldots, F_k(x)],$

where $F_i(x) = l_i(x) + \frac{f_i(x)}{m_i(x)},$

$x \in X.$

and, $l_i(x) = p_i^T x + \theta_i,$ $f_i(x) = c_i^T x + \alpha_i,$ $m_i(x) = d_i^T x + \beta_i,$ are real valued function on $X,$ where $X = \{x : Ax (\leq, =, \geq) b, x \geq 0, x \in R^n, b \in R^m, A = (a_{ij})_{m \times n}, \theta_i, \alpha_i, \beta_i \in R\},$ and $d_i^T x + \beta_i > 0$ ($i = 1, 2, \ldots, k$) $\forall x \in X.$

Here, $X$ is assumed to be non-empty convex bounded set in $R^n.$

If an uncertain aspiration level is introduced to each of the objectives of MOLLFP, then these fuzzy objectives are called fuzzy goals. The Fuzzy multiobjective linear plus linear fractional programming problem (FMOLLFP) can be defined as

Find $X(x_1, x_2, \ldots, x_n)$ such that

$F_i(x) \leq g_i$ or $F_i(x) \geq g_i \forall (i = 1, 2, \ldots, k)$

subject to

$x \in X = \{x \in R^n, Ax \leq b, x \geq 0$ with $b \in R^m,$ $A \in R^m \times n\}$

$F_i(x) = (p_i^T x + \theta_i) + \frac{c_i^T x + \alpha_i}{d_i^T x + \beta_i}$

where $g_i$ is the aspiration level of the $i^{th}$ objective $F_i$ and $\leq,$ $\geq$ indicate fuzziness of the aspiration level. The membership function $\mu_i(x)$ must be described for each fuzzy goal. A membership function can be explained as given below:
If $F_i(x) \lesssim g_i$, then

$$
\mu_i(x) = \begin{cases} 
1 & \text{if } F_i(x) \lesssim g_i \\
\frac{t_i - F_i(x)}{t_i - g_i} & \text{if } g_i \lesssim F_i(x) \lesssim t_i \\
0 & \text{if } F_i(x) \gtrsim t_i 
\end{cases}
$$

If $F_i(x) \gtrsim g_i$, then

$$
\mu_i(x) = \begin{cases} 
1, & \text{if } F_i(x) \gtrsim g_i \\
\frac{F_i(x - t_i)}{g_i - t_i}, & \text{if } t_i \lesssim F_i(x) \gtrsim g_i \\
0, & \text{if } F_i(x) \lesssim t_i 
\end{cases}
$$

and $t_i$ and $t_j$ are the upper tolerance limit and lower tolerance limit, respectively, for the $i^{th}$ fuzzy goal.

Multiobjective linear plus linear fractional programming (MOLLFP) are applied to different disciplines such as transportation, problem of optimizing enterprise capital, the production development fund and social, cultural and construction fund. Multiobjective linear plus linear fractional programming problem have been extensively studied by authors and the research is based on the theoretical background of fractional programming. As a matter of fact, many ideas and approaches have their foundation in the theory of fractional programming (See [1, 15]).

Teterev [10] pointed out this type of problem and he derived optimality criteria for (LLFP) using simplex type algorithm. Shaible [1, 2] has pointed out for LLFP that for these problems a local maximum is not a global one and an optimal solution is not attained at an extreme point of polyhedron in general. Chaddha [8] presented a dual of a maximization problem for linear plus linear fractional programming problem under linear constraints. His approach is based on assertion of Teterev [10] which have been already proven to be erroneous. In [4], Hirche clarified some deficiencies of Chaddha’s [8] proposed duality and illustrated the fact about the behavior of the objective function. Singh, Gupta and Bhatia [5] studied multiparametric sensitivity analysis for LLFP using the concept maximum volume in the tolerance region. They constructed critical regions for simultaneous and independent perturbations in the objective function coefficients and in the right-hand-side vector in the given problem. They derived necessary and sufficient conditions to classify perturbation parameter as focal and non-focal. In [6], Gupta and Singh studied multiparametric sensitivity analysis under perturbations in multiple rows or columns of the constraint matrix in linear plus linear fractional programming problem.

Recently, Jain and Lachhawani [3] have given the solution procedure for sum of linear plus linear fractional multiobjective programming problem fuzzy under rule constraint. They suggested the use of if - then fuzzy reasoning method to determine the crisp functional relationship between the objective function and decision variables under the assumptions that the denominator of the fractional part of the objective function is non-zero on the constraint set and finally solved the resulting programming problem to find a pair of optimal solution of original problem. Kheirfam [13] studied classical sensitivity analysis, when the coefficients of the objective function and right hand side are parameterized. Recently Singh [14] derived optimality and duality conditions for transportation with linear plus linear fractional programming problem. He also established weak duality and strong duality theorem for the dual model.

In this paper, we propose an algorithm to the solution of multiobjective linear plus linear fractional programming problem (MOLLFP) using goal programming procedure. In the Goal Programming (GP) model formulation of the problem, first the objectives are transformed in to fuzzy goals by means of assigning an aspiration level to each of them. Then achievement of the highest membership value (unity) to the extent possible of each of the fuzzy goals is considered. In the solution process, the under- and over-deviation variables of the membership goals associated with the fuzzy goals are introduced to transform the proposed model into an equivalent non - linear goal programming (NLGP) model to solve the problem using Wolf - Frank method of approximation programming ( MAP). Our attempt is to give simple solution procedure for MOLLFP.

## 2 Goal Programming Problem

The concept of goal programming (GP) was first introduced by Charnes and Cooper in 1961 [20] as a tool to resolve infeasible linear programming problems. There after, significant methodological development of GP was made by Ignizio [19] and among others. The overall purpose of GP is to minimize the deviations between the achievement of goals and their
applications. A typical GP is expressed as follows

\[
\text{Minimize } \sum_{i=1}^{k} |F_i(x) - g_i|
\]
subject to
\[
x \in X = \{ x \in \mathbb{R}^n; Ax \leq b, x \geq 0 \}.
\]

Where \( F_i \) is the linear function of the \( i^{th} \) goal and \( g_i \) is the aspiration level of \( i^{th} \) goal.

Let \( F_i(x) - g_i = d_i^{+} - d_i^{-} \), \( d_i^{-}, d_i^{+} \geq 0 \). Equation \( (6) \) can be formulated as follows

\[
\text{Minimize } \sum_{i=1}^{k} (d_i^{+} + d_i^{-})
\]
subject to
\[
F_i(x) - d_i^{+} + d_i^{-} - g_i = 0, i = 1, 2, \ldots k
\]
\[
x \in X = \{ x \in \mathbb{R}^n; Ax \leq b, x \geq 0 \}.
\]

Where \( d_i^{-} \geq 0, d_i^{+} \geq 0 \) are, respectively under - and over - deviations of \( i^{th} \) goal.

Problem \( (7) \) has been applied to solve many real world problems.

### 2.1 Fuzzy Goal Programming

In fuzzy goal programming approaches, the highest degree of membership function is 1. So, for the defined membership function in \( (4) \) and \( (5) \), the flexible membership goals with aspiration levels 1 can be expressed as

\[
\frac{F_i(x) - t_i}{g_i - t_i} + d_i^{-} - d_i^{+} = 1
\]
\[
\text{or}
\frac{t_i - F_i(x)}{t_i - g_i} + d_i^{-} - d_i^{+} = 1
\]

Where \( d_i^{-} \geq 0, d_i^{+} \geq 0 \) with \( d_i^{-}, d_i^{+} = 0 \) are, respectively under - and over - deviations from the aspiration levels.

In conventional GP, the under- and over-deviational variables are included in the achievement function or minimizing them and that depend upon the type of the objective functions to be optimized.

In this approach, only the under - deviational variable \( d_i^{-} \) is required to be achieve the aspired levels of the fuzzy goals. It may be noted that any over - deviation from fuzzy goal indicates the full achievement of the membership value. Recently, B. B. Pal. et.al [17] proposed an efficient goal programming (GP) method for solving Fuzzy multiobjective linear fractional programming problems. In this paper, the idea of B.B.Pal for FMOLFP is extended to FMOLLFP.

### 3 Linear Approximations of Nonlinear Programs [2]

Algebraic procedures such as pivoting are so powerful for manipulating linear equalities and inequalities that many nonlinear programming algorithms replace the given problem by an approximating linear problems. Separable programming is a prime example and also one of the most useful of these procedures. As in separable programming these non - linear algorithms usually solve several linear approximations by letting the solution of the last approximation suggest a new one.

By using different approximation schemes, this strategy can be implemented in several ways. We are restricted ourself only Frank - Wolf Algorithm and its extension.

#### 3.1 Frank - Wolf Algorithm [2]

Let \( x^{(0)} = (x_1^0, x_2^0, \ldots x_n^0) \) be any feasible solution with linear constraints:

\[
\text{Max } F(x_1, x_2, \ldots x_n)
\]
subject to
\[
\sum_{j=1}^{m} a_{ij} x_j \leq b_i, \quad (i = 1, 2, \ldots m)
\]
\[
x_j \geq 0, \quad (j = 1, 2, \ldots n).
\]

Here \( x^{(0)} \) might be determined by Phase I of the simplex method. This algorithm forms a linear approximation at the point \( x^{(0)} \) by replacing the objective function with its current value plus a linear correction term; that is by the linear objective

\[
F(x^{(0)}) + \sum_{j=1}^{n} c_j (x_j - x_j^0)
\]

where \( c_j \) is the slope, or partial derivative of \( F \) with respect to \( x_j \), evaluated at the point \( x^{(0)} \). Since \( f(x^{(0)}), c_j \), and \( x_j \) are fixed, maximizing the objective function is equivalent to maximizing

\[
Z = \sum_{j=1}^{n} c_j x_j.
\]

This linear approximation problem is solved, giving an optimal solution \( y = (y_1, y_2, \ldots y_n) \). At this point
the algorithm recognizes that, although the linear approximation problem indicates that the objective improves steadily from $x^{(0)}$ to $y$. Therefore, the algorithm uses a procedure to determine the maximum value for $F(x_1, x_2, \ldots, x_n)$ along the line-segment joining $x^{(0)}$ to $y$. Letting $x^1 = (x^1_1, x^1_2, \ldots, x^1_n)$ denote the optimal solution of the line-segment optimization problem, we repeat at $x^1$. Continuing in this way, we determine a sequence of points approach $x^1 = (x^1_1, x^1_2, \ldots, x^1_n)$ any point $x^* = (x^*_1, x^*_2, \ldots, x^*_n)$ that these point approach in the limit is an optimal solution to the original problem. The Frank-Wolf algorithm is convergent computationally because it solves linear programs with the same constraints as the original problem.

### 3.2 MAP (Method of Approximation) [2]

The Frank-Wolf algorithm can be extended to general nonlinear programs by making linear approximations to the constraints as well as the objective function. When the constraints are highly nonlinear, however, the solution to the approximation problem can become far removed from feasible region since the algorithm permits large moves from any candidate solution. The Method of approximation programming (MAP) is a simple modification of this approach that limits the size of any move. As a result, it is sometimes referred to as a small-step procedure. Let $x^{(0)} = (x^{(0)}_1, x^{(0)}_2, \ldots, x^{(0)}_n)$ be any candidate solution to the optimization problem:

Max $F(x_1, x_2, \ldots, x_n)$

subject to,

$g_i(x_1, x_2, \ldots, x_n) \leq 0, (i = 1, 2, \ldots, m)$

Each constraint can be linearized, using its current value $g_i(x^{(0)})$ plus a linear correction term, as:

$$g_i(x) = g_i(x^{(0)}) + \sum_{j=1}^{n} a_{ij}(x_j - x^0_j) \leq 0,$$

where $a_{ij}$ is the partial derivative of constraint $g_i$ with respect to variable $x_j$ evaluated at the point $x^{(0)}$. This approximation is a linear inequality, which can be written as

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i^0 = \sum_{j=1}^{n} a_{ij}x^0_j - g_i(x^{(0)}).$$

since the terms on the right hand side are all constants. The MAP algorithm uses these approximations, together with the linear objective function approximation and solve the linear programming problem:

Maximize $Z = \sum_{j=1}^{n} c_j x_j$

subject to

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i^0, (i = 1, 2, \ldots, m), x_j \geq 0, (j = 1, 2, \ldots, n),$$

We might expect then that the additional work required by the line-segment optimization of Frank-Wolf algorithm is not worth the slightly improved solution that it provides. MAP operates on this premise, taking the solution to the linear programs as the new $x^1$. The partial derivative data $a_{ij}$, $b_i$, and $c_j$ is recalculated at $x^1$, and the procedure is repeated. Continuing in this manner determines points $x^1, x^2, \ldots, x^k, \ldots$ and as in Frank-Wolf procedure any point $x^* = (x^*_1, x^*_2, \ldots, x^*_n)$ that these points approach in the limit is considered a solution.

### 4 Solution Method

We consider the multiobjective linear plus linear fractional programming problem of the form (2).

Assume fuzzy aspiration level $g_i$ and tolerance limit $(\bar{l}_i, \bar{u}_i)$ for each objective function $F_i$, then we construct membership function for each objective using Zimmermann Max-Min approach [16], then the problem (2) becomes FMOFFLP (3). The membership function $\mu_i$ must be described for each fuzzy goal as equation (4) and (5).

The proposed algorithm can be explained in three steps:

**Step 1:** Determine $x^*_i = \{x^*_1, x^*_2, \ldots, x^*_n\}$ for which the $i^{th}$ membership function $\mu_i$ is to be constructed, associated with $i^{th}$ objective function $F_i(x)$ $\forall (i = 1, 2, \ldots, k)$, where $n$ is the number of variable.

**Step 2:** The $i^{th}$ membership goal (8) can be written as

$$H_iF_i - H_i\bar{l}_i + d_i^* - d_i^+ = 1, \quad H_i = \frac{1}{g_i - \bar{l}_i}. \quad (9)$$

Substituting the expression for $F_i$

$$H_i\{(p_i^T x + \theta_i)(d_i^T x + \beta_i) + (c_i^T x + \alpha_i)\}$$

$$+ d_i^- (d_i^T x + \beta_i) - d_i^+ (d_i^T x + \beta_i) \quad (10)$$

$$= H'_i(d_i^T x + \beta_i),$$

where $H'_i = 1 + H_i \bar{l}_i$.

Similar expression for other membership goal can also be obtained. However, for model simplification, the expression in (10) can be considered as a general form of goal expression for any type of the stated membership goals. Using the method of variable change as presented by Kornbluth and Steuer [21], the goal expression in (10) can be written as follows: The simplified form of the expression in (10) is obtained
as

\[ C_i x^2 + F_i x + D_i^- - D_i^+ = G_i \]  

where \( G_i = H_i p_i^T \beta_i + H_i \alpha_i - H_i^\prime \beta_i \)

\[ C_i = H_i p_i^T d_i^T \]

\[ F_i = H_i p_i^T d_i^T + H_i p_i^T \beta_i + c_i^T H_i - d_i^T H_i^\prime \]

\[ D_i^- = d_i^- (d_i^T x + \beta_i) \]

\[ D_i^+ = d_i^+ (d_i^T x + \beta_i) \]

with \( D_i^- , D_i^+ \geq 0 \) and \( D_i^- . D_i^+ = 0 \),

since \( d_i^- , d_i^+ \geq 0 \), \( d_i^- (d_i^T x + \beta_i) \geq 0 \).

Step 3: Now in making decision, minimization of \( d_i^- \)
means \( \frac{D_i^-}{(d_i^T x + \beta_i)} \), which is also a non-linear one.

It may be noted that when a membership goal is fully achieved, \( d_i^+ = 0 \) and when its achievement is zero, \( d_i^- = 1 \) are found in the solution.

So involvement of \( d_i^- \leq 1 \) in the solution leads to impose the following constraint to the model of the problem.

\[
\frac{D_i^-}{(d_i^T x + \beta_i)} \leq 1, \\
i.e. \quad -d_i^T x + D_i^- \leq \beta_i. 
\]  

(12)

It may be pointed out that any such constraint corresponding to \( d_i^+ \) does not arise in the formulation and simplest version of GP (i.e. minsum GP)(17) is introduced to formulate the model of the problem under consideration, then the GP model formulation becomes:

Minimize \( \bar{F} = \sum_{i=1}^{k} w_i^- D_i^- \)

also satisfy \( C_i x^2 + F_i x + D_i^- - D_i^+ = G_i \)

subject to \( x \in X = \{x \in \mathbb{R}^n, A x \leq b, x \geq 0 \} \)

\( -d_i^T x + D_i^- \leq \beta_i \).

\( D_i^- , D_i^+ \geq 0, \quad i = 1, 2 \ldots k \)

\[ w_i^- = \begin{cases} 
\frac{1}{g_i - \mu_i} & \text{for } \mu_i(x) \text{ in (5)} \\
1 & \text{for } \mu_i(x) \text{ in (4)}.
\end{cases} 
\]

Where \( \bar{F} \) represents the fuzzy achievement function consisting of the weighted under-deviation variables, and the numerical weights \( w_i^- \geq 0 \), \( i = 1, 2, \ldots k \) represent the relative importance of achieving the aspired level of the respective fuzzy goals subject to the constraints sets of the decision situation.

Now above non-linear GP can be solve easily using Wolf-Frank method of approximation (MAP) satisfying the nonlinear constraints.

### 5 Numerical Example

Consider a MOLLFP with two objective functions:

\[
\text{Max}\{ F_1(x) = (-x_1 - 1) + \frac{-5x_1 + 4x_2}{2x_1 + x_2 + 5}, \quad F_2(x) = (x_2 + 1) + \frac{9x_1 + 2x_2}{7x_1 + 3x_2 + 1}\}
\]

subject to

\[
x_1 - x_2 \geq 2 \\
4x_1 + 5x_2 \leq 25 \\
x_1 \geq 5 \\
x_1, x_2 \geq 0.
\]

It is observed that \( F_1 < 0, F_2 \geq 0, \) for each \( x \) in the feasible region.

If the fuzzy aspiration levels of the two objectives are \(-7.31, 3.21\), find \( x \) in order to satisfy the following fuzzy goals.

\( F_1(x) \gtrless -7.31, \quad F_2(x) \gtrless 3.21. \)

The tolerance limits for the two fuzzy goals are \((-9.04, 2.21)\) respectively. The membership function for the two fuzzy goals are i.e.

\[
\mu_1(x) = \begin{cases} 
1, & \text{if } F_1(x) \gtrless -7.31 \\
\frac{(x_1 - 1) + \frac{5x_1 + 4x_2}{2x_1 + x_2 + 5} + 9.04}{1} & \text{if } -9.04 \lesssim F_1(x) \lesssim -7.31 \\
0, & \text{if } F_1(x) \lesssim -9.04
\end{cases}
\]

(15)

\[
\mu_2(x) = \begin{cases} 
1, & \text{if } F_2(x) \gtrless 3.21 \\
\frac{(x_2 + 1) + \frac{9x_1 + 2x_2}{7x_1 + 3x_2 + 1} - 2.21}{1} & \text{if } 2.21 \lesssim F_2(x) \lesssim 3.21 \\
0, & \text{if } F_2(x) \lesssim 2.21.
\end{cases}
\]

(16)
Then the membership goal can be expressed as
\[
\frac{(-x_1 - 1) + \frac{-5x_1 + 4x_2}{2x_1 + x_2 + 5} + 9.04}{1.73} + d_1^{-} - d_1^{+} = 1
\]
\[
\frac{(x_2 + 1) + \frac{-9x_1 + 2x_2}{7x_1 + 3x_2 + 1} - 2.21}{1} + d_2^{-} - d_2^{+} = 1
\]
where, \( d_i^{-}, d_i^{+} \geq 0, \) with \( d_i^{-}.d_i^{+} = 0, \) \( i = 1, 2, \ldots k. \) Following the procedure, the membership goals are restated as
\[
-2x_1^2 - x_1 x_2 + 2.54x_1 + 10.27x_2 + \quad (19)
\]
\[
D_1^{-} - D_1^{+} = -31.35,
\]
\[
3x_2^2 + 7x_1 x_2 - 6.41x_1 + 3x_2 + \quad (20)
\]
\[
D_2^{-} - D_2^{+} = 8.84
\]

Now the restrictions \( d_1^{-} \leq 1 \) and \( d_2^{-} \leq 1 \) gives
where
\[
D_1^{-} - 3.46x_1 - 1.73x_2 \leq 8.65
\]
\[
D_2^{-} - 7x_1 - 3x_2 \leq 1
\]
Thus the final equivalent GP formulation is obtained as

Find \( X(x_1, x_2) \)
\[
\text{Min } \left( \frac{1}{1.73} D_1^{-} + D_2^{-} \right)
\]
subject to
\[
-2x_1^2 - x_1 x_2 + 2.54x_1 + 10.27x_2 +
\]
\[
D_1^{-} - D_1^{+} = -31.35
\]
\[
3x_2^2 + 7x_1 x_2 - 6.41x_1 + 3x_2 +
\]
\[
D_2^{-} - D_2^{+} = 8.84
\]
subject to
\[
D_1^{-} - 3.46x_1 - 1.73x_2 \leq 8.65
\]
\[
D_2^{-} - 7x_1 - 3x_2 \leq 1
\]
\[
x_1 - x_2 \geq 2
\]
\[
4x_1 + 5x_2 \leq 25
\]
\[
x_1 \geq 5
\]
\[
x_1, x_2 \geq 0.
\]
\[
D_1^{-}, D_1^{+}, D_2^{-}, D_2^{+} \geq 0, \text{ with, } D_i^{-}.D_i^{+} = 0
\]
\( i = 1, 2, \ldots k. \)

Now apply Frank - Wolf method of approximation (MAP). If we assume the initial solution is \( x_1 = 5, \) \( x_2 = 0, \) \( D_1^{-} = 0, D_1^{+} = 0, D_2^{-} = 0, D_2^{+} = 0 \) from the feasible region, then the non-linear problem transformed in to linear approximation program as follows

Find \( X(x_1, x_2) \)
\[
\text{Min } \left( \frac{1}{1.73} D_1^{-} + D_2^{-} \right)
\]
subject to
\[
-17.46x_1 + 5.27x_2 + D_1^{-} - D_1^{+} = -81.35
\]
\[
-6.41x_1 + 38x_2 + D_2^{-} - D_2^{+} = -72.9
\]
\[
D_1^{-} - 3.46x_1 - 1.73x_2 \leq 8.65
\]
\[
D_2^{-} - 7x_1 - 3x_2 \leq 1
\]
\[
x_1 - x_2 \geq 2
\]
\[
4x_1 + 5x_2 \leq 25
\]
\[
x_1 \geq 5
\]
\[
x_1, x_2 \geq 0
\]
\[
D_1^{-}, D_1^{+}, D_2^{-}, D_2^{+} \geq 0, \text{ with, } D_i^{-}.D_i^{+} = 0
\]
\( i = 1, 2, \ldots k. \)

The optimal solution of the problem (23) is at the point \( x_1 = 5, \) \( x_2 = 1, \) \( D_1^{-} = 0.68, D_1^{+} = 0, D_2^{-} = 0, D_2^{+} = 78.85, \) and minimum value is 0.39. Now repeat the process for the point \( x_1 = 5, \) \( x_2 = 1, \) \( D_1^{-} = 0.68, D_1^{+} = 0, D_2^{-} = 0, D_2^{+} = 78.85, \) and the new LPP is obtained

Find \( X(x_1, x_2) \)
\[
\text{Min } \left( \frac{1}{1.73} D_1^{-} + D_2^{-} \right)
\]
subject to
\[
-18.46x_1 + 5.27x_2 + D_1^{-} - D_1^{+} = -86.35
\]
\[
0.59x_1 + 44x_2 + D_2^{-} - D_2^{+} = 46.84
\]
\[
D_1^{-} - 3.46x_1 - 1.73x_2 \leq 8.65
\]
\[
D_2^{-} - 7x_1 - 3x_2 \leq 1
\]
\[
x_1 - x_2 \geq 2
\]
\[
4x_1 + 5x_2 \leq 25
\]
\[
x_1 \geq 5
\]
\[
x_1, x_2 \geq 0
\]
\[
D_1^{-}, D_1^{+}, D_2^{-}, D_2^{+} \geq 0, \text{ with, } D_i^{-}.D_i^{+} = 0
\]
\( i = 1, 2, \ldots k. \)

The optimal solution of the problem (24) is at the point \( x_1 = 5, \) \( x_2 = 1, \) \( D_1^{-} = 0.68, D_1^{+} = 0, D_2^{-} = 0, D_2^{+} = 0.11, \) and minimum value is 0.3944. Since we obtained the same value for \( x_1 = 5, \) \( x_2 = 1. \) So \( x_1 = 5, \) \( x_2 = 1 \) is the final solution for the problem.
The solution for the original problem is given by $x_1 = 5$, $x_2 = 1$, $F_1 = -7.31$, $F_2 = 3.21$. The membership function values at $(5, 1)$ indicate that goal $F_1$ and $F_2$ are satisfied 100% and 100% respectively, for the obtained solution.

6 Conclusion

Various fuzzy approaches have been proposed for the solution of multiobjective linear plus linear fractional programming problem and most of the approaches have computational burdensome. Our approach is to give simple procedure for the solution of multiobjective linear plus linear fractional programming problem using fuzzy set theory, goal programming and method of approximation (MAP).

References:


