Hybrid Solutions for International Marketing Decision-Making: 
Mathematical Description, Computational Modelling, Knowledge 
Automation and Software Examples

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Abstract: Aiming at supporting the international marketing decision-making process and improving 
its outcomes, the authors develop and put forward a novel hybrid mathematical, computational and 
knowledge automation framework for integrating and hybridising the strengths of Web-enabled 
Monte Carlo simulation algorithms, fuzzy logic, knowledge automation expert system, and online 
relational data algebraic operations. The value of the framework is illustrated using software 
examples.

Key-Words: International marketing planning; digital marketing; computational modelling; hybrid 
knowledge automation; simulation; fuzzy logic; expert system; decision support

1 Introduction
Over the past decades, computer-based systems 
that assist with strategic marketing planning have 
obtained attention from both researchers and 
practitioners [9, 10]. Studies have also been 
conducted to investigate the use of computers in 
support of different facets of the international 
marketing decision making process for entering 
and competing in the global markets using various 
techniques and technologies including expert 
systems [1, 2, 14], fuzzy logic [8], software agents 
[12, 15], and hybrid systems [10, 11, 12, 13]. 

Previous studies, nonetheless, are mainly 
concerned with the development and evaluation of 
relevant software systems. In contract, as the first 
attempt, we propose and develop a hybrid 
mathematical, computational and knowledge 
automation framework for the process of 
international marketing decision-making. We will 
also show and discuss associated software 
examples created by the authors.

2 Mathematical Description, 
Computational Modelling and 
Knowledge Automation Framework
A system can be defined as a multifaceted 
complex of interacting components [21]. A 
hybrid system [4, 5] is an effective solution to 
many complex problems. Our framework aims 
at assisting the key stages of international 
marketing planning: 1) “go versus no go” 
decisions; 2) “how to go” decisions, and 3) 
marketing strategy formulation. Diverse 
decision support and artificial intelligence 
techniques and technologies are integrated into 
one framework, while their specific advantages 
are utilised to deal with the particular aspects of 
the planning problem. The following elements 
will be used in our mathematical description 
and computational formulation:

- The simplified international marketing 
decision making process, I, which focuses on 
the following three key stages: go 
versus no go decisions (G), how to go 
decisions (entry mode selection) (H), and 
marketing strategy formulation (S).
• Human judgement, personal vision, intuition and creativity, J.
• Decision support and artificial intelligence techniques and technologies, T, such as analytic hierarchy process (AHP), Internet-enabled intelligent software agents (IA), Monte Carlo simulation (MCS), fuzzy logic (FL), Web-based knowledge automation expert systems (KAES), online databases (DB), and other techniques (OT).

The key stages of the international marketing decision making process may be formulated as: $I = (G, H, S)$. The supporting techniques and technologies can be defined as: $T = (AHP, IA, MCS, FL, KAES, DB, OT, Web)$. Managerial judgement can is described as: $J = (Judgement, Intuition, Creativity)$.

The outcome of the international marketing decision making process can be represented as the following equation:

$$\text{OUTCOME} = f_I (G, H, S) + f_T (AHP, IA, MCS, FL, KAES, DB, OT, Web) + f_J (Judgement, Intuition, Creativity)$$

where $f_I$, $f_T$ and $f_J$ are implicit functions for the decision making process, supporting techniques and human judgement, respectively. Here, symbol “+” indicates logical join, union, integration or hybridisation with relevant interaction.

Within this mathematical framework, the Web-based hybrid intelligent decision support system, WHS, is expressed as

$$\text{WHS} = \text{AHP} \bowtie \text{MCS} \bowtie \text{FL} \bowtie \text{KAES} \bowtie \text{DB} \bowtie \text{OT} \bowtie \text{Web}$$

in which the symbol $\bowtie$ is intercommunicating hybridisation and integration operator and OT denotes other techniques and associated sub-systems.

### 2.1 The analytic hierarchy process

The analytic hierarchy process (AHP) [17, 18] can be used to structure the decision making problem in a hierarchical form and incorporate human judgement and preferences [20]. It enables the user to build pair-wise comparison matrices and sum up the weights or relative importance of decision variables or factors to gain an overall ranking [7]. AHP can be applied to estimate the weights for the factors or criteria influencing international marketing planning.

### 2.2 Web-based Monte Carlo simulation

Monte Carlo simulation utilises probability and random numbers to model and deal with uncertainty and stochastic permutation [16]. The algorithm for Monte Carlo simulation employs uniformly distributed random numbers and the inverse function of a symmetric or asymmetric cumulative distribution of the triangular probability distribution. In this study, the algorithm published on the Web site http://www.brighton-webs.co.uk/distributions/triangular.asp (accessed on 01/12/2008) has been adapted and extended to suit the computing needs of simulation analysis within the proposed framework.

### 2.3 Fuzzy logic and the space of strategic variables

According to page 338 of Zadeh [24], “a fuzzy set is a class of objects with a continuum of grades of membership”. Let $U$ be a universe of discourse, a collection of objects $\{u\}$. A fuzzy set $A$ in $U$ is symbolized by a compatibility or membership function $\mu_A$ taking values in the range $[0, 1]$. $A$ in $U$ is denoted as $\{ (u, \mu_A(u)) | u \in U \}$ where $U$ can be continuous or discrete.

The space of strategic variables or factors is defined below.

For the cases of making “go versus no go” decisions, the set of variables or factors (values ranging from 1 to 10) that influence the choices can be represented as an vector $Z$, on the basis of Fung et al [3]’s work:

$$Z = (Z_1, Z_2, \ldots, Z_m) \text{ in the fuzzy space of } U$$

For the input fuzzy vector $Z$, there exist $s$ a real vector $w$ that represents the weights or relative importance for the strategic factors or variables, such that $w = (w_1, w_2, \ldots, w_m)$ which can be determined using the AHP method.

We then aggregate the values of $Z$ and $w$ to compute the value of an object, $u$, using the formula:

$$u = Z_1 \cdot w_1 + Z_2 \cdot w_2 + \ldots + Z_m \cdot w_m$$

Similarly, for the cases of using four-box or nine-box matrix for strategy formulation, a set of criteria or variables (values ranging from 1 to 10)
determining any dimension of a strategic grid/matrix can be modelled and calculated in the same way.

2.4 Fuzzification of the variables or factors affecting international marketing decisions

Trapezoidal membership/compatibility functions are employed to fuzzify the aggregated scores for “go versus no go” variables or strategic grid dimensions. For real numbers $a \leq b \leq c \leq d$, the trapezoid $\tilde{T}(a, b, c, d)$ with amplitude one is defined by Levy and Yoon [8] as:

$$\tilde{T}(u) = \begin{cases} 0 & \text{if } u \leq a; \\ \frac{u-a}{b-a} & \text{if } a < u \leq b; \\ 1 & \text{if } b < u \leq c; \\ \frac{d-u}{d-c} & \text{if } c < u \leq d; \\ 0 & \text{if } d < u; \\ 1 & \text{if } a = b; \\ \frac{d-c}{d-c} & \text{if } c = d. \end{cases}$$

Here, $u \in U$.

2.5 Fuzzification of the strategic models

Firstly, inputs for each strategic factor from the AHP and simulation components and human decision-makers are collected and aggregated. An overall score for each dimension of the strategic matrix or grid is then calculated. The calculated scores are then converted into fuzzy membership or compatibility functions. Fig. 1 illustrates one example where trapezoidal membership functions are used. Other models may be fuzzified using a similar method.

2.6 Advising “go versus no go” options

Fuzzy variable “go” and “no go” are recommended with levels of confidence simply computed using the above-mentioned trapezoidal membership functions, on the basis of the aggregated scores for pertinent variables or criteria concerned. See Fig.2.

2.7 Entry mode selection

Entry modes are determined by “if-then” rules on the basis of the user’s inputs to the following two criteria: the level of risk the company is wishing to take, and the degree of market control.

2.8 Evaluation of the fuzzy rules for strategic grids or models for advising marketing strategies

Based upon the theory for the management of uncertainty in expert systems pioneered by Zadeh [26], the mechanism for the evaluation fuzzy rules for international marketing strategy recommendation is proposed below. Let $D$ denote the dimension (e.g. Internet connectivity or market attractiveness) of a strategic matrix and $O$ denote a list of strategic options. For a two-dimensional strategic matrix, the general form of a fuzzy inference rule can be expressed as:

$$R_i: \text{If } (D_1 \text{ is } d_1 \text{ and } D_2 \text{ is } d_2), \text{ then } O_i \text{ is } o_i$$

where $d_1$ and $d_2$ are the linguistic variables corresponding to the specific values (e.g. low, or high) of the two-dimensions in a strategic grid and $o_i$ is the linguistic variable corresponding to a particular strategic option.

2.9 Evaluating the premise of a fuzzy rule for international marketing strategy development

The level of confidence or grade of certainty of the predicates in the premise (condition part) of the rule $R_i$ is given by:

The level of confidence of ‘$D_1$ is $d_1$’ is $l_{i1}$

The level of confidence of ‘$D_2$ is $d_2$’ is $l_{i2}$
The overall grade of certainty in the premise of $R_i$ can be expressed in the form of conjunctive rule:

$$l_i = (l_i_1) \land (l_i_2) = \min (l_i_1, l_i_2)$$

In real-world decision-making situations, to avoid same levels of confidence for different strategic options caused by the above expression, we can apply Bayes theorem to a fuzzy extension [22]. Because the two dimensions of a strategic grid are independent to each other, the overall degree of confidence or certainty factor in the premise of $R_i$ may also be denoted as a joint fuzzy probability:

$$l_i = (l_i_1) \cdot (l_i_2)$$

2.10 Determining the degree of confidence for the consequent of the rule

For fuzzy rule $R_i$, the level of confidence of its consequent will be the same as the overall degree of confidence of its premise. The grade of certainty of the consequent $O_i$ is also equal to $l_i$.

2.11 Strategic recommendations

After execution of the fuzzy rules, the automated system produces one or more strategic alternatives in the form of:

$$O_j \rightarrow o_{j_1}, l_{j_1}$$

$$O_j \rightarrow o_{j_2}, l_{j_2}$$

... ...

$$O_j \rightarrow o_{j_k}, l_{j_k} \quad \text{where} \quad k \leq 4 \quad \text{for a four-cell or nine-cell strategic grid/matrix.}$$

The outputs are not aggregated because we hope to give a list of strategic options/alternatives with corresponding grades of certainty or degrees of confidence. The decision-makers then re-examine the system’s outputs, evaluate which alternative is preferred, and choose a particular one.

2.12 Online databases

Following Silberschatz, Korth and Sudarshan [19]’s mathematical notations and representation for manipulative operations on the relations of databases, our target hybrid system mainly include Web-enabled insertion, deletion and updating data.

Insertion into a Web-based relation, $r$, can be represented in the following relational algebra form:

$$r \leftarrow r \cup E \quad \text{where} \quad \cup \quad \text{is the union of sets and} \quad E \quad \text{is an expression.}$$

A deletion manipulation can be stated as:

$$r \leftarrow r - E$$

Choosing tuples from relation $r$ and modify them when necessary:

$$r \leftarrow \prod_{F_1, F_2, \ldots, F_n} (\sigma_{P}(r)) \cup (r - \sigma_{P}(r))$$

in which each $F_i$ is the $i$th data field of $r$ while $P$ symbolizes the condition that looks up which tuples to alter. Here, $\prod$ is the projection operator on Web-based database server via the Internet.

Selecting tuples and displaying data:

$$\sigma_{\text{selection predicate}}(r) \quad \text{in which} \quad \sigma \quad \text{stands for selection manipulation.}$$

2.13 Roles of managerial intuition, judgement and creativity

The decision makers are required to provide their judgemental inputs to the variables or factors influencing such decisions as “go versus no go”, “how to go” and marketing strategy formulation. They should also apply their intuition, judgement and creativity when making a final choice on the basis of the system-generated “go/no go” advice, entry modes, strategic alternatives, considering various degrees of confidence.

3 Examples

3.1 The AgentsInternational system

The main objective of this system is to aid the following key stages of international marketing decision making discussed in Section 2. Following the mathematical, computational and knowledge automation framework proposed in the previous sections, AgentsInternational [12] was developed by the authors using WIN-Prolog packages and Chimera Agents Toolkits.

The system consists of various multiple agents-enabled components: a chief coordination agent created to manage interaction and communication between the user and connected intelligent agents; a “go vs. no go” agent designed to make “go” or “no go” recommendations by helping review the variables or criteria in different categories and at different hierarchies; a Monte Carlo simulation element programmed to model the market changes
and uncertainties; an entry mode selection agent
developed to advise “how to go” options in line
with the levels of risk and control; and strategy
formulation agents developed to represent diverse
analytical models, domain expertise and guidelines,
and conduct fuzzy reasoning for generating and
advising marketing strategies, global marketing
strategies, Internet strategies, Sun Tze’s Art of War
guidelines, and Porter’s generic competitive
strategies, with calculated grades of certainty.

The multi-agent system was tested by carrying out
evaluation work in October and November of 2008
with three company directors and managers, and
five university course leaders. The participants
were asked to apply the AgentsInternational system
to take international marketing decisions for their
own cases. They then were requested to answer an
evaluation questionnaire that contained both closed
and open-ended questions.

The system was assessed as very effective in terms
of helping understand relevant factors, offering
marketing expertise, handling uncertainty, and
improving the quality of decision making. In
particular, it was described as: “reinforcing my
confidence and consideration in decision making”,
“providing useful ideas/suggestions”, and
“expanding thinking of areas and issues to be
considered.”

3.2 The WebInternational system
On the basis of the computational and knowledge
automation framework, the WebInternational
system [13] was created by the authors to aid the
process of international marketing decision-making.
It was programmed by the authors using PHP
(Hypertext Preprocessor), Ajax, JavaScript and
MySQL and is based on server-side
implementation architecture.

The elements of WebInternational include: a Web-
based database component for saving and
retrieving data entries to different types and
different levels of decision-making variables; a
knowledge base for containing rules and fuzzy
rules which represent the domain expertise for
“go/no go” choices, entry mode decisions,
recommendations and advice on strategies; a Web-
based inference function for reasoning and
producing intelligent outputs; and an user interface
for user-system communications.

The efficiency and effectiveness of
WebInternational were evaluated with five
company directors and managers, and five
university course leaders in 2008, 2009 and 2010,
employing case-based questionnaire survey method.
The system was seen to significantly enhance
decision-making efficiency in terms of easy access,
speed and saving effort. It was also considered as
highly effective in combining analysis with
judgement and creativity, improving quality of
decision-making outcomes, confidence and
satisfaction. In addition, it was evaluated as “more
user friendly and easier to access”, and “reviewing
the issues, providing focus on key questions,
encouraging different thinking”.

4 Conclusions
This is the first study that seeks to establish a
hybrid mathematical, computational and
knowledge automation framework for international
marketing planning. We have developed a
systematic method for modelling and automating
the process of international marketing decision-
making, through combining the powers of online
databases, Monte Carlo simulation, expert systems,
fuzzy rules and approximate reasoning under
uncertainty to complement human intuition and
creativity, and support the key stages of the
planning process. Software examples have been
presented to demonstrate the usefulness of such a
framework.

Our research work and evaluation findings show
that the international marketing planning process
can be described mathematically and supported
effectively. The application of Web-based hybrid
intelligent framework and relevant software
systems to the international marketing cases unveil
that it is possible for using this type of hybrid
solutions to improve the decision-making
procedure and its output.

A priority for further work is to create digital
marketing strategy development functions.
Digital marketing is emerging as a hot topic in
the business world. It is concerned with
advertising products or services via such digital
channels as Web, the Internet, digital TV,
mobile phones, wireless devices, etc. to reach
and attract individual consumers through two-
way personalised interaction [27, 28, 29].
Relevant work is being undertaken to include
Web-based intelligent decision support for
digital marketing/e-marketing strategy
development in the strategy formulation stage
of the international decision-making process.
Mathematical and computational modelling with approximate reasoning and knowledge automation framework is being developed on the basis of S. Li and J. Z. Li’s work [27,28], which will be reported in a separate paper.

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