Impulse Noise Suppression in Blind Channel Equalization

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Abstract: In this paper, robust blind channel equalization is addressed in which the combination of order statistics (OS) and thresholding techniques to suppress impulse noise is considered. We implement nonlinear adaptation with the OS operation based on the Sato error criterion. The threshold is calculated using the variance of the input signal. Computer simulations demonstrate that a significant improvement in both the bit error rate and mean square error is achieved for a communication channel model with a mixture of additive white Gaussian noise and impulse noise.

Key–Words: Blind channel equalization, impulse noise, order statistics, threshold

1 Introduction

Digital communication channels suffer from inter-symbol interference (ISI) due to multipath fading and band limitation. To compensate for ISI, adaptive equalization techniques can be used [1]. Usually, the equalization techniques are training sequence based. The communication capacity, however, degrades with the length of the training sequence. Hence, blind equalization [2] can be a solution to overcome such a situation, in which the equalizer works without any training sequence. Most of the blind equalizers use a linear adaptive algorithm (with a nonlinear error criterion) and assumes that the additive noise is Gaussian. However, the Gaussian assumption of the noise is not always valid in practical communications systems, because sometimes the noise can be impulsive [3]. Linear filters may not be suitable there. A large amplitude of the impulse noise adversely affects both signal restoration and adaptation [4]. This has prompted a great deal of research in nonlinear filtering techniques that are more robust against impulse noise.

Order statistics (OS) filters are known for suppressing impulse noise [5]-[7]. The $L$-filter [5] forms a new input vector by sorting the elements of input vector. This filter works well in image processing, but it cannot perform well in channel equalization due to the loss of temporal information after the ranking operation. The $Ll$-filter [6] provides good signal reconstruction and noise removal by considering both time and rank orders. In [7], the performance of the $Ll$-filter was investigated for the purpose of (training based) channel equalization in the presence of impulse noise, and it was shown that the $Ll$-filter behaves better than the linear filter in terms of mean square error (MSE). The C-filter [8] also utilizes the rank and temporal information in processing inputs with impulse noise, which is also applicable to the channel equalization purpose. In [8], however, the task of channel equalization was not studied. On the other hand, the multirate optimizing order statistics equalizer (MOOSE) has a two-stage architecture, consisting of an order statistical impulse removal pre-filter and a (training based) channel equalizer [9]. For the MOOSE, the use of the impulse removal prefilter does not provide inherently a solution to the equalization problem in impulse noise without the fractionally spaced technique. This is because the impulse removal prefilter is a general filter to suppress impulse noise, which is applicable to a variety of fields containing impulse noise. Like the above, there are several works about the training based equalization techniques in impulse noise. However, blind equalization in impulse noise has not been discussed.

The Sato equalizer [10] is a pioneer work of blind equalization, which is a simple and effective blind equalizer and has been widely used for applications. The performance of the Sato equalizer in impulse noise is not known. In this paper, we investigate the performance of the Sato equalizer in impulse noise and visualize that the Sato equalizer is severely affected in an impulse noise environment. In order to improve the performance of the Sato equalizer in impulse noise, we need to suppress the impulse noise effect in adaptation process. One idea to accomplish this is to use the principle of the C-filter in the adaptation process of blind equalizer. From this point of view, we derive an OS based blind equalizer with the...
Sato error criterion, where the time and rank orders are taken into account.

An adaptive threshold technique has been used in the training based equalizer to handle significant outliers in [11]. This technique is also applicable to the blind equalizer. In this paper, thus, we investigate the effectiveness of the adaptive threshold [11] for blind equalization with the Sato error criterion. The threshold is calculated with the variance of the input signal for every iteration. Unlike the trimming with the C-filter [8], the thresholding calculation is kept isolated from the adaptation process.

The organization of this paper is as follows. The channel model and the blind equalization system are described in Section 2. The proposed OS based blind equalization with the Sato error criterion (OS-Sato) is described in Section 3. In Section 4, the adaptive threshold technique is described and a threshold based blind equalizer is derived. Section 5 shows computer simulation results to validate the proposed blind equalizers. Section 6 concludes the paper with a brief summary.

2 Channel Model and Blind Equalization

We consider a digital communication system where the transmitted sequence is corrupted by both the additive white Gaussian noise (AWGN) and impulse noise. The channel is described by

\[ x(n) = \sum_{i=0}^{L-1} h_i(n)u(n-i) + g(n) + I(n), \]  

where \( h_1, h_2, \ldots, h_{L-1} \) are the channel coefficients, \( u(n) \) is the transmitted sequence, \( g(n) \) is the AWGN, and \( I(n) \) is the impulse noise. The transmitted sequence \( u(n) \) is assumed to be an independent random binary sequence of \( \pm 1 \) with an equal probability. For the mixture noise, the impulse noise \( I(n) \) with the values of +10 or -10 is added to the AWGN. The probability of the generation of the impulse noise \( I(n) \) is defined by \( P_i \).

Figure 1 shows a system model of the blind equalizer considered in this paper, where the Sato algorithm is employed as the adaptation scheme of the transversal filter. The Sato equalizer minimizes a non-convex cost function:

\[ J = E[(\hat{u}(n) - y(n))^2], \]  

where \( \hat{u}(n) \) is an estimate of \( u(n) \), which is given by

\[ \hat{u}(n) = \gamma \text{sgn}[y(n)] \]  

with the sign function \( \text{sgn}[] \) returning the sign of its argument. For the sign function, \( \text{sgn}[y(n)] \) is 1 if \( y(n) \) is positive, 0 if \( y(n) \) is zero and -1 if \( y(n) \) is negative. The dispersion coefficient \( \gamma \) in (3) controls the output scaling, which is computed as

\[ \gamma = \frac{E[u^2(n)]}{E[|u(n)|]} \]  

The Sato equalizer updates its coefficient vector by the least mean squares (LMS) algorithm. The adaptation equations are given by

\[ y(n) = x(n)^T \mathbf{c}(n), \]  

\[ e(n) = \hat{u}(n) - y(n) = \gamma \text{sgn}(y(n)) - y(n), \]  

\[ \mathbf{c}(n+1) = \mathbf{c}(n) + \mu e(n)x(n). \]  

where \( x(n) = [x(n), x(n-1), \ldots, x(n-L+1)]^T \) is the input vector, \( \mathbf{c}(n) = [c(n), c(n-1), \ldots, c(n-L+1)]^T \) is the coefficient vector, and \( e(n) \) is the estimation error. The parameter \( \mu \) controls the convergence of the equalizer.
3 OS Based Blind Equalization

In this section, the proposed OS-Sato equalizer is described where the OS operation is incorporated into the Sato equalizer.

The adaptation of the OS-Sato equalizer is done based on the OS of the input vector, where a combination of temporal and rank orders information is used. The OS-Sato equalizer utilizes the filter output to estimate the transmitted sequence blindly with the Sato error criterion.

The OS-Sato equalizer is implemented at the part of transversal filter with the Sato algorithm in Figure 1, where the coefficient vector \( c(n) \) is nonlinearly adapted based on the order of the input sequence \( x(n) \). Due to the OS operation, the impulse noise appears only on dominated elements of the coefficient vector and hence provides robustness against the impulse noise effect.

The OS-Sato equalizer prepares a coefficient matrix instead of a coefficient vector as follows

\[
C(n + 1) = \\
\begin{bmatrix}
  c_{0,0}(n) & c_{0,1}(n) & \cdots & c_{0,L-1}(n) \\
  c_{1,0}(n) & c_{1,1}(n) & \cdots & c_{1,L-1}(n) \\
  \vdots & \vdots & \ddots & \vdots \\
  c_{L-1,0}(n) & c_{L-1,1}(n) & \cdots & c_{L-1,L-1}(n)
\end{bmatrix}
\]  

(8)

In (8), the elements \( c_{i,j}(n), i, j=0, 1, \cdots, L-1 \) are initialized to zeros at \( n=0 \). For the adaptation of the OS-Sato equalizer, among the \( L \) by \( L \) elements of \( C(n) \), only \( L \) elements are selected using the OS operation of the input vector \( x(n) \) and then updated. Specifically, for \( n^{th} \) iteration, only \( c_{l(j),j}(n), j=0, 1, \cdots, L-1 \) are selected. And a coefficient vector

\[
c_{OS-Sato}(n) = [c_{l(0),0}(n), c_{l(1),1}(n), \cdots, c_{l(L-1),L-1}(n)]^T
\]  

(9)

is formed and updated where \( l(j) \) corresponds to the order when the input vector, \( x(n) = [x(n), x(n-1), \cdots, x(n-L+1)]^T \), is transformed into the order statistic vector as follows

\[
S(n) = [s_0(n), s_1(n), \cdots, s_{L-1}(n)]^T,
\]  

(10)

\[
s_0(n) < s_1(n) < \cdots s_{L-1}(n).
\]  

(11)

The order \( l(j) \) in (9) is determined for \( i, j=0, 1, \cdots, L-1 \) by

\[
l(j) = i \text{ if } s_i(n) = x(n-j).
\]  

(12)

The adaptation equations for the OS-Sato equalizer are given by

\[
y(n) = x(n)^T c_{OS-Sato}(n),
\]  

(13)

\[
e(n) = \hat{u}(n) - y(n) = \gamma \text{sgn}(y(n)) - y(n),
\]  

(14)

\[
c_{OS-Sato}(n + 1) = c_{OS-Sato}(n) + \mu e(n)x(n).
\]  

(15)

The updated coefficient vector \( c_{OS-Sato}(n + 1) \) is then inserted into the coefficient matrix \( C(n + 1) \). Such a process corresponds to one iteration for the OS-Sato equalizer.

4 Threshold Based Blind Equalization

In this section, the threshold based adaptation for blind equalization is discussed. A block diagram

![Figure 2: Threshold based blind equalizer](image-url)
of the use of the adaptive thresholding for the blind equalizer is shown in Figure 2.

To mitigate the impulse noise effect, the threshold based adaptation [11] is applicable to the Sato equalizer. The resulting equalizer is termed the threshold Sato (T-Sato) equalizer in this paper. The coefficient vector of the T-Sato equalizer will not be updated for the full length of the input vector when an outlier is present in the input vector. This operation is equivalent to that the step size is set to zero in the above regions. Otherwise, the equalizer will update the coefficient vector using (5), (6) and (7). The no-adaptation combats the impulse noise effect.

The adaptive thresholding is also applicable to the OS-Sato equalizer, which results in the threshold OS-Sato (T-OS-Sato) equalizer. The coefficient vector of the T-OS-Sato equalizer is selected with the OS operation and updated with threshold based adaptation using the Sato error criterion. If there is no outlier present in the input vector, the selected coefficient vector will be updated using (13), (14) and (15), and the impulse noise effect is suppressed by no-adaptation. In the T-Sato and T-OS-Sato equalizers, commonly the input signal corrupted by only the AWGN is effectively used for the equalizer coefficient adaptation.

The adaptive thresholding technique is described in details here. To implement the thresholding, the squared deviation \( \Gamma(n) \) is calculated for each element of the input vector \( x(n) \) as

\[
\Gamma(n) = |x(n) - \bar{x}(n)|^2, \tag{16}
\]

where \( \bar{x}(n) \) is the average up to \( n^{th} \) input signal where the input signals corrupted by the AWGN only are used. This averaging is implemented in such a way that it does not include potential impulse candidates. In general, observations having high order and low order are considered to be potential impulse candidates. Hence by eliminating the first and last elements, \( s_0(n) \) and \( s_{L-1}(n) \), of the OS vector, the influence of the impulse noise can be reduced. To deal with the AWGN only, by eliminating these two elements, \( \bar{x}(n) \) is adaptively obtained as follows

\[
\bar{x}(n) = \frac{1}{n(L-2)}(x(n-1) + \sum_{i=1}^{L-2} s_i(n)). \tag{17}
\]

The following threshold value is then evaluated to judge whether impulses are included in the input vector as

\[
\Omega(n) = \alpha V(n), \tag{18}
\]

where \( \alpha \) is a scaling parameter and \( V(n) \) is the estimate of the variance of the input signal corrupted by the AWGN only. \( V(n) \) can be calculated adaptively as

\[
V(n) = \frac{1}{n(L-2)} \left[ (V(n-1) + \sum_{i=1}^{L-2} s_i(n)^2) + (\bar{x}(n))^2 \right]. \tag{19}
\]

The variance comparator shown in Figure 2 compares \( \Gamma(n) \) with \( \Omega(n) \). If

\[
\Gamma(n) > \Omega(n), \tag{20}
\]

then the observation is considered to be affected by impulse noise and the coefficient vector is not updated until the \( (n + L - 1) \)th iteration. Otherwise, it will be updated. Rather than trimming of high amplitude impulse noise in [8], the thresholding in (20) just consider whether the observation is impulse or not. When the relationship in (20) is satisfied, it is expected that impulses are included in the input vector and they are then suppressed by no-adaptation.
5 Simulation Results

The OS-Sato, T-Sato and T-OS-Sato equalizers are implemented and compared for simulation experiments. At first, we assume a minimum phase channel whose transfer function is given by

\[ H(z) = 1 + 0.5z^{-1}. \]  

(21)

The additive noise is a white-impulse mixture noise. The impulse noise is generated from a binary sequence with the values of +10 and -10. Three different generation probabilities \( P_i = 0.0001, 0.001, \) and 0.01 are investigated. The impulses are generated individually for each individual trial. The scaling parameter \( \alpha \) in (18) is commonly set to 5.

Figures 3, 4 and 5 show the BER performance on Channel 1 with different \( P_i \). The data number \( N=100000 \) is used. The equalizer length is set to \( L=4 \) and the step size is set to \( \mu=0.04 \) commonly. Individual trials of 100 are implemented for each performance plot. From these figures, it is observed that the OS-Sato equalizer can suppress the impulse noise with lower \( P_i \). On the other hand, the T-Sato equalizer can mitigate the impulse effect well for the entire \( P_i \) and it is effective in higher SNR conditions. Moreover, the T-OS-Sato equalizer outperforms for the entire \( P_i \) and for a wide range of SNR.

The MSE performance on Channel 1 is shown in Figure 6 and an enlarged view of Figure 6 is shown in Figure 7, where \( P_i \) is set to a moderate value of 0.001, the SNR is set to 20 dB and the step size is set to 0.13 commonly. Individual trials of 100 are implemented for each plot. Figure 7 clearly demonstrates that the OS-Sato equalizer can suppress the impulse effect up to a certain level. The T-Sato equalizer provides successfully an MSE level with significant impulse noise suppression. The T-OS-Sato equalizer outperforms the other equalizers and provides a further MSE improvement of about 5 dB than the T-Sato equalizer.

The performance of the OS-Sato, T-Sato and T-OS-Sato equalizers is investigated also on a raised co-
sine channel whose transfer function is given by
\[ H(z) = \sum_{i=1}^{3} \frac{1}{2} (1 + \cos \frac{2\pi (i-2)}{W} ) z^{-i}. \]
(22)
The parameter \( W \) is set to 3. Figures 8, 9 and 10 show the BER performance on Channel 2 with different \( P_i \). The data number \( N=100000 \) is used. The equalizer length is set to \( L=7 \), delay is 5 and the step size is set to \( \mu=0.018 \) commonly. Individual trials of 100 are implemented for each performance plot.

From Figures 7, 8 and 9, it is observed that the OS-Sato equalizer can suppress the impulse noise for the entire \( P_i \) on Channel 2. Besides this, the OS-Sato equalizer is more effective with lower \( P_i \). On the other hand, the T-Sato equalizer provides significant impulse noise suppression with higher \( P_i \). The T-OS-Sato equalizer outperforms for the entire \( P_i \) and specifically in low SNR conditions on Channel 2.

The MSE performance on Channel 2 is shown in Figure 11 and an enlarged view of Figure 11 is shown in Figure 12, where \( P_i \) is set to a moderate value of 0.001, the SNR is set to 20 dB and the step size is set to 0.13 commonly. Individual trials of 100 are implemented for each performance plot. Figure 12 shows that the OS-Sato equalizer can suppress the impulse noise up to a level on Channel 2. Further MSE improvement is achieved with the T-Sato equalizer. The T-OS-Sato equalizer outperforms the other equalizers and provides a further 5 dB MSE improvement than the T-Sato equalizer on Channel 2.

From the computer simulation results, it is observed that the OS based nonlinear adaptation can suppress impulse noise up to a level. The threshold based adaptation can provide a further performance improvement. Especially, the OS based approach along with thresholding technique provides significant impulse noise suppression where the suppression of dominant impulse noise is achieved by the adaptive thresholding and the remaining minor impulse effect is handled with the nonlinear OS operation.
6 Conclusion

This paper provides novel insight of the OS operation for the Sato equalizer. The adaptive thresholding can significantly suppress the impulse noise regardless of its generation probability. Both BER and MSE measurements demonstrate the performance superiority of the OS-Sato, T-Sato and T-OS-Sato equalizers in impulse noise environments. Future work aims at automatic setting of the scaling parameter for the adaptive thresholding.

References: