Increasing the Image Quality of Atomic Force Microscope by Using Improved Double Tapered Micro Cantilever

Ali Sadeghi
Department of Mechanical Engineering
Damavand Branch, Islamic Azad University
Damavand, Tehran, Iran
a_sadeghi@damavandiau.ac.ir

Abstract: The resonant frequency of flexural vibration for a double tapered atomic force microscope (AFM) cantilever has been investigated for increasing the sensitivity to the normal contact stiffness. This sensitivity controls the image contrast, or image quality. The differential Quadrature method (DQM) is employed to solve the nonlinear differential equations of motion. The results show that the resonant frequency decreases when Timoshenko beam parameter or cantilever thickness increases and high order modes are more sensitive to it. The first frequency is sensitive only in the lower range of contact stiffness, but the higher order modes are sensitive to the contact stiffness in a larger range.

Key-Words: Atomic Force Microscope; Sensitivity Analysis; Tapered Timoshenko Beam; AFM Cantilever; Resonant Frequency

1 Introduction
The atomic force microscope (AFM) has been proven by Quate and Gerber [1] in 1986, to be a powerful tool for studying the surface topography of conductors and insulators on a nanometer scale. During scanning an AFM cantilever is vibrated near the contact-resonance frequency. These resonances depend on the tip-sample contact position, shape of the cantilever, height of the tip, and thickness of the beam. There are some types of cantilevers that can be used for atomic force microscope, generally the rectangular and V shaped cantilevers are used for atomic force microscope. Besides providing high-resolution topographic images of sample surfaces, the AFM can be used as a cutting tool and powerful manipulator in micro / nano electromechanical system. A tapered AFM cantilever can be used for this requirement; hence, the investigation about the dynamic behavior of tapered AFM cantilevers is necessary. Turner and Wiehn [2] have studied the sensitivity of the vibration modes of the rectangular AFM cantilevers and derived a closed-form expression. They have assumed that the cantilever is parallel to the sample surface and the tip is exactly located at the end of the cantilever. Chang and Chu [3] have investigated the analytical solution of flexural vibration responses on tapered atomic force microscope. Again they have assumed that the cantilever is parallel to the sample surface. Lee, et al [4] have studied the flexural sensitivity of a V shaped cantilever of an atomic force microscope, taking into account the angle between the cantilever and the surface without considering the tip dimensions, assuming that the tip located at the end of the cantilever.

2 Problem Formulation
A tapered AFM cantilever is shown in figure 1 and a conical tip is located at near the end of the cantilever. The beam tapers linearly from a height \( h_0 \) to \( h_1 \) and then to \( h_2 \) and from a breadth \( b_0 \) to \( b_1 \) and then to \( b_2 \). By neglecting the axial load the governing differential equation of motion based on Timoshenko beam theory for the left and right side of a double tapered cantilever is written as [5]:

\[
\rho A_1 \frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial x} \left[ kG A_1 \left( \frac{\partial w}{\partial x} - \phi \right) \right] = 0
\]  

(1)

\[
\rho I_{y1} \frac{\partial^2 \phi}{\partial t^2} - \frac{\partial}{\partial x} \left[ E I_{y1} \left( \frac{\partial \phi}{\partial x} \right) - kG A_1 \left( \frac{\partial w}{\partial x} - \phi \right) \right] = 0
\]  

(2)

for \( 0 \leq x \leq L_2 \)

\[
\rho A_2 \frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial x} \left[ kG A_2 \left( \frac{\partial u}{\partial x} - \phi \right) \right] = 0
\]  

(3)

\[
\rho I_{y2} \frac{\partial^2 \phi}{\partial t^2} - \frac{\partial}{\partial x} \left[ E I_{y2} \left( \frac{\partial \phi}{\partial x} \right) - kG A_2 \left( \frac{\partial u}{\partial x} - \phi \right) \right] = 0
\]  

(4)

The transverse deflection and corresponding bending angle for the right and left sides of the tip is presented with \( u, \phi \) and \( w, \Phi \) respectively. Considering the double tapered beam model given in figure 1, the following assumptions are made:
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where \( m_{tip} \) is the tip mass and \( k = \frac{5(1 + \nu)}{6 + 5\nu} \) is the shear coefficient. Now we introduce \( k_c = \frac{3EIy_0}{L_1^3} \) as contact stiffness of the cantilever. Also

\[
\lambda^4 = \frac{\rho A_0L^4}{EI_y0} \omega^2, \quad r = \frac{I y_0}{A_0L^2}, \quad \Lambda_n = \frac{k_n}{k_c} \quad \text{(as relative normal contact stiffness)}
\]

3.1 Differential Quadrature Method (DQM)

The differential quadrature method is a numerical method to solve nonlinear differential equations [7]. In this method, the derivative of the function at a given point is approximated by a weighted linear summation of the function values at all discrete grid points in the domain. Consider a function \( F(x) \) in a domain \( D \), the \( n \)th order derivative of the function \( F(x) \) at point \( x_i \) is approximated by [6, 7]

\[
\frac{d^n F(x_i)}{dx^n} = \sum_{j=1}^{N} k_{ij}^{(n)} F(x_j), \quad i = 1, 2, ..., N; \quad n = 1, 2, ..., N - 1
\]

where \( N \) is the number of total discrete grid points in the domain. The weighting coefficient can be written as follows

\[
k_{ij}^{(n)} = \prod_{m=1, m \neq i}^{N} \left( \frac{(x_j - x_m)}{(x_i - x_m)} \right), \quad i, j = 1, 2, ..., N \quad \text{and} \quad j \neq i
\]

\[
k_{ij}^{(r)} = r \left[ k_{ii}^{(r-1)} k_{ij}^{(1)} - k_{ij}^{(r-1)} \right] / x_i - x_j, \quad 2 \leq r \leq N - 1
\]

Usi

\[
k_{ij}^{(m)} = - \sum_{j=1, j \neq i}^{N} k_{ij}^{(m)}, \quad m = 1, 2, ..., N - 1
\]

\[
\prod_{j=1, j \neq i}^{N} (x_i - x_j)
\]

ng a cosine distribution of discrete grid points is a useful and efficient method for selecting the discrete grid points in the domain.

\[
x_i = \frac{1}{2} \left[ 1 - \cos \left( \frac{i - 1}{N - 1} \pi \right) \right], \quad i = 1, 2, ..., N
\]

4 Results and Discussion

In this paper, dynamic behavior of a double tapered AFM cantilever has been investigated using DQM and based on Timoshenko beam theory, taking into account the effects of various parameters like the influence of thickness of the beam, contact position and the normal and lateral contact stiffness. In all cases, we consider
that the tip mass is neglected and $\nu = 0.28$ as Poisson's ratio.

4.1 The effect of the contact position on the resonant frequency
The effects of contact position on the frequency is shown in figure 2 for
\[ \Lambda_s = 30, \frac{H}{L} = 0, \alpha = 0, \frac{k_l}{k_n} = 0.3, D_h = C_h = D_b = C_b = 0.1 \text{ In} \]
this figure the first and second vibrational modes as the most important modes have been investigated. It can be seen that the variation of the first and second modes are fast when the contact position is lower than 0.3, so after this point, the regime of variation will be slower than before. It is clear that the frequency is sensitive to the contact position and the amount of the resonant frequency can be controlled by the contact position. This effect can not be ignored for designing the AFM cantilever.

4.2 The effect of Timoshenko beam parameter on the resonant frequency
The frequency of the first four modes as function of the Timoshenko beam parameter is shown in figure 3 for
\[ \Lambda_s = 30, \frac{k_l}{k_n} = 0.8, \frac{H}{L} = \frac{20}{300}, \alpha = 10, \frac{L_2}{L_1} = 0.03, C_h = D_h = 0.3, C_b = D_b = 0.4 \]
Frequency decreases by increasing Timoshenko beam parameter or cantilever thickness. It can be seen that the high order modes are more sensitive to the Timoshenko beam parameter or cantilever thickness. This figure shows that the influences of shear deformation and rotatory inertia are more important for higher order modes.

4.3 The effect of normal and lateral contact stiffness on the resonant frequency
The influence of the contact stiffness on the frequency is investigated in figure 4 with
\[ \frac{k_l}{k_n} = 0.9 \quad (1), \quad \frac{k_l}{k_n} = 0.3 \quad (2), \quad \frac{H}{L} = \frac{12}{200}, \alpha = 15^\circ, \]
\[ \frac{L_2}{L_1} = 0.03, C_h = D_h = 0.3, C_b = D_b = 0.4 \]
and various Timoshenko beam parameter. Figure 4 shows that all vibrational modes are sensitive to the normal contact stiffness for both states. The second and third vibrational modes are sensitive to the normal contact stiffness in all ranges of contact stiffness, but the first mode is sensitive to the contact stiffness when the contact stiffness is lower than 40 and here, this amount for normal contact stiffness is called the critical contact stiffness, which controls the order of the sensitivity. Increasing the lateral contact stiffness increases the sensitivity to the normal contact stiffness after critical contact stiffness and this effect is higher for the first mode, but when the contact stiffness is lower than critical contact stiffness, the situation is reversed.

References:
Figure 1. Schematic of a double tapered AFM cantilever. The interaction with the surface is modeled by normal and horizontal springs.

Figure 2. Relation between the frequency and contact position for the first and second modes at various Timoshenko beam parameter for $A_1 = 30, \frac{H}{L} = 0, \alpha = 0^\circ, \beta = 0, D_4 = C_4 = D_5 = C_5 = 0.1$
Figure 3. Relation between the frequency and Timoshenko beam parameter for $\Lambda = 30, \frac{k_l}{k_w} = 0.8, \frac{H}{L} = \frac{20}{300}, \alpha = 10^5, \frac{L_2}{l_1} = 0.03, C_k = D_k = 0.3, C_b = D_b = 0.4$.

Figure 4. Relation between the frequency and contact stiffness for the first three modes for a double tapered AFM cantilever at various Timoshenko beam parameter with solid line for $\frac{k_l}{k_w} = 0.1$ (2) and dashed line for $\frac{k_l}{k_w} = 0.9$ (1).