PERIODIC ORBITS AROUND $L_4$ IN THE PHOTOGRavitATIONAL
REstricted Problem WITH OBLATE PRIMARIES

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ABSTRACT: - In this paper, we have studied periodic orbits generated by $L_4$ in the restricted three body problem when both the primaries are radiating oblate bodies. We have determined periodic orbits for different values of $\mu$, $h$, $\sigma_1$, $\sigma_2$, $p_1$ and $p_2$ ($h$ is energy constant, $\mu$ mass ratio of the two primaries, $\sigma_1$ and $\sigma_2$ are oblateness factors and $p_1$ and $p_2$ are radiation parameters). These orbits have been determined by giving displacements along the tangent and normal to the mobile coordinates as defined in our papers (Mittal et. al. [3, 4]). We have also studied the effect of oblateness and radiation pressures on periodic orbits by taking some values of $\mu$, $\sigma_1$, $\sigma_2$, $p_1$ and $p_2$.

Key Words: - Restricted Three-Body Problem; Periodic Orbits; Oblate Body; Radiation Pressure.

1 Introduction
This paper is the extension of our papers, Mittal et. al. [3, 4]. In the first paper, we have studied the periodic orbits of the restricted problem of three bodies around $L_4$ when the smaller primary is an oblate rigid body while in the second paper, we have studied the periodic orbits of the restricted problem of three bodies around $L_4$ when the smaller primary is an oblate body and the bigger primary is a source of radiation.

Hadjidemetriou [1] has discussed the continuation of periodic orbits from the restricted to the general three-body problem. Karimov and Sokolsky [2] have studied the periodic motions generated by Lagrangian solutions of the circular restricted three-body problem by using mobile co-ordinates and by taking displacements along the tangent and the normal. We have given the development of this problem in detail in our earlier papers [3, 4].

In this paper, we have determined the periodic orbits around $L_4$ of the restricted three-body problem. We have taken both the primaries as radiating oblate bodies. We studied the periodic orbits by giving the tangential and normal displacements to the mobile co-ordinates. These orbits have been drawn by taking into consideration the equations of motion along with the variational equations and using the predicted-corrector method. We have also determined the family of periodic orbits by fixing $\mu$ (the mass ratio of the two primaries), $\sigma_1$ and $\sigma_2$ (the oblateness parameters) and $p_1$ and $p_2$ (the radiation parameters) and varying $h$ (the energy constant). We have also studied the effect of oblateness parameters and the radiation pressure on the energy constant ($h$).

Most of the natural and artificial bodies moving in space are not point masses or spherical, rather they are oblate bodies. Many authors have not taken into account the effect of the solar radiation pressure in the motion of the third body where as we have taken both the primaries as radiating oblate bodies. Besides taking both the primaries as oblate bodies and the source of radiation, we have used mobile-coordinates and given the displacement along the normal and the tangent to the orbit which has wider applications in space dynamics. We have drawn the periodic orbits by using the predictor-corrector method, given in detail in our paper [3].

2 Equations of Motion
Following the procedure of our papers [3, 4], we can find the Lagrangian function $L$ and the equations of motion of the infinitesimal mass in the restricted three-body problem when both the primaries are oblate rigid bodies and source of radiation in the synodic co-ordinate system (Fig.1).
These are given by
\[ \frac{d}{dt}(L_x) - \frac{\partial L}{\partial x} = 0, \]
\[ \frac{d}{dt}(L_y) - \frac{\partial L}{\partial y} = 0, \]
where
\[ L = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + n(x\dot{y} - \dot{x}) + \frac{n^2}{2}(x^2 + y^2) + \mu(1-p_1)\left(\frac{1}{r_1} + \frac{\sigma_1}{2r_1^3}\right) + (1-\mu)(1-p_2)\left(\frac{1}{r_2} + \frac{\sigma_2}{2r_2^3}\right) + U, \]
(x, y) = the synodic rectangular dimensionless co-ordinates of the infinitesimal mass. Here we have assumed both the primaries, with masses \( m_1 \) and \( m_2 \) as oblate bodies and source of radiation as well, \( n = \text{mean motion} = 1 + \frac{3}{4}\sigma_1 + \frac{3}{4}\sigma_2, \sigma_1 \) and \( \sigma_2 \) are the oblateness parameters (\( \sigma_1 << 1 \) and \( \sigma_2 << 1 \)).

It may be observed that \( n \) is independent of \( p_1 \) and \( p_2 \) (the radiation parameters),
\[ \begin{align*}
    p_1 &= \text{Radiation pressure due to the bigger primary} \\
    &= \text{Gravitational force due to the bigger primary}, \\
    p_2 &= \text{Radiation pressure due to the smaller primary} \\
    &= \text{Gravitational force due to the smaller primary},
\end{align*} \]
\[ \begin{align*}
    \sigma_1 &= \frac{a_1^2 - c_1^2}{5R^2} \quad \text{and} \quad \sigma_2 = \frac{a_2^2 - c_2^2}{5R^2} \quad \text{here} \quad \sigma_1, \sigma_2, p_1 \quad \text{and} \quad p_2 << 1, \\
    a_1, c_1 &= \text{the lengths of the semi axes of the oblate body of mass} \ m_1, \\
    a_2, c_2 &= \text{the lengths of the semi axes of the oblate body of mass} \ m_2, \\
    R &= \text{the dimensional distance between the primaries}, \\
    r_1^2 &= (x + \mu)^2 + y^2, \\
    r_2^2 &= (x - \mu - 1)^2 + y^2, \\
    \mu &= \frac{m_1}{m_1 + m_2}, \quad (m_1 > m_2), \\
    U &= \text{constant to be so chosen such that} \ h \ \text{will vanish at} \ L_4 \ \text{(libration point) and} \\
    h &= \text{energy constant.}
\end{align*} \]

It may be noted that total force exerted on \( m \) due to \( m = F_1(1-p_1) \) along PA,

\begin{align*}
    \bar{F} &= \text{Solar radiation pressure on} \ m \ \text{due to} \ m \ \text{along} \ AP, \quad (i=1 \ to \ 2).
\end{align*} \]

The coordinates of the libration point \( L_4 \) are
\[ \begin{align*}
    x_{L_4} &= \frac{1}{2}(1 - 2\mu) + \frac{1}{2}(\sigma_1 - \sigma_2)^{1/3}(p_1 - p_2), \\
    y_{L_4} &= \frac{1}{2}\sqrt{3}(1 - \frac{1}{3}(\sigma_1 + \sigma_2)) + \frac{1}{3\sqrt{3}(p_1 + p_2)}
\end{align*} \]
and
\[ U = \frac{1}{2} \left( 3 - \mu + \mu^3 \right) - \frac{\sigma_1}{4} \left( 3 - \mu + 3\mu^3 \right) - \frac{\sigma_2}{4} \left( 5 - 5\mu + 3\mu^3 \right) + \mu p_1 + (1-\mu)p_2. \]

Therefore the equations of motion can also be written as:
\[ \begin{align*}
    \ddot{x} - 2\dot{y} &= W_x, \\
    \ddot{y} + 2\dot{x} &= W_y, \\
    \ddot{x} &= W_x, \\
    \ddot{y} &= W_y, \quad \ldots(2)
\end{align*} \]
where
\[ \begin{align*}
    W &= \frac{n^2}{2}(x^2 + y^2) + \mu(1-p_1)\left(\frac{1}{r_1} + \frac{\sigma_1}{2r_1^3}\right) \\
    &\quad + (1-\mu)(1-p_2)\left(\frac{1}{r_2} + \frac{\sigma_2}{2r_2^3}\right) + U + h.
\end{align*} \]
The Jacobi integral is
\[
C = \frac{1}{2} \left( \dot{x}^2 + \dot{y}^2 \right) - \frac{n^2}{2} \left( x^2 + y^2 \right) - \mu \left( 1 - p_1 \right) \left( \frac{1}{r_1} + \frac{\sigma_1^2}{2r_1^3} \right) - \left( 1 - \mu \right) \left( 1 - p_2 \right) \left( \frac{1}{r_2} + \frac{\sigma_2^2}{2r_2^3} \right) - U \equiv h.
\]

... (3)

3 Normal and Tangent Variables

The generalized coordinates \( Q = (x, y) \) depend upon six parameters \( P = (\mu, \sigma_1, \sigma_2, p_1, p_2, h) \). The corresponding differential equations are given by the system of Equation (2) with Jacobi intergral given by (3). We consider the solutions of the Equations (2) for which \( C \) is zero. If we consider the solutions of the Equations (2) given by (4) for some fixed parameters values \( P = (\mu, \sigma_1, \sigma_2, p_1, p_2, h) \) then there may exist neighbouring solution given by (5) with parameters values \( P^* = (\mu^*, \sigma_1^*, \sigma_2^*, p_1^*, p_2^*, h^*) \) close to \( P \).

We have
\[
x = x(t, \mu, \sigma_1, \sigma_2, p_1, p_2, h),
\]
\[
y = y(t, \mu, \sigma_1, \sigma_2, p_1, p_2, h),
\]
\[
\dot{x} = \dot{x}(t, \mu, \sigma_1, \sigma_2, p_1, p_2, h),
\]
\[
\dot{y} = \dot{y}(t, \mu, \sigma_1, \sigma_2, p_1, p_2, h),
\]
\[
x^* = x(t, \mu, \sigma_1^*, \sigma_2^*, p_1^*, p_2^*, h^*),
\]
\[
y^* = y(t, \mu, \sigma_1^*, \sigma_2^*, p_1^*, p_2^*, h^*),
\]
\[
\dot{x}^* = \dot{x}(t, \mu, \sigma_1^*, \sigma_2^*, p_1^*, p_2^*, h^*),
\]
\[
\dot{y}^* = \dot{y}(t, \mu, \sigma_1^*, \sigma_2^*, p_1^*, p_2^*, h^*).
\]

... (4)

Solution (5) will reduce to the solution (4) as \( P^* \rightarrow P \).

We give the displacements by the formulae:
\[
\Delta P = P^* - P \quad \text{and} \quad \xi = Q^* - Q,
\]
where \( Q^* = (x^*, y^*) \) and \( \xi = (\xi_1, \xi_2) \).

We consider \( \Delta P \) and \( \xi \) are small quantities of the same order. Then following the procedure of our papers [3, 4], we have obtained the following variational equations
\[
\ddot{\xi}_1 = W_{xx} \xi_1 + W_{xy} \xi_2 + 2n \dot{\xi}_2 + W_{x\mu} \Delta \mu + \sum_{i=1} \left( W_{sx} \Delta \sigma_i + W_{s\mu} \Delta p_i \right) + W_{sh} \Delta h,
\]
\[
\ddot{\xi}_2 = W_{xy} \xi_1 + W_{yy} \xi_2 - 2n \dot{\xi}_1 + W_{y\mu} \Delta \mu + \sum_{i=1} \left( W_{sy} \Delta \sigma_i + W_{s\mu} \Delta p_i \right) + W_{sh} \Delta h.
\]

The integral constructed from the Equation (3) and retaining only the first order terms, we get
\[
C = \dot{x}^2 \dot{\xi}_1 + \dot{y}^2 \dot{\xi}_2 - W_{x\mu} \xi_1 - W_{y\mu} \xi_2 - W_{\mu} \Delta \mu + \sum_{i=1} \left( W_{sx} \Delta \sigma_i + W_{sy} \Delta p_i \right) - W_{sh} \Delta h.
\]

... (6)

The modulus of the momentary velocity on the orbit is defined by
\[
V(t) = \sqrt{\dot{x}^2 + \dot{y}^2}.
\]

We assume that (5) is not corresponding to the equilibrium state, i.e., \( V(t) \neq 0 \) and we further assume that \( V(t) \neq 0 \) on the whole orbit. Therefore, \( x \) and \( y \) become the mobile coordinates. We will, now, use this mobile co-ordinate system to draw the periodic orbits by resolving one of the axis along the velocity vector and the other axis along the normal vector.

We define the transition matrix \( S \) as in our papers [3, 4]. So, we have
\[
S = \begin{bmatrix} r \\ s \end{bmatrix}, \quad \dim(r) = 2 \times 1, \quad \dim(s) = 2 \times 1, \quad \text{so that} \quad \dim(S) = 2 \times 2,
\]
i.e.,
\[
S = \begin{bmatrix} \dot{y} & \dot{x} \\ \dot{y} & \dot{x} \end{bmatrix}, \quad r = \begin{bmatrix} 1 \\ \dot{x} \end{bmatrix}, \quad s = \begin{bmatrix} 1 \\ \dot{y} \end{bmatrix}.
\]

We may further define,
\[
r^* = r^T = \begin{bmatrix} \dot{y} & \dot{x} \end{bmatrix}, \quad \text{the first row of} \ S^{-1},
\]
\[
s^* = s^T = \begin{bmatrix} \dot{x} & \dot{y} \end{bmatrix}, \quad \text{the last row of} \ S^{-1}.
\]

In the new coordinate system, we write \( \alpha \), the vector of local coordinates, as follows:
\[
\alpha = \begin{bmatrix} N \\ M \end{bmatrix}, \quad \dim(N) = 1 \times 1 \quad \text{and} \quad \dim(M) = 1 \times 1,
\]
where \( N \) is the displacement along the normal to the orbit and \( M \) is the displacement along the tangent to the orbit. So that \( N \) and \( M \) become the normal and the tangent coordinates.

The relation between the old and the new coordinates are given by:
\[
\xi = S \alpha = \begin{bmatrix} r & s \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix} = rN + sM, \quad \alpha = S^{-1} \xi,
\]
\[
N = r^* \xi, \quad M = s^* \xi.
\]
Substituting these values into the integral (6), we have
\[
\ddot{\xi}_1 = W_{xx} \xi_1 + W_{xy} \xi_2 + 2n \dot{\xi}_2 + W_{x\mu} \Delta \mu + \sum_{i=1} \left( W_{sx} \Delta \sigma_i + W_{s\mu} \Delta p_i \right) + W_{sh} \Delta h.
\]

... (6)
\[
C = \frac{2W}{V^2} \left( MV - MV' \right) + \frac{1}{V} \left( W_y \dot{y} - W_x \dot{x} + \dot{x} y - \dot{y} \right) \nabla
- W_x \Delta \mu - \sum_{i=1}^2 \left( W_{i, \sigma} \Delta \sigma_i + W_{i, \mu} \Delta \mu_i \right) - W_h \Delta h \equiv 0
\]

The Equation (7) gives
\[
M = \frac{MV}{\dot{V}} - \frac{1}{2W} \left( W_y \dot{y} - W_x \dot{x} + \dot{x} y - \dot{y} \right) \nabla
+ \frac{1}{V} \left( W_x \Delta \mu + \sum_{i=1}^2 \left( W_{i, \sigma} \Delta \sigma_i + W_{i, \mu} \Delta \mu_i \right) + W_h \Delta h \right)
\] ...

The Equations of motion (2) for the new coordinates are
\[
\dot{\alpha} = S \left( F_N N + F_N N + F_M \Delta P + \frac{\dot{V}}{V} M s \right),
\]
where
\[
F_{AP} = \left( \begin{array}{c}
F_{1, \mu} \ F_{2, \sigma} \ F_{3, \mu} \ F_{4, \sigma} \ F_{5, \mu} \ F_{6, \sigma}
\end{array} \right),
\]
and the tangent coordinates are given by:
\[
F_{\alpha} = \left( \begin{array}{c}
\frac{W_{x, \alpha}}{W} \left( n y + \frac{\dot{V}}{V} x - \dot{x} \right) \\
\frac{W_{y, \alpha}}{W} \left( - n x + \frac{\dot{V}}{V} \dot{y} - \dot{y} \right)
\end{array} \right),
\]
\[l = 1 \text{ to } 6, \]
\[\alpha_1 = \mu, \alpha_2 = \sigma_1, \alpha_3 = \sigma_2, \alpha_4 = \sigma_1, \alpha_5 = \sigma_2, \alpha_6 = h, \]
\[W_{x, \mu} = 0 \text{ and } W_{y, \mu} = 0 \text{ and } F_N \text{ and } F_N \text{ are the same as in our paper [3].}
\]

The equations of motion in the normal and the tangent coordinates are given by:
\[
\dot{N} = \frac{1}{V^2} \left( W_x \dot{x}^2 - 2 W_y \dot{x} \dot{y} + W_y \dot{y}^2 \right)
+ \frac{1}{W} \left( \dot{W}_x \dot{x} + \dot{W}_y \dot{y} \right) \left( - n V^2 + \dot{x} \dot{y} - \dot{y} \dot{y} \right)
- 2n \left( \dot{x} \dot{y} - \dot{y} \dot{y} \right) - \frac{1}{W} \left( \dot{x} \dot{y} - \dot{y} \dot{y} \right) \left( - n V^2 + \dot{x} \dot{y} - \dot{y} \dot{y} \right)
+ \dot{x}^2 + \dot{y}^2 - \dot{V}^2 \right) \nabla
+ \frac{1}{V} \left( - W_{y, y} + W_{x, x} \right) \left( - n V^2 + \dot{x} \dot{y} - \dot{y} \dot{y} \right)
\]
\[\Delta \mu + W \Delta \sigma_i + W \Delta P_i + W \Delta h \equiv 0 \]
\[
M = \frac{MV}{\dot{V}} - \frac{1}{2W} \left( W_y \dot{y} - W_x \dot{x} + \dot{x} y - \dot{y} \right) \nabla
+ \frac{1}{V} \left( W_x \Delta \mu + \sum_{i=1}^2 \left( W_{i, \sigma} \Delta \sigma_i + W_{i, \mu} \Delta \mu_i \right) + W_h \Delta h \right)
\]
\[\Delta \mu + W \Delta \sigma_i + W \Delta P_i + W \Delta h \equiv 0 \]
\[
\dot{M} = \frac{\dot{V}}{V} M + s \left( F_N N + F_N N + F_M \Delta P \right).
\]

Thus, we have derived the equation in \( \dot{N} \) which possesses the remarkable property: the Equation (7) for the normal coordinate (N) is independent of the tangent coordinate (M). Moreover, instead of the differential equation of the second order (10), we can use the first order differential equation (8). If the investigated motion (4) is periodic, then the matrix \( S(t) \) can be taken as periodic and the Equations (9) and (10) will have the periodic coefficients at \( \Delta P \equiv 0 \).

4 PERIODIC ORBITS

For determining the periodic orbits, we require the equations of motion and the variational equations. These are given as follows:
\[
\dot{x} = 2n y + W_x, \quad \text{...(11)}
\]
\[\dot{y} = -2n x + W_y, \quad \text{...(12)}
\]
\[
\dot{Z}_j = \begin{pmatrix} 0 & 1 \\ r F_N^* & r F_N^* \end{pmatrix} Z_j, \quad Z_j(0) = e_j, \quad (j = 1, 2, ..., 2J)
\]
\[\dot{\mu}_j = \frac{\dot{V}}{V} \mu_j - \frac{V}{2W} g_{z_j} Z_j, \quad \mu_j(0) = 0, \quad \text{...(13)}
\]
\[
\dot{v}_{\Delta \mu} = \begin{pmatrix} 0 & 1 \\ r F_N^* & r F_N^* \end{pmatrix} v_{\Delta \mu}, \quad \dot{v}_{\Delta \mu}(0) = 0, \quad (k = 1, 2, ..., K), \quad K = 6, \quad \text{...(15)}
\]
\[
\dot{\mu}_{\Delta \mu} = \frac{\dot{V}}{V} \mu_{\Delta \mu} - \frac{V}{2W} \left( g_{\mu_{\Delta \mu}} + g_{r_{\mu_{\Delta \mu}}} \right), \quad \mu_{\Delta \mu}(0) = 0, \quad \text{...(16)}
\]

where \( Z(t) \) is the matrix of solutions of a homogeneous system with initial condition \( Z(0) = I_{2J} \) and \( v_{\Delta \mu}(t) = \text{a particular solution of the equations with zero initial conditions, i.e.,} \), \( v_{\Delta \mu}(0) = 0 \). The row-vector \( \mu(t) \) and \( \mu_{\Delta \mu}(t) \) are the solutions of Cauchy problem (14) with (16).

So for finding the new periodic motion it is necessary to integrate the above system from \( t = 0 \) and \( t = T \). It may be observed that the form of the variational equations is the same as in our papers [3, 4] but they differ as now \( K = 6 \), whereas in our earlier papers [3, 4] it was 3 or 4. In the formulae (11) to (16), it may be noted that \( I_{2J} = (e_1, e_2, ..., e_{2J}), \quad Z = (Z_1, Z_2, ..., Z_{2J}) \),
\(\mu = (\mu_1, \mu_2, \ldots, \mu_{2l})\) and the initial conditions \(x(0), y(0), \dot{x}(0), \dot{y}(0)\) are known. The order of this system is twenty-eight.

After solving the above equations of motion (11) and (14) and the variational equations (13)-(16) and applying the predictor-corrector method, we have determined the periodic orbits in various cases, the results of which are given in Table 1.

We have drawn the periodic orbits for the following:

(i) for fixed \(\mu = 0.001, \sigma_1 = 0.001, \sigma_2 = 0.001, p_1 = 0.0\) and \(p_2 = 0.0\) (Fig. 2),

(ii) for fixed \(\mu = 0.001, \sigma_1 = 0.001, \sigma_2 = 0.0, p_1 = 0.0\) and \(p_2 = 0.0001\) (Fig. 3),

(iii) for fixed \(\mu = 0.001, \sigma_1 = 0.001, \sigma_2 = 0.001, p_1 = 0.001\) and \(p_2 = 0.001\) (Fig. 4),

(iv) for fixed \(\mu = 0.001, \sigma_1 = 0.001, \sigma_2 = 0.002, p_1 = 0.01\) and \(p_2 = 0.01\) (Fig. 5),

(v) for fixed \(\mu = 0.001, \sigma_1 = 0.002, \sigma_2 = 0.003, p_1 = 0.01\) and \(p_2 = 0.02\) (Fig. 6).

In each figure, we have drawn 5 periodic orbits corresponding to the different values of \(h\). These values are given on the left hand top of each figure. These orbits have been numbered 1, 2, 3, 4 and 5 corresponding to different values of \(h\) mentioned in each figure. We have investigated the family upto the member which touches the point \(L_4\). Since the final orbit in each case is non-symmetrical, the family can be further continued whereas in the case of Karimov and Sokolsky [2] model, the final orbit is symmetrical and the family terminates at \(L_4\).

The above analysis is summed up in Table 1.
and source of radiation, the difference in the behavior of the values of \( h \) is obvious.

<table>
<thead>
<tr>
<th>( \mu = 0.001 )</th>
<th>Fig. 2</th>
<th>Fig. 3</th>
<th>Fig. 4</th>
<th>Fig. 5</th>
<th>Fig. 6</th>
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<td>( \sigma_1 = 0.001 )</td>
<td>( \sigma_2 = 0.001 )</td>
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<td>( \sigma_1 = 0.001 )</td>
<td>( \sigma_2 = 0.002 )</td>
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</table>

5 Conclusion
Karimov and Sokolsky [2] has studied periodic orbits in the restricted three body problem by giving the displacements along the normal and the tangent to the orbit at the mobile co-ordinates. They have taken both the primaries as point masses while in our paper [3], we have taken one of the primaries as an oblate body. In our paper [4], we have taken bigger primary as an oblate body and smaller primary as source of radiation pressure.

In this paper besides taking both the primaries as oblate bodies, we have also taken both the primaries as source of radiation pressure. In this paper, we have again determined five periodic orbits in a family for fixed values of the mass parameter \( \mu \), the oblateness parameters \( \sigma_1 \) and \( \sigma_2 \) and the radiation parameters \( p_1 \) and \( p_2 \) with varying energy constant \( h \).

We have observed the following effects on the periodic orbits and on the energy constant \( h \) due to oblateness and radiation pressure if we compare it with the results of Karimov and Sokolsky [2] and our paper [3, 4]. The energy constant \( h \) increases in a family for fixed oblate parameters \( \sigma_1, \sigma_2 \) and radiation parameters \( p_1, p_2 \).

1. As we increase the radiation parameters \( p_1, p_2 \), the energy constant \( h \) increases whereas the periodic orbits shrink a little.
2. The periodic orbits go away from the libration point \( L_4 \) as we increase oblate parameters \( \sigma_1, \sigma_2 \) and radiation parameters \( p_1, p_2 \) whereas energy constant \( h \) increase.

We have investigated the family up to the member which touches the point \( L_4 \). Since the final orbit in each case is non-symmetrical, the family can be further continued.

References: