Analysis of Operating Characteristics of Multivalent Systems Using Renewable Energy Sources in Terms of Mathematical Modelling

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Abstract: - The aim of the paper is to show the possibilities of mathematical modelling of thermal processes in the programming environment of Wolfram Mathematica regarding the design and analysis of operating characteristics of multivalent heating systems using renewable energy sources. Each step of a mathematical description of the thermal system consisting of a heat pump, solar system and accumulation system is based on real measurements on a physical model of the combined system. In the paper there are in detail described chosen procedures of mathematical description of the thermal system and the results of modelling are presented for the heat pump and thermal accumulation tank. The paper also presents the measured performance characteristics of combined energy system with heat pump and the comparison with the results of mathematical analysis.

Key-Words: - Heat Accumulator, Heat Pump, Mathematical Modelling, Renewable Energy, Coefficient of Performance, Environment, Multivalent System

1 Introduction
Continuous development of renewable power sources and efforts on decentralized production of electricity and thermal energy, are placing ever more importance to proper design and implementation of such power sources. Regarding the continuous increase in prices of primary energy sources there appear more often multivalent energy systems using heat pumps and solar systems in the Czech Republic, such systems are primarily designed for the thermal energy supply in small objects. In cooperation with industry there were in the laboratories of the Centre of Research and Utilization of Renewable Energy Sources (CRURES) compiled mathematical models for energy systems with heat pumps and heat accumulation. Compiled simulation of heat transfer is then used to assemble a physical model of the combined system, which consists of a heat pump, solar hybrid system and a heat accumulator with suppressed convection. Design of a functional energy system is a complex process. The primary objective of the energy system is to provide all the energy needs of the building. In this field there merge the knowledge of several technical areas – power engineering, civil engineering, technical building equipment, heating etc. For applications with alternative power sources such as solar systems or heat pumps, it is very important to ensure that project design attends to the mutual relations between energy sources and used building materials, which have a considerable influence on the energy needs of building [1].

Systems with heat pump are regarding operating characteristics very specific. The heat pump as an energy source using low-potential heat exchangers can be operated in two basic circuits [2]. The first option is to run heat pump as a monovalent power source, but this option is in terms of energy efficiency less suitable. The reason for this finding is the fact that such system with a heat pump is due to the high installed performance of heat pump working in optimal mode for a very short period of time during the year. Other possible use of heat pumps in the energy system is so called bivalent (multivalent) connection. In this system, the involvement of
the heat pump is supplemented by another (bivalent) source, which may be e.g. solar system, electric power boiler, etc. The thermal performance of heat pump is in the multivalent system designed to cover approximately 60% of total heat loss of the building. The remaining part of the required energy is supplied by an auxiliary energy source, which involvement due to the heat pump can be parallel or alternatively parallel [2], [3].

To obtain information about physical processes during operation of heat pump there is for the mathematical simulation selected connection shown in Figure 1. In this energy scheme heat pump will be intermittently switched (simulation of operational cycles), and will supply hot water to the accumulation tank which will supply the heating system of building.

![Diagram of heat pump system](image1.png)

Fig. 1 Basic application of air-to-water heat pump for simulation of operation characteristics

![Graph of temperature variations](image2.png)

Fig. 2

2 Description of Mathematical Simulations on the Model of Heat Pump

Analysis of operating states is developed in the programming environment of Wolfram Mathematica and as starting data there are used real operational values obtained from measuring on the operand multivalent thermal energy system. Performed mathematical simulations of operation of the assembled power system (Fig. 1) should give an answer to the question, what value will reach COP of heat pump during the entire heating season. The basic argument for such an evaluation is the assumption that the drop in temperature in the accumulation tank must be kept within defined limits.

2.2 Mathematical Description of Heat Pump

For the simulation the operating characteristics of the system (Fig. 1) there are compiled mathematical equations describing the physical processes in the thermal system. Equation (1) - (5) describe these physical processes:

- equation (1) describes the equality of the output temperature at the outlet of the heat pump and heating power \( Q_{HP} \) of the heat pump, “k” to determine the state of the heat pump \((k = 1 – on, k = 0 – off)\),
- equation (2) describes the heat pump – thermal energy on input together with power supplied \( W_{EP} \) from the network is equal to the total output energy,
- equation (3) describes the behaviour of the total power supplied to the heating system, depending on the temperature difference \( T_2 \) and \( T_4 \),
- equation (4) describes the accumulation reservoir – the temperature difference at the input and the output is proportional to the size of the accumulation reservoir \( m_{Ac} \) and to a derivative of output temperature \( T_2 \),
- equation (5) describes the heating system – the overall heating power supplied to the heating system is equal to the average value of input temperature \( T_2 \) and output temperature of water \( T_4 \), room temperature \( T_{room} \) and the heating constant \( k_{heat} \).
A prerequisite for the mathematical modelling is to determine the limiting conditions when the heat pump is in operation and when it is out of operation - definition of heat pump cycles. To solve the assembled differential equations there is used Runge-Kutta numerical method.

So far there has not been mentioned that the basic operational indicator of the heat pump is the coefficient of performance (COP). COP is defined as the ratio of the obtained heat $Q_{TP}$ (heating capacity) and the energy required for heat re-pumping $W_{EP}$. It expresses how many times we get more energy than in the form of drive energy (electricity) we put in. Based on the available values of COP of the existing heat pump there is defined dependence of COP defined on outlet temperature of the heat pump [7], [2].

The next step is to compile a mathematical script for a definition of the characteristic curves of temperature behaviour during the year in the Czech Republic and the subsequent determination of the duration of the heating season. Parameter for the start of the heating season was selected outdoor air temperature $t_{outheat} = 13 \, ^\circ C$. This temperature is also a parameter at which the heat pump starts to operate. The outputs of this mathematical model are defined temperature behaviour during a period of time indicating the duration of the heating season (Fig. 2).

The simulation is performed for the parameter of the required temperature in the accumulation tank $T_2 = 45 \, ^\circ C$. As it is apparent in the figure the heating season begins in September and ends in May. For details of the procedures for the actual simulation of the heat pump there is presented a sample of the compiled script.

The following step is to determine the total heat energy required for heating ($Q_{TP}$), determination of the total amount of electricity needed to drive the heat pump ($W_{EP}$) and the calculation of mean energy coefficient of performance ($COP_{ave}$) of heat pump. The result of the simulation is shown in Figure 3 and Figure 4. The resulting characteristics clearly define the operation characteristics of the heat pump. In the picture below there can be noticed the dependence of COP on the ratio of the produced heat energy to supplied electricity.
2.2 Mathematical Description of Heat Accumulator

The accumulation systems are used to heat accumulation and subsequent distribution of the accumulated heat. Its structure is variable. From simple forms with one heat exchanger to the hybrid tanks combining several heat exchangers. Currently, as a storage medium there is mostly used liquid water, in future there is a perspective counting with the application of substances with a phase change.

In a hybrid system (heat accumulator with solar panels and independent heat pump) there is a storage container with associated media very important part of the combined system. Generally speaking, the accumulation systems, but mainly the heat-transfer agents, are subjected to high requirements in terms of corrosion resistance, high heat capacity, low thermal expansion, acceptable density and related viscosity.

To simulate the operating conditions there has been chosen an accumulation tank in the shape of cuboid. In this case there is water used as an accumulation substance. The actual accumulation tank, which has been subsequently tested in connection with combined heat system, is shown in Figure 5.

From the measured data of water input and output temperatures there are compiled the regression curves of heating model. Using mathematical modelling there has been generated area, which corresponds to the amount of accumulated heat during monitored time unit (Fig. 6). During the simulation there is not re-examined the initial container temperature, hence the result of simulation is the speed of start and time of stabilization of the storage medium temperature. Transition states during the accumulation of thermal energy (red dashed curve) are caused by opening and closing the thermostatic valve. Behaviour of the start speed and time of temperature stabilization in the storage tank are in simulation characterized by polynomial function (equation 6). Similarly there is described temperature behaviour in the storage tank after stabilization (equation 7). It is assumed that the input temperature curve in the system is linear (equation 8). For the determination of the quantity of accumulated heat there has been used integration according to the equation (9). Simulation of heat transfer in the heat storage tank is based on volumetric flow of heat transfer medium through the pump $Q_v = 0.4 \, \text{L.s}^{-1}$, water density $\rho = 985.2 \, \text{kg.m}^{-3}$, specific heat capacity of water $c = 4180 \, \text{J.\text{kg}^{-1}.\text{K}^{-1}}$ and from equation (10) which expresses the amount of accumulated heat.
\[ f[x] = n_1 + n_2^x + \ldots + n_i^x \quad (6) \]
\[ g[x] = n_1 + n_2^x + \ldots + n_i^x \quad (7) \]
\[ h[x] = a \cdot x \quad (8) \]
\[ a = \int_{x_1}^{x_2} f[x] dx + \int_{x_2}^{x} g[x] dx - \int_{x_1}^{x} h[x] dx \quad (9) \]
\[ Q = V \cdot \rho \cdot c \cdot \Delta \Theta \quad (10) \]

3 Operation Characteristics of Combined System Physical Model

The above described mathematical models of the combined heating system has been used as a basis to assemble a physical model, on which are then analyzed various operating conditions, and consequently there are obtained data compared with results of mathematical modelling. Technological scheme of the physical model can be seen in Figure 7. In the Figure 8 there is shown the secondary part of the assembled system. On the assembled model there are measured operating characteristics and these are compared with the results of simulations. The following pictures (9 and 10) show behaviour of the coefficient of performance (COP) and electricity (\(W_{EP}\) – power input) required to drive the heat pump. The behaviour of the COP is presented in response to change of the heat transfer fluid input temperature to heat pump (\(T_{IN}\)) and as a parameter of such dependence there is used the output temperature of the heat pump (\(T_{OUT}\)).

In a similar manner there is also evaluated power dependence of the heat pump (\(W_{EP}\)), which is part of the physical model. \(W_{EP}\) dependence on the output temperature of the heat pump (\(T_{OUT}\)) is shown in Figure 10. As a parameter there is used the air temperature in the vicinity of the heat pump (\(T_{IN-AIR}\)). Even with this dependence there can be seen that the real physical model in terms of operating characteristics behave similarly, as assembled mathematical model. In the context of comparative analysis of the results of mathematical simulation on mathematical model of the combined system (Figure 4) there can be stated that the results of this simulation conform to real operational characteristics of the heat pump. Chosen mathematical description is useful for initial evaluation of the characteristics of the combined system regarding its practical application.

4 Conclusion

The results presented in this paper show that mathematical modelling together with the experience of real operation of heating energy systems leads to optimization of energy system design. The advantage of applied simulations is the ability to verify the operational characteristics of the designed energy system and its subsequent optimization. Currently, these procedures are particularly needed with regard to the increasing installation of energy systems using renewable energy sources and that are currently designed as multivalent.

Analysis of the results of the mathematical modelling of operation of energy systems shows that, one assembled energy system will have different operational characteristics when installed in different locations. By comparing the results obtained from mathematical modelling with real values measured on a physical model of the energy system, there can be stated that the chosen mathematical description of the heat pump is very close to the real parameters. Assembled mathematical model is presently used as a support for designing multivalent energy systems for specific objects.

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