

# Usage of Peak Functions in Heat Load Modeling of District Heating System

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**Abstract:** - This paper describes the usage of peak functions in the heat load modeling of district heating system. Heat load is approximated by the sum of time dependent and temperature dependent components. The temperature dependent component is approximated using sum of two peak functions and temperature dependent component is approximated using generalized logistic function. The model parameters are estimated using Particle Swarm Algorithm.

**Key-Words:** District heating, Heat load, Modeling, Peak function, Particle swarm, Approximation

This paper describes the usage of peak functions in heat load modeling of heat distribution and consumption in municipal heating network. Heat load is approximated by the sum of time dependent and temperature dependent components. There are several approaches for heat load modeling [1][2][3][4][5]. We have proposed new method, where the temperature dependent component is approximated using sum of two peak functions. We use the Hybrid of Gaussian and truncated exponential functions (EGH) [6]. The temperature dependent component is approximated using generalized logistic function. The model parameters are estimated using Particle Swarm (PSO) Algorithm [7]. Method was implemented as algorithm in JAVA language and was evaluated on the data of two combined heat and power plants. This paper presents calculation of delivered heat load and it's approximation model, PSO variant, stopping criterion and related cost function. Finally, the experiment results are presented.

## 1 Introduction

## 2 Methods

### 2.1 Time delays

Time delay of the supply line

$$R = \Delta t \sum_{t-T_{D1}}^t \dot{m}(t) \quad (1)$$

Where

$\Delta t$	s	is the sample period,
$R$	kg	is the mass volume,
$T_{D1}$		is the supply line delay,
$\dot{m}(t)$	$\frac{kg}{s}$	is the mass flow.

Time delay of the return line

$$R = \Delta t \sum_{t-T_{D1}}^t \dot{m}(t) \quad (2)$$

Where

$T_{D1}$		is the supply line delay.
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### 2.2 Delivered Heat load

$$P(t) = \dot{m}(t)c \left( \vartheta_1 \left( t - \frac{T_{D1}}{2} \right) - \vartheta_0 \left( t + \frac{T_{D0}}{2} \right) \right) \quad (4)$$

Where

$P(t)$	W	is the heat load,
$c$	$\frac{J}{kg K}$	is the specific heat capacity,
$\vartheta_1$	$^{\circ}C$	is input temperature,
$\vartheta_0$	$^{\circ}C$	is return temperature.

### 3 Heat load approximation

Heat load is approximated by the sum of time dependent and temperature dependent components.

$$f_P(t, \vartheta_{ex}) = f_{time}(t) + f_{temp}(\vartheta_{ex}) \quad (4)$$

Where

$f_{time}(t)$	is the time dependent, component,
$t_0$	is the time offset,
$\vartheta_{ex}$	$^{\circ}\text{C}$ is the outdoor temperature,
$f_{temp}(\vartheta_{ex})$	is the outdoor temperature, dependent component.

#### 3.1 Temperature dependent component

Temperature dependent component is approximated using generalized logistic function.

$$f_{temp}(\vartheta_{ex}) = A + \frac{K - A}{(1 + Qe^{-B(\vartheta_{ex}-M)})^{\frac{1}{v}}} \quad (4)$$

Where

$A$	is the lower asymptote,
$K$	is the upper asymptote,
$Q$	is the depend on the value $f_{temp}(0)$ ,
$B$	is the growth rate,
$v$	Affects near which asymptote maximum growth occurs,
$M$	is the time of maximum growth if $Q = v$ .

#### 3.2 Time dependent component

The time dependent component is approximated by the sum of two peak functions. The Hybrid of Gaussian and truncated exponential function (EGH) [6] was selected as most the convenient function.

Hybrid of Gaussian and truncated exponential function is defined as

$$d = 2\sigma^2 + \tau(t - t_m) \quad (4)$$

$$f_{EGH}(t) = \begin{cases} H \exp\left(\frac{-(t - t_m)^2}{d}\right), & d > 0 \\ 0, & d \leq 0 \end{cases}$$

Where

$H$	is the peak height,,
$\sigma$	is the standard deviation of the parent Gaussian peak,
$\tau$	is the time constant of the precursor exponential decay,
$k_L$	is the parameter of the speed of the fall of the leading trail,
$t_m$	is the time of the peak.

$f_{time}(t)$  is the sum of two EGH functions:

$$f_{time}(t) = f_{EGH1}(t) + f_{EGH2}(t) \quad (4)$$

### 3.3 Parameter estimation

#### 3.3.1 Cost function

Cost function using EGH functions is defined as

$$\min_{\beta} \sum_t (P(t) - f_P((t - t_0) \bmod 24, \vartheta_{ex}, \beta))^2$$

$$f_P(0, \vartheta_{ex}, \beta) = f_P(24, \vartheta_{ex}, \beta)$$

$$0 < t_m^{EGH1} < t_m^{EGH2} < 24$$

$$0 < H^{EGH1}$$

$$0 < H^{EGH2} \quad (4)$$

Where

$\beta$	is vector of EGH1 and EGH2 functions parameters.
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#### 3.3.4 Particle Swarm Algorithm

The Particle swarm algorithm [7] was chosen as the numeric optimization algorithm suitable for problem without explicit knowledge of the gradient of function to be optimized. We use MaxDistQuick as a stopping criterion as described in [8]. The optimization is stopped if the maximum distance of the major part of particles is below a threshold  $eps$  or the maximum number of iteration is reached.

We use these PSO variant:

$$v'_{i,j} = \omega v_{i,j} + c_1 r_1 (\text{global best}_j - x_{i,j}) + c_2 r_2 (\text{local best}_{i,j} - x_{i,j}) \quad (1)$$

$$x'_{i,j} = x_{i,j} + v'_{i,j} \quad (2)$$

Where

$n$	is the number of particles, $i = 1, \dots, n$ ,
$m$	is the dimension, $j = 1, \dots, m$ ,
$x_{i,j}$	is the particle position,
$x'_{i,j}$	is the updated particle position,
$v_{i,j}$	is the particle velocity,
$\omega$	is the inertia component,
$c_1$	is the social component,
$c_2$	is the cognitive component,
$r_1, r_2, r_3$	are uniform random numbers $\langle 0,1 \rangle$ ,
$\text{global best}_j$	is the best global position,
$\text{local best}_{i,j}$	is the best local particle position.

The number of particles  $n$  we usually set two times more than dimension  $m$ . Inertia component  $\omega$  is set about 0.8, social component  $c_1$  is set about 1.4 and cognitive component  $c_2$  is set about 0.6.

## 4 Results

Method was evaluated on data from two CHP plant in Czech Republic. Figure shows comparison between return temperatures TR of reference day, simulation and measured data. Table 1 shows approximation results as Root Mean Square Error (RMSE), Percentage Average Relative Error (PARE) and Percentage Normalized Root Mean Square Error (PNRMSE).

	Komořany	Dětmarovice
	1.11.2009	1.11.2008
	1.3.2010	1.3.2010
RMSE [KW]	7395.8	3200.5
PARE [%]	7.34	10.18
PNRMSE [%]	5.99	6.98

Table 1: Approximation results

## 4 Conclusion

The new method was evaluated on the data of two combined heat and power plant. The results prove the suitability of this method. Next research will be the classification of daily patterns by means of EGH parameters.

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