Control of the serial production system

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Abstract: - The paper highlights the problem of mathematical modelling of the production process in which production stands are arranged in series. Heuristic algorithms are implemented in order to solve this highly complex mathematical task consisting in minimizing the total order realization time, maximizing the production output and minimizing the loss of unused tool capacity. The given optimization criteria are responsible for controlling the production process. Elements of the order vector are realized subsequently.

Key-Words: - mathematical modelling, optimization, heuristic algorithm, computer simulation, production systems,

1 Introduction

Minimizing production costs has always been one of the most important goals in mathematical modelling of every production activity [4]. However, minimizing production costs may lead to the increase of the total production time [2]. Maximizing the production output is another vital issue [7]. In consequence, this approach increases the loss of the residual capacity [5]. To avoid this and other obstacles emerging from analysing the production environment, there is a need to introduce optimization criteria which will be responsible for channelling the flow of material [3]. E-commerce systems form a standard environment and support of business activities and can be characterized by a wide range of parameters. Parameters describe individual e-commerce system components, are important indicators of e-commerce system status and are of great importance for Business Process Management (BPM). All e-commerce system parameters must be continuously monitored, more precisely measured. Different parameters can be categorized [10]. Mathematical modelling forms the background for preparing control of each production environment [9]. The situation of logistic control for production schedule is usually more complicated in practice and we need to use an integrated approach to solve the problem [1]. New modelling and system design techniques are required for information technologies that can support the enterprise in achieving and sustaining the necessary flexibility [8]. The paper analyses the production system in which charge elements are passed through the production stands arranged in series. The process continues as long as all elements of the order vector are realized. There are heuristic algorithms given. Each heuristic algorithm determines an element of the order vector. This element is later passed through the subsequent production stands. Optimization criteria are responsible for choosing the production strategy. The main goal of each production strategy is to optimize a certain area of production activity.

2 General assumption

Let us assume there is a production system in which the machines M_i , $i=1,...,I$ are arranged in series (see Fig. 1). The vectors of charges W as well as the vector of orders Z are given. It is assumed that each product can be manufactured from any element of the charge vector. We also assume that used charge vector elements are immediately supplemented which means that we treat it as infinite source of energy.

$$\boxed{\mathbf{W} \to \mathbf{M_1} \to \dots \to \mathbf{M_i} \to \dots \to \mathbf{M_I} \to \mathbf{Z}}$$

Fig.1: The scheme of the serial production system

Each machine in the production system carries out a different production operation. These operations are realized subsequently. Let us introduce the vector of orders in the form (1).

$$Z = \left[z_n\right],\ n = 1 \ldots N \tag{1}$$

where: z_n - the n-th production order

The structure of the manufacturing system is shown in the vector form (2).

$$\left[e_1 \quad e_2 \quad \cdots \quad e_i \quad \cdots \quad e_I\right] \tag{2}$$

It is assumed that the manufacturing process takes place in subsequent stands on condition that each stand is used in the manufacturing process

$$\left.\begin{bmatrix} z_1^{k-1} \\ \ldots \\ z_n^{k-1} \\ \ldots \\ z_N^{k-1} \end{bmatrix}\begin{matrix} \rightarrow \\ \ldots \\ \rightarrow \\ \ldots \\ \rightarrow \end{matrix}\right\} \Rightarrow e_1^k \rightarrow \ldots \rightarrow e_i^k \rightarrow \ldots \rightarrow e_I^k \tag{3}$$

The above assumption is made on condition that each i-th production stand is directly employed in the production process. Otherwise, another aspect assuming that not each production stand is employed in realizing the n-th order has to be taken into account. This case makes us propose the following structure in the form (4).

$$\left.\begin{bmatrix} z_1^{k-1} \\ \ldots \\ z_n^{k-1} \\ \ldots \\ z_N^{k-1} \end{bmatrix}\begin{matrix} \rightarrow \\ \ldots \\ \rightarrow \\ \ldots \\ \rightarrow \end{matrix}\right\} \Rightarrow e_{n,1}^k \rightarrow \ldots \rightarrow e_{n,i}^k \rightarrow \ldots \rightarrow e_{n,I}^k \tag{4}$$

The element $e_{n,i}^k$ takes the values in accordance to the form (5).

$$e_{n,i}^k = \begin{cases} 1 & \text{if the } n\text{th product is realized by the } i\text{th production stand at the } k\text{th stage,} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Let $D = \left\lfloor d_{n,i} \right\rfloor$ be the matrix of procedures to be carried out to realize the n-th element of the vector Z.

The elements of this matrix take the values in accordance to the form (6).

$$d_{n,i} = \begin{cases} 1 & \text{if the } n\text{th product is realized by the } i\text{th production stand,} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

The stage k, $k = 1,\ldots,K$ is the moment at which the manufacturing process at any production stand begins. We need to consider that decisions are made at the stage $k-1$, $k = 1,\ldots,K$.
Having assumed the above we can introduce the life matrix of the serial production system for a brand new tool in the form (7)

$$G = \left\lfloor g_{n,i} \right\rfloor,\quad n = 1 \ldots N,\quad i = 1 \ldots I \tag{7}$$

Where $g_{n,i}$ the number of the nth product units which can be realized in the ith production stand before the tool in this stand is completely worn out and requires an immediate replacement (as a result of this usage the stand $i-1$, $i = 2,\ldots,I$ is brought to a standstill).
The life matrix is shown in the matrix form (8).

$$G = \begin{bmatrix} g_{1,1} & \cdots & g_{1,i} & \cdots & g_{1,I} \\ \vdots & & \vdots & & \vdots \\ g_{n,1} & \cdots & g_{n,i} & \cdots & g_{n,I} \\ \vdots & & \vdots & & \vdots \\ g_{N,1} & \cdots & g_{N,i} & \cdots & g_{N,I} \end{bmatrix} \tag{8}$$

The elements of the matrix G take the values according to the form (9).

$$g_{n,i} = \begin{cases} \psi & \text{if the } n\text{th product is realized in the } i\text{th production stand, } \psi = 1 \ldots \Psi, \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

If the number ψ is reached for the given nth element of the vector Z, the tool has to be replaced by a new one.
Let $S^{k-1} = \left\lfloor s_{n,i}^{k-1} \right\rfloor$ be the vector of state of the production system at the stage $k-1$ where $s_{n,i}^{k-1}$ is the number of units of the nth product already realized in the ith stand with the use of the installed tool.
Let $P^{k-1} = \left\lfloor p_{n,i}^{k-1} \right\rfloor$ be the vector of flow capacity of the production system at the stage $k-1$, where variable $p_{n,i}^{k-1}$ is the number of units of the nth product which still can be realized in the ith stand.

On the basis of the above assumptions we can determine the flow capacity of the ith production stand for the nth element of the order vector Z in the form (10).

$$p_{n,i}^{k-1} = g_{n,i} - s_{n,i}^{k-1} \tag{10}$$

These assumptions are made for simplicity reasons. However, in the production process, after completing realizing the nth product in the ith stand, we check if there is a possibility to manufacture another product in the discussed stand. If so, another element from the order vector is realized in the ith stand as long as the flow capacity of the stand enables the whole nth product realization. We assume that production in the ith stand is resumed only then if there is enough flow capacity to manufacture at least one element n. If there is not enough capacity to realize the whole element n, the tool in the ith stand is replaced with a new one and the production process is resumed.

Let us propose manufacturing procedures:

1. Orders are realized in sequence (manufacturing of the order may begin when the previously realized one leaves the serial production system). This procedure guarantees us that no product blocks manufacturing another element from the order vector. Its disadvantage consists in the need of awaiting for completing manufacturing of a certain product before resuming the manufacturing process. This results in not using the available capacity of the whole production system. Moreover, during the production course tools must be replaced.
2. Orders are realized simultaneously which means that if a manufactured element n leaves a production stand, the next one can enter it on condition that there is no need to replace the tool in the given stand responsible for accepting the element of the order vector. Otherwise, it has to await for entering possibility in the preceding production stand.

Let us define the production times of the nth product in the ith production stand in the matrix form (11).

$$T^{pr} = \begin{bmatrix} \tau_{1,1}^{pr} & \dots & \tau_{1,i}^{pr} & \dots & \tau_{1,I}^{pr} \\ \vdots & & \vdots & & \vdots \\ \tau_{n,1}^{pr} & \dots & \tau_{n,i}^{pr} & \dots & \tau_{n,I}^{pr} \\ \vdots & & \vdots & & \vdots \\ \tau_{N,1}^{pr} & \dots & \tau_{N,i}^{pr} & \dots & \tau_{N,I}^{pr} \end{bmatrix} \tag{11}$$

If $\tau_{n,i}^{pr} \le \tau_{n,i+1}^{pr}$, then the ith production stand becomes blocked and the nth product is kept in this stand. If $\tau_{n,i}^{pr} > \tau_{n,i+1}^{pr}$, then the production stand $i+1$ accepts the nth product.

The elements of the matrix T^{pr} take the following values specified in the form (12).

$$\tau_{n,i}^{pr} = \begin{cases} \psi' & \text{the } n\text{th product realization time in the } i\text{th production stand}, \psi' = 1 \dots \Psi', \\ 0 & \text{if the product is not realized in the } i\text{th production stand} \end{cases} \tag{12}$$

Let us define the vector (13) of replacement times for the tool in the ith production stand, where: τ_i^{repl} is the replacement time of the tool in the ith production stand.

$$T^{repl} = \begin{bmatrix} \tau_1^{repl} & \dots & \tau_i^{repl} & \dots & \tau_I^{repl} \end{bmatrix} \tag{13}$$

Let us calculate the total manufacturing time of all elements from vector Z. It is given by (14) for products realized in sequence and in the form (15) for products realized simultaneously.

$$T = \sum_{n=1}^{N} \sum_{i}^{I} \tau_{n,i}^{pr} + \sum_{k=0}^{K} \sum_{i}^{I} y_i^k \tau_i^{repl} \tag{14}$$

$$T = \sum_{n=1}^{N} \sum_{i}^{I} \tau_{n,i}^{pr} + \sum_{k=0}^{K} \sum_{i}^{I} y_i^k \tau_i^{repl} - \Delta\tau^{sim} \tag{15}$$

Variable $\Delta\tau^{sim}$ represents the total time during which two or more elements n, ($n = 1,\dots,N$) are manufactured simultaneously during the whole production process, however in different stands.

The coefficient y_i^k takes the values according to the specification (16).

$$y_i^k = \begin{cases} 1 & \text{if the replacement procedure of the tool in the } i\text{th stand is carried out}, \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

3 Production criteria

The criteria presented hereby are to either maximize the production output or minimize the lost flow capacity of the production stands or minimize the tool replacement time. Let us propose production criteria for the serial production system. Firstly it is possible to define the production maximization criterion in the form (17), where x_n^k is the number

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of units of the nth element of the order vector realized at the kth stage.

$$Q_1 = \sum_{k=1}^{K} q_1^k = \sum_{k=1}^{K}\sum_{n=1}^{N} x_n^k \rightarrow \max \tag{17}$$

Secondly, it is possible to use the lost flow capacity criterion in the form (18), where parameter y_i^k is the lost flow capacity of the ith stand at the kth stage.

$$Q_2 = \sum_{k=1}^{K} q_2^k = \sum_{k=1}^{K}\sum_{i=1}^{I} y_i^k \sum_{j=1}^{J_i} p_{n,i}^k \rightarrow \min \tag{18}$$

Finally, we can use the minimal tool replacement time criterion in the form (19), where variable τ_i^{repl} is the replacement time of the used tool in the ith stand.

$$Q_3 = \sum_{k=1}^{K}\sum_{i=1}^{I} y_i^k \tau_i^{repl} \rightarrow \min \tag{19}$$

4 Equations of state

The state of the discussed serial production system changes in the production course according to scheme (20).

$$S^0 \rightarrow S^1 \rightarrow \ldots \rightarrow \ldots \rightarrow S^k \rightarrow \ldots \rightarrow S^K \tag{20}$$

The state of the production stand in case of production the nth product changes according to the form (21). This state take the values specified in the form (22).

$$s_{n,i}^0 \rightarrow s_{n,i}^1 \rightarrow \ldots \rightarrow s_{n,i}^k \rightarrow \ldots \rightarrow s_{n,i}^K \tag{21}$$

$$s_{n,i}^k = \begin{cases} s_{n,i}^{k=1} & \text{if no } n\text{th product is realized in the } i\text{th stand at the } k\text{-}1 \text{ stage,} \\ s_{n,i}^{k=1} + x_n^{k=1} & \text{otherwise} \end{cases} \tag{22}$$

Let b be the tool to be replaced with a new one, $1 \le b \le I$. The state of the production stand in case of replacement of tools changes in the way according to the specification (23).

$$s_{n,i}^k = \begin{cases} s_{n,i}^{k=1} & \text{if } i \ne b \text{ at the } k\text{-}1 \text{ stage,} \\ 0 & \text{if } i = b \text{ at the } k\text{-}1 \text{ stage,} \end{cases} \tag{23}$$

If tools in all stands are totally worn out, then $S = G$ and need an immediate replacement to enable the production process.
The order vector Z changes after every production decision according to scheme (24).

$$Z^0 \rightarrow Z^1 \rightarrow \ldots \rightarrow Z^k \rightarrow \ldots \rightarrow Z^K \tag{24}$$

The order vector is modified after every decision about production accordance with specification (25).

$$z_n^k = \begin{cases} z_n^{k-1} - x_n^k & \text{if the number of units } x_n^k \text{ of the } n\text{th order is realized at the } k \text{ stage,} \\ z_n^{k-1} & \text{otherwise} \end{cases} \tag{25}$$

5 Heuristic

In order to control the logistic process we need to implement heuristics which determine elements from the vector Z for the production process. The following control algorithms are put forward.

5.1 The algorithm of the maximal order

This algorithm chooses the biggest order vector element characterized by the biggest coefficient γ_n^{k-1} in the state S^{k-1}.
To produce element a, the condition in the form (26) must be met, where $\gamma_n^{k-1} = z_n^{k-1}$.

$$(q^k = a) \Leftrightarrow \left[\gamma_a^{k-1} = \max_{1 \le n \le N} \gamma_n^{k-1} \right] \tag{26}$$

The above approach is justified by avoiding excessive bringing the production line to a standstill in order to change an element to be manufactured. If in state S^{k-1} only minimal orders were chosen, in consequence the number of orders might be reduced. Such control is favorable because the serial production line will not be blocked and must be stopped only in order to replace the tools in certain stands (on condition that the replacement process disturbs the flow of the material).

5.2 The algorithm of the minimal order

This algorithm chooses the smallest order vector element characterized by the smallest coefficient γ_n^{k-1} in the state S^{k-1}. To produce element a, the condition (27) must be met, where $\gamma_n^{k-1} = z_n^{k-1}$.

$$(q^k = a) \Leftrightarrow \left[\gamma_a^{k-1} = \min_{1 \le n \le N} \gamma_n^{k-1} \right] \tag{27}$$

The above approach is justified by the need to eliminate the elements of the order vector Z which could be sent to the customer just after the *n*th product leaves the production line on condition that the customer sets such a requirement.

6 Conclusion

The problem presented in the paper discusses the issue of the serial production system. The system delivers ready products corresponding with the elements of the order vector. The main goal is to fulfil the task set by the criteria introduced in the paper. There is also a possibility to implement a two- or three-criterion model. Such models may lead to delivering a solution which would satisfy criteria included in the discussed model only partly as there should be bounds added. Heuristic algorithms proposed in the paper enable the operator to choose the satisfactory production sequence on the basis of which a certain element of the order vector is determined to be realized. The use of one specified algorithm does not mean that we will achieve the result satisfying the given criterion. We should try to verify another algorithm and decide which one minimizes the order realization time, the loss of residual capacity, and the total replacement time or satisfies a hypothetical customer's demand not specified hereby. Another idea already used in previous works is to simulate the combination of heuristic algorithms [6]. By means of this method, we can combine two or more algorithms. It also seems reasonable to draw products for manufacturing. To achieve the satisfactory result a big number of simulations must be carried out. In conclusion, it must be admitted that a simulator imitating real environment should be built to continue this work. Simulation experiments carried out in the synthetic environment may deliver an answer which heuristic approach is the expected one for a certain criterion

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