Uncertainty Modelling in Time-Delay Systems: Parametric vs. Unstructured Approach

RADEK MATUŠŮ, ROMAN PROKOP, LIBOR PEKAŘ
Department of Automation and Control Engineering
Faculty of Applied Informatics
Tomas Bata University in Zlín
nám. T. G. Masaryka 5555, 760 01 Zlín
CZECH REPUBLIC
{rmatusu; prokop; pekar}@fai.utb.cz

Abstract: - This contribution is focused on comparison of two basic approaches to uncertainty modelling and corresponding robust stability analyses for a system with uncertain time-delay. A paper bleaching process, described both as a system with parametric uncertainty and in the form of unstructured multiplicative uncertainty model, is considered as a testing plant. The robust stability of closed control loop with selected controller and appropriate uncertain model of controlled plant is verified and obtained results are compared.

Key-Words: - Uncertainty Modelling, Time-Delay Systems, Parametric Uncertainty, Unstructured Uncertainty, Robust Stability Analysis

1 Introduction

The whole classical control theory as well as many contemporary methods use some form of mathematical model of a controlled system for a controller design. The crucial problem, however, is that assumed ideal mathematical model, due to many reasons, practically never exactly matches the real behaviour of the plant. One of possible approaches how to overcome this discrepancy grounds in utilization of an uncertain model and subsequent robust controller design.

There are two principal ways of uncertainty modelling in the literature [1], [2], [3] – parametric or unstructured approach. Both of them have their advantages and drawbacks. Consequently, each of approaches is more suitable for different situations. This contribution presents the comparison of uncertainty modelling and subsequent robust stability analyses for a fist order system with uncertain time-delay term. The tests are performed by means of the simulation examples with a paper bleaching process [4].

2 Uncertainty Modelling

The introductory part has already foreshadowed that difference between real process and its mathematical model is the fundamental and omnipresent control problem. For example, the parameters of controlled plant need not to be known exactly or they can be even time-variant (however, only "slowly" from the robust control point of view). Then, nonlinearity in controlled system can be neglected and consequently discrepancy could originate in linear approximation in given operational point. Or a simplified model can be intentionally used instead of originally very complex system (e.g. caused by neglecting the fast dynamic effects due to system order reduction, assumption of a distributed-parameter system as a lumped-parameter one, or time-delay neglect) because of easier calculations.

In robust control, respecting these factors in mathematical description leads to the use of uncertain model. In other words, not only one nominal model, but the whole family of models given by some neighborhood of the nominal one is defined. The "size" of this neighborhood can be described in two main ways – as a parametric or unstructured uncertainty. The combination of both main methods is also possible. Then one speaks about mixed uncertainty.

The real parametric uncertainty is utilized if the structure of system is known but its actual physical parameters are not. On the contrary, unstructured uncertainty does not require even knowledge of structure (order) of model. Parametric uncertainty is defined through intervals which the imprecisely known parameters lie within. The unstructured uncertainty description is based on restriction of the area of possible appearance of frequency characteristics.

However, the terminology used in this paper is not the one and only possible. The scientific literature presents also different nomenclatures, e.g. structured (=parametric) vs. nonparametric (=unstructured) or possibly parametric vs. dynamic, which are subsequently divided into unstructured and structured (with different meaning than in the previous case). Thus one has to be careful about the terminology of each author. This paper

ISBN: 978-1-61804-004-6

adopts probably the most frequent version, i.e. parametric vs. unstructured uncertainty [5].

It is known that robustness means preservation of a selected property of control loop not only for one nominal system but also for the whole family of systems given by the uncertain model and appropriate boundary. Generally, the most important control problem consists in ensuring the stability and so, quite naturally, one of the typical robust control problems is robust stability analysis. It investigates if the closed-loop stability is assured for all possible systems from the family. If this if fulfilled, then the system is called as robustly stable. Furthermore, the aim of robust synthesis is to find a controller which guarantee robustness (robust stability, robust performance, etc.) of the closed control loop. This contribution is focused only on analysis of robust stability.

3 Robust Stability Analysis for a Paper Bleaching Process

A bleaching process in a paper-making machine is adopted from the work [4] where it is modelled as a first order plant with uncertain time-delay. More specifically, it describes the dependency of lignin amount on chlorine flow-rate. The known part of time-delay results from sensor placement while the unknown one originates in neglect of fast dynamics of the chemical process. Thus, the nominal model of the controlled process is defined as:

$$G_0(s) = \frac{1}{2s+1} e^{-0.1s} \tag{1}$$

and the class of uncertain models can be described by:

$$G(s) = \left\{ \frac{1}{2s+1} e^{-(0.1+\Theta)s} : 0 \le \Theta \le 0.9 \right\} \tag{2}$$

The task is to verify if this system is robustly stabilized by the following PI controller:

$$C(s) = \frac{1.5s + 0.5}{s} \tag{3}$$

by means of parametric and unstructured uncertainty modelling approach.

3.1 Parametric Uncertainty Approach

First, the controlled plant is assumed as a transfer function with single uncertain parameter (time-delay term):

$$G(s, \tilde{\Theta}) = \frac{1}{2s+1} e^{-\tilde{\Theta}s} = \frac{1}{2s+1} e^{-[0.1,\,1]s} \tag{4}$$

The robust stability of systems with parametric uncertainty can be tested for example using the graphical method, which relies on depiction of the closed-loop characteristic (quasi)polynomial value sets and application of the zero exclusion condition. An interested reader can find a lot of information about robustness of systems with parametric uncertainty and related topics in [1], [2], [3], [6].

The closed-loop characteristic polynomial of the circuit with plant (4) and controller (3) can be simply expressed as:

$$\begin{aligned} & p_{CL}(s, \tilde{\Theta}) = (2s+1)s + e^{-\tilde{\Theta}s}(1.5s + 0.5); \\ & \tilde{\Theta} \in \langle 0.1, 1 \rangle \end{aligned} \tag{5}$$

Roughly speaking, the value set for one fixed frequency ω can be obtained by substitution of s for $j\omega$ in the family (5) and letting the time-delay term $\tilde{\Theta}$ range over the prescribed set. The fig. 1 shows such value sets plotted in complex plane for several non-negative frequencies starting from 0 to 2.4 with step 0.05.

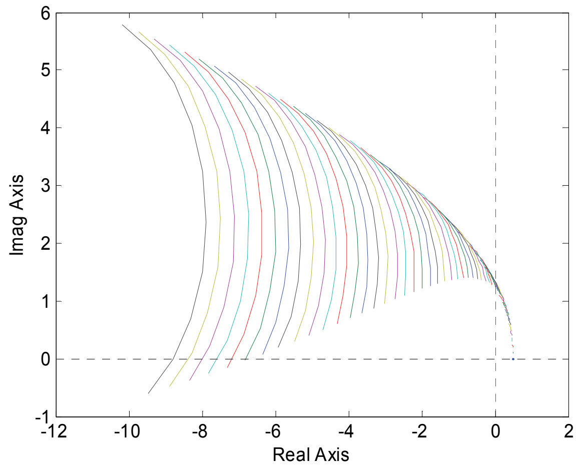

Fig. 1: The value sets of uncertain quasipolynomial (5)

Due to the fact that the family has a stable member and the origin of the complex plane is excluded from the value sets, one can conclude that the quasipolynomial (5) and thus also the whole control system with controller (3) and time-delay plant (4) is robustly stable.

3.2 Unstructured Uncertainty Approach

In the second case, the system is considered to be described as an unstructured multiplicative uncertainty model, generally:

$$G(s) = \left[1 + W_M(s)\Delta_M(s)\right] G_0(s) \tag{6}$$

where $G(s)$ represents a perturbed model, $G_0(s)$ stands for a nominal model, $W_M(s)$ is a (stable) weight

ISBN: 978-1-61804-004-6

function representing uncertainty dynamics, i.e. the distribution of the maximum amplitude of the uncertainty over the frequency, and $\Delta_M(s)$ means the uncertainty (uncertain information about actual magnitude and phase of perturbation), which can be an arbitrary stable function fulfilling the inequality:

$$\left\|\Delta_M(s)\right\|_\infty \le 1 \quad \Rightarrow \quad \left|\Delta_M(j\omega)\right| \le 1 \quad \forall\omega \tag{7}$$

For multiplicative uncertainty, it holds true:

$$\left|\frac{G(j\omega)}{G_0(j\omega)} - 1\right| \le \left|W_M(j\omega)\right| \quad \forall\omega \tag{8}$$

Moreover, many theoretical tools for analysis and synthesis require that $G(s)$ and $G_0(s)$ have to have the same amount of poles for all $\Delta_M(s)$.

Under assumption of multiplicative uncertainty, the closed-loop system is robustly stable if and only if:

$$\left\|W_M(s)T_0(s)\right\|_\infty < 1 \tag{9}$$

where $T_0(s)$ is a complementary sensitivity function:

$$T_0(s) = \frac{C(s)G_0(s)}{1 + C(s)G_0(s)} \tag{10}$$

The condition (9) practically says that the envelope of Nyquist diagrams with radius $\left|W_M(j\omega)L_0(j\omega)\right|$ and centre $L_0(j\omega)$ must not include the critical point [-1, 0j]. The term $L_0(j\omega)$ represents open-loop frequency transfer function:

$$L_0(j\omega) = G_0(j\omega)C(j\omega) \tag{11}$$

The normalized perturbation of the plant (2) can be obtained using (8):

$$\left|e^{-\Theta j\omega} - 1\right| \le \left|W_M(j\omega)\right| \quad \forall\omega \tag{12}$$

The object of interest is just the amplitude of perturbation. The phase is not restricted. So, the suitable weight function is chosen in [4] as:

$$W_M(s) = \frac{2.1s}{s+1} \tag{13}$$

The fig. 2 shows the comparison of Bode plots of the absolute values of the weight (12) and normalized perturbations for three values of time-delay ($\Theta = 0.9$, $\Theta = 0.1$, and $\Theta = 0.01$). It can be seen how $\left|W_M(j\omega)\right|$ approximates even the worst case of $\Theta = 0.9$ from the upper side.

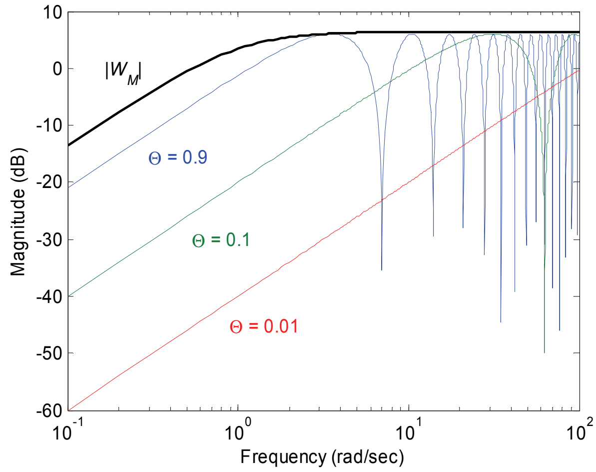

Fig. 2: Bode plots – envelope of the uncertainty

Thus, remind that the aim is to analyze the robust stability of closed-loop with the family of systems:

$$\begin{aligned} &G(s) = \left[1 + W_M(s)\Delta_M(s)\right]G_0(s) \\ &\left\|\Delta(s)\right\|_\infty \le 1 \\ &G_0(s) = \frac{1}{2s+1}e^{-0.1s} \\ &W_M(s) = \frac{2.1s}{s+1} \end{aligned} \tag{14}$$

and with the controller (3).

The envelope of uncertainty given by circles with radius $\left|W_M(j\omega)L_0(j\omega)\right|$ around Nyquist diagram of $L_0(j\omega)$ (red curve) is plotted in fig. 2 (with frequency step 0.1). It shows that the point [-1, 0j] is excluded from the envelope which means robust stabilization of the closed-loop with controller (3) and family of systems (13), i.e. also the plant (2), which is the same result as for the previously used model with parametric uncertainty.

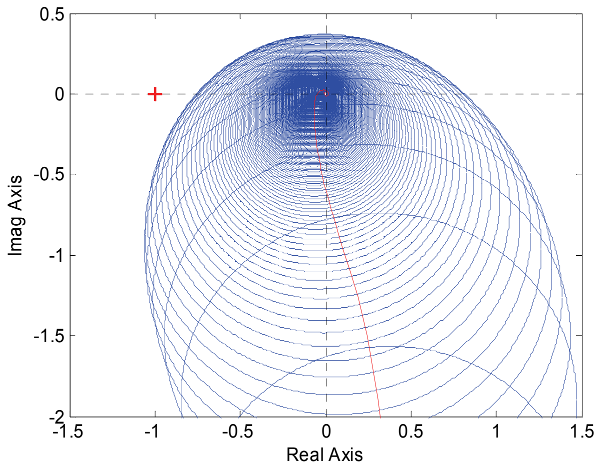

Fig. 3: Graphical interpretation of stability condition

4 Conclusion

The paper has dealt with comparison of parametric and unstructured approaches to uncertainty modelling and robust stability analysis. The parametric way of description seems to be more natural and comprehensive and robust stability analysis is also relatively straightforward under parametric uncertainty scenario. On the other hand, application of unstructured uncertainty model allows taking advantage of wider range of more sophisticated controller design methods.

Acknowledgements:

The work was supported by the Ministry of Education, Youth and Sports of the Czech Republic under the Research Plan No. MSM 7088352102 and by the European Regional Development Fund under the project CEBIA-Tech No. CZ.1.05/2.1.00/03.0089. This assistance is very gratefully acknowledged.

References:

[1] J. Ackermann, *et al.*, *Robust control – systems with uncertain physical parameters*, Springer-Verlag London, Great Britain, 1993.

[2] B. R. Barmish, *New Tools for Robustness of Linear Systems*, Macmillan, New York, USA, 1994.

[3] S. P. Bhattacharyya, H. Chapellat, L. H. Keel, *Robust control: The parametric approach*, Prentice Hall, Englewood Cliffs, New Jersey, USA, 1995.

[4] V. Kučera, Robustní regulátory (Robust Controllers), *Automa*, Vol. 7, No. 6, 2001, pp. 43-45. (In Czech).

[5] P. Dostál, R. Matušů, *Stavová a algebraická teorie řízení* (*State-Space and Algebraic Control Theory*), Faculty of Applied Informatics, Tomas Bata University in Zlín, Czech Republic, 2010. (In Czech).

[6] M. Šebek, *Robustní řízení*, PDF slides for course “Robust Control”, ČVUT Prague, Available from: http://dce.felk.cvut.cz/ror/prednasky_sebek.html. (In Czech).