Solving of non-stationary heat transfer in a plane plate

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Abstract In this paper we present software application destined for study of heat transfer problems that is a part of education of subject Process engineering taught at the Tomas Bata University in Zlin. The application we use as a teaching aid for calculation and visualization of temperature distribution in the plane plate body during its heating or cooling. Calculation accuracy of our application enables it to be used for real engineering computing.

Key-Words: - Non-stationary Heat Transfer, Maple, Temperature Field, Software Application

1 Introduction

Non-stationary heat transfer problem is a part of study subject Process engineering which is taught at Faculty of Technology and at Faculty of Applied Informatics of the Tomas Bata University in Zlín. But study and calculation relating to these problems are in many cases relatively complicated and also time-consuming. Moreover, using mathematical software is often required to obtain sufficiently accurate calculations. Therefore we make software applications which help students to study and solve selected technological problems.

In this paper we present software application that is designed for non-stationary heat conduction in a solid plane plate body. We made this application by using computer algebraic system Maple. The application can calculate and visualize the temperature field in the homogeneous plane plate during its heating or cooling affected by free flowing surrounding fluid as often occurred phenomena in the processing industry.

2 Theory

2.1 Symmetric temperature field

We will solve the problem of non-stationary heat conduction in the solid plate under the assumptions:

- The plate of initial temperature t_p is suddenly exposed double-sided heat action of ambient temperature t_o.
- heat conduction in the plate is affected by heat effect of surrounding fluid,
- the plate is made from homogeneous material,
- length or high of plate is much greater than its thickness.

Geometry sketch of the mentioned problem is in Fig. 1.One-dimensional heat conduction can be described by relations (1) – (6)

$$\frac{\partial t(x,\tau)}{\partial \tau} = a\frac{\partial^2 t}{\partial x^2}(x,\tau) \quad 0 < x < b;\ \tau > 0 \qquad (1)$$

$$-\lambda\frac{\partial t}{\partial x}(b,\tau) = \alpha(t - t_o) \qquad (2)$$

$$t(x,0) = t_p \qquad (3)$$

$$t(x,\tau\to\infty) = t_o \qquad (4)$$

$$t(b,\tau) = t_o \qquad (5)$$

$$\frac{\partial t(0,\tau)}{\partial x} = 0 \qquad (6)$$

Equation (1) describes a non-stationary temperature

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field in the plate. Heat balance equation (2) represents heat transfer between plane and surrounding fluid (2). Equation (3) is initial conditions. Equations (4) and (5) represent boundary conditions. Equation (6) is condition of symmetry.

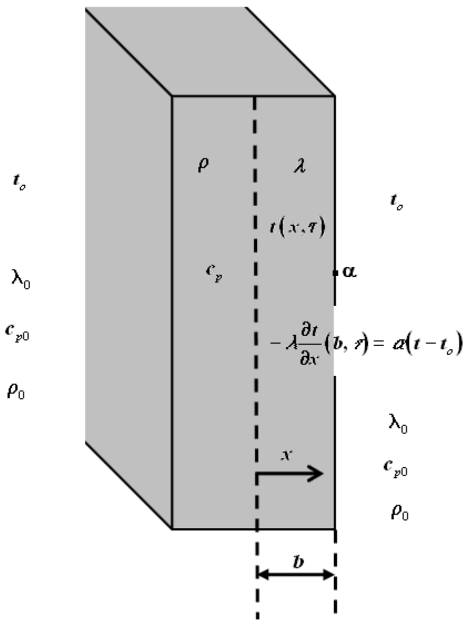

Fig. 1 Geometry sketch of the non-stationary symmetric heat conduction in a plane plate

By use of Laplace transformation we obtained analytical solution of the formulated model. Temperature field in a wall during heating (cooling) $t(x, \tau)$ is given by equation (7):

$$t(x,\tau)=\left(\left(t_p-t_0\right)2\sum_{n=1}^{\infty}\frac{\sin(q_n)}{q_n+\sin(q_n)\cos(q_n)}\cos\left(\frac{x}{b}q_n\right)e^{-\frac{a\tau}{b^2}q_n^2}\right)+t_0 \tag{7}$$

where a is thermal diffusivity of the heated (cooled) body

$$a=\frac{\lambda}{\rho\cdot c_p} \tag{8}$$

q_n are roots of the following equation

$$\cot(q)=\frac{q}{Bi} \tag{9}$$

where symbol Bi represents Biot number

$$Bi=\frac{\alpha\cdot b}{\lambda} \tag{10}$$

Value of the Biot number (10) strongly depends on a process of heat transfer between surface of the plate and surrounding liquid. In this paper we study two cases of the surrounding liquid convection - free convection and turbulent flow around the wall of plate.

In the case of free convection Grashof number Gr, Prandtl number Pr and Nusselt number Nu at average temperature t_m have to be computed for the heat transfer coefficient determination:

$$t_m=\frac{t_p+t_o}{2} \tag{11}$$

Grashof number Gr:

$$Gr=\frac{g\cdot d^3\cdot\beta_o\cdot\Delta t}{\nu_o^2} \tag{12}$$

Prandtl number Pr:

$$Pr=\frac{\rho_o\cdot\nu_o\cdot c_{po}}{\lambda_o} \tag{13}$$

Nusselt number Nu:

$$Nu=C\cdot(Gr\cdot Pr)^K \tag{14}$$

where values of parameters C and K depend on value of product $Gr\cdot Pr$ as you can see in following Table 1.

Table 1 Values of coefficients C and K of equation (14) for Nusselt number calculation

$Gr\cdot Pr$	C	K
$< 10^{-2}$	0.500	0.000
$[10^{-2}; 5\cdot10^{2}]$	1.180	0.125
$[5\cdot10^{2}; 2\cdot10^{7}]$	0.540	0.250
$[2\cdot10^{7}; 5\cdot10^{13}]$	0.135	0.333

The heat transfer coefficient can be then computed according to equation:

$$\alpha=\frac{Nu\cdot\lambda_o}{d} \tag{15}$$

In the case of turbulent flow of surrounding liquid around the wall of plate we solved Nusselt number Nu at average temperature t_m according to following relation (16):

$$Nu = 0.023 Re^{0.8} Pr^{0.4} \qquad (16)$$

where symbol Re is Reynolds number:

$$Re = \frac{v \cdot d}{\nu_o} \qquad (17)$$

3 The software application for solving

We made our application in the computer algebraic system Maple. By using the Maple programming language, we created a user interface in the Maplet form, which is a special graphical user interface launched from a Maple session. It allows a user to combine packages and procedures with interactive windows and dialogs [6]. This form is also suitable for e-learning and other web applications.

Our application contains several windows with the specific functions, textbox regions, and other visual interfaces, which gives a user point-and-click access to the power of Maple.

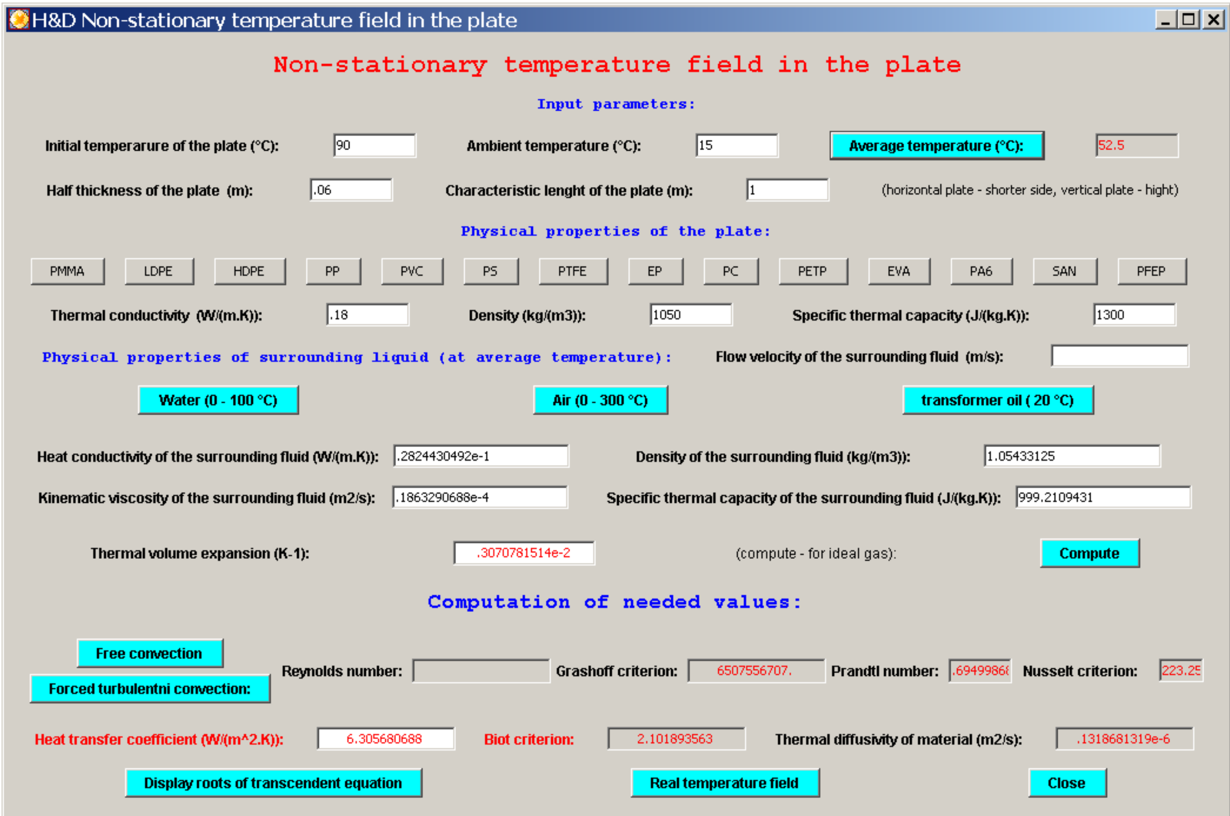

Fig. 2 The initial window of software application

The calculation proceeds step by step to students obtain notion of its principle. Presentation of our application you can see in following figures 2 – 4. After the program starts, the initial window will open (Fig. 2). There, the user inserts required conditions of the heating or cooling process. After them, the needed parameters, heat-transfer coefficient, temperature conductivity of solid material and one hundred roots of transcendent equation (11) are computed and displayed (Fig. 3). Computing of real temperature field is shown on the figure 4.

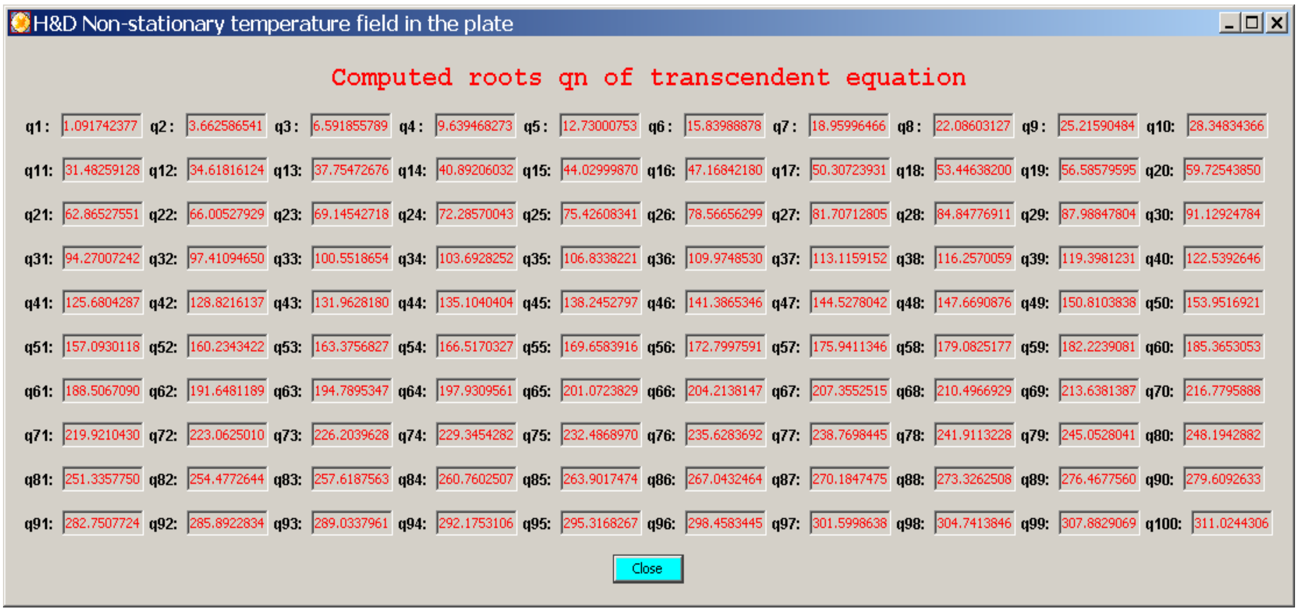

Fig. 2 The window with computed one hundred roots of transcendent equation

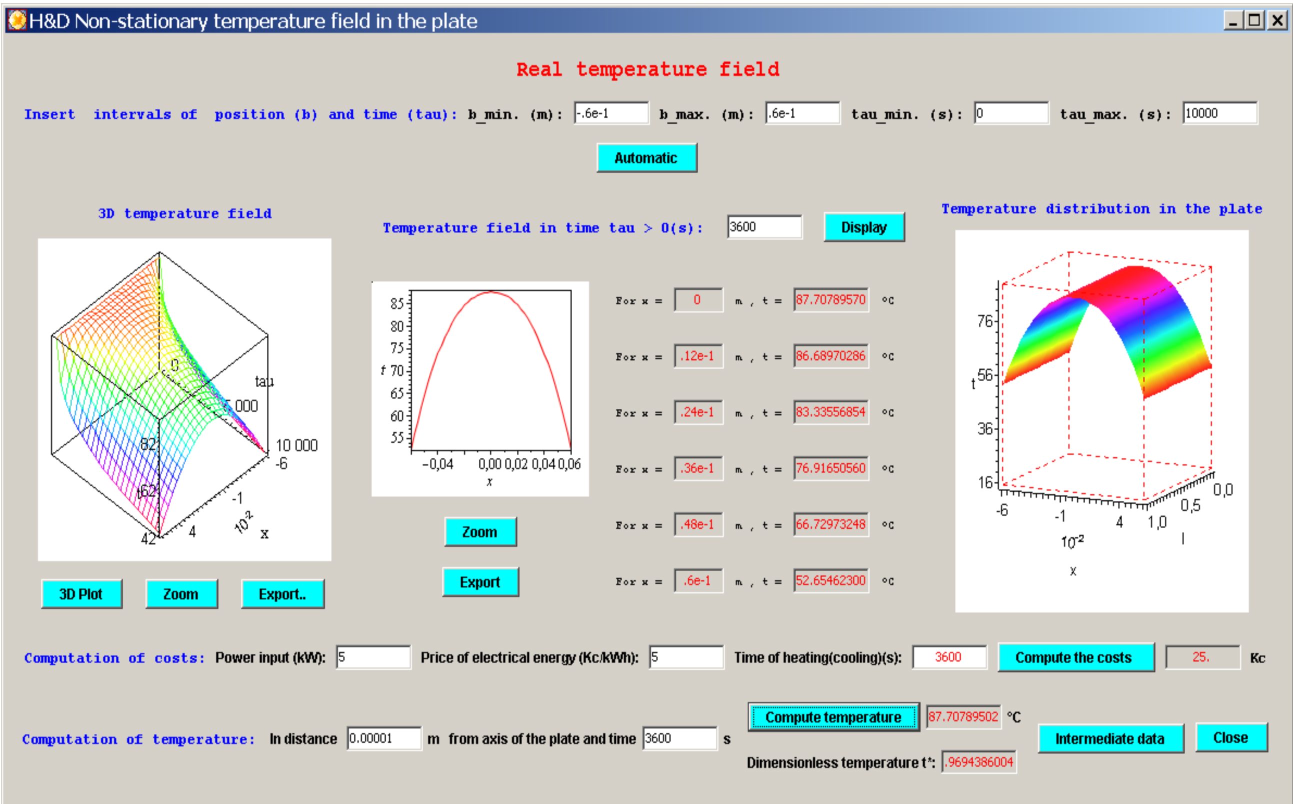

Fig. 4 The window for real temperature field computing

4 Conclusion

Analytical solution of above described mathematic model of non-stationary heat transfer in the solid a plane plate enabled us to made application for study of heating or cooling course inside this body affected by free convection surrounding fluid. The application we made by use of computer algebraic system Maple as a teaching aid. Illustration of the relevant non-stationary heat conduction problems, speeding of computing of non-stationary heat conduction in a plane plate and visualization of temperature field in 2D and 3D projection at the lectures and seminars are main benefits of the application. It can be also used for study by mans of Internet. Furthermore, the accuracy of our application enables it to be used for engineering computing in the processing industry.

List of symbols

a - thermal diffusivity of the heated (cooled) material, [$m^2.s^{-1}$];
b - half thickness of the plate, [m];
Bi - Biot number, [1];
d -characteristic size of the plate, [m];
C - coefficient of relation for Nusselt number calculation, [1];
c_p - specific thermal capacity of the heated (cooled) plate, [$J.kg^{-1}.K^{-1}$];
c_{p0} - specific thermal capacity of surrounding fluid, [$J.kg^{-1}.K^{-1}$];
Gr - Grashof number, [1];
K - coefficient of relation for Nusselt number calculation, [1];
Nu - Nusselt number, [1];
Pr - Prandtl number, [1];
q - positive root of the transcendent equation (9), [1];
Re - Reynolds number, [1];
t - temperature of the heated (cooled) body, [°C];
t_o - ambient temperature [°C];
t_p - initial temperature of the plate, [°C];
x - space coordinate, [m];
α - heat transfer coefficient, [$W.m^{-2}.K^{-1}$];
λ - thermal conductivity of the heated (cooled) plate, [$W.m^{-1}.K^{-1}$];
λ_0 - thermal conductivity of surrounding fluid, [$W.m^{-1}.K^{-1}$];
ρ - density of the heated (cooled) plate, [$kg.m^{-3}$];
ρ_0 - density of surrounding fluid, [$kg.m^{-3}$];
ν_o - kinematic viscosity of surrounding fluid, [$m^2.s^{-1}$];
τ - time, [s]
β_o - thermal volume expansion of surrounding fluid, [K^{-1}];

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