Modeling of Hydraulic Control Valves

PETR CHALUPA, JAKUB NOVAK, VLADIMIR BOBAL
Department of Process Control, Faculty of Applied Informatics
Tomas Bata University in Zlin
nam. T.G. Masaryka 5555, 760 01 Zlin, Czech Republic
CZECH REPUBLIC
chalupa@fai.utb.cz

Abstract: The paper deals with development of a mathematical model of valves of a hydraulic system. A three tank laboratory model (Amira DTS200) was investigated and characteristics of its valves were measured and a process of creating a mathematical model of the valves is described in detail. Although the three tank system is a classical modeling task this paper focuses on nonlinearities which are present in real system and other differences between ideal mathematical model and real-time system. The valves contain hysteresis and other nonlinearities. Despite the fact that all valves the system is equipped with are of the same type, big differences were observed between their characteristics. The approach to modeling of the system is not restricted to the particular system but can be used for many real-time hydraulic systems.

Key-Words: Control valve, Modeling, Identification

1 Introduction

Almost all current control algorithms are based on a model of a controlled plant [1]. Some information about controlled plant is necessary for design of a controller with satisfactory performance. A plant model can be also used to investigate properties and behaviour of the modelled plant without a risk of damage of violating technological constraints of the real plant. There are two basic approaches of obtaining plant model: the black box approach and the first principles modelling.

The black box approach [2], [3] is based on analysis of input and output signals of the plant. Usage the same identification algorithm for wide set of different controlled plants is the main advantages of this approach. The knowledge of physical principle of controlled plant and solution of set of mathematical equation is not required. Main drawback of a black box model persists in fact that it is generally valid only for signals it was calculated from.

The first principle modelling provides general models valid for wider range of plant inputs and states. The model is created by analyzing the modelled plant and combining physical laws [4]. But there is usually a lot of unknown constants and relations when performing analysis of a plant.

The paper uses combination of both methods. Basic relations are derived using mathematical physical analysis. Values of model parameters are identified on the basis of real-time measurements. The goal of the work was to obtain a mathematical model of the valves of DTS200 Three-Tank System [5] and to design the models in MATLAB-Simulink environment. The DTS200 laboratory equipment was developed by Amira Gmbh, Duisburg, Germany and serves as a real-time model of different industrial systems concerning liquid transport.

The models of valves serve as a part in process of creating a model of whole DTS200 system. The major reason for creating the model of this laboratory equipment are big time constants of the plant and thus time consuming experiments. A model, which represents the plant well, can considerably reduce testing time of different control approaches. Then only promising control strategies are applied to the real plant and verified.

The paper is organized as follows. Section 2 presents the modelled system – Amira DTS200. Derivation of initial ideal using first principles modelling is carried out in Section 3. Section 4 and 5 presents characteristics and calibration water level sensors and pumps respectively. Section 6 consists or results of measurements of valves.

2. The DTS200 System

The photo of main part of Amira DTS200 system is shown in Fig. 1. The system consists of three interconnected cylindrical tanks, two pumps, six valves, pipes, water reservoir in the bottom, measurement of liquid levels and other elements. Both pumps pump water from the bottom reservoir

ISBN: 978-1-61804-004-6

to the top of the left and right tanks Valve positions are controlled and measured by electrical signals, which allow precision setting of their position.

Fig. 1. Amira DTS200 – three tank system

A simplified scheme of the system is shown in Fig. 2. The pump P_1 controls the inflow to tank T_1 while the pump P_2 controls the liquid inflow to tank T_2. There is no pump connected to the middle tank T_s. The characteristic of the flow between tank T_1 and tank T_s can be affected by valve V_1, flow between tanks T_s and T_2 can be affected by the valve V_2 and the outflow of the tank T_2 can be affected by valve V_3. The system also provides the capability of simulating leakage from individual tanks by opening the valves V_4, V_5 and V_6.

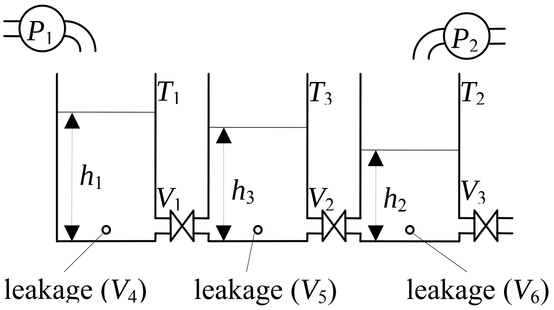

Fig. 2. Scheme of three tank system Amira DTS200

Pumps are controlled by analogue signal in range from -10V to 10V. Heights of water level are measured by pressure sensors. Each valve is operated by two digital signals which control motor of particular valve. First signal orders to start closing of the valve while the second signal is used for opening of the valve. If none of the signals is activated the valve remains in its current position. Each valve also provides three output signals: analogue voltage signal correspond to the current position of the valve and two informative logical signals which states that the valve is fully opened or fully closed respectively.

The overall number of inputs to the modelled plant DTS200 is 14:

- 2 analogues signals controlling the pumps,
- 12 digital signals (2 for each of the 6 valves) for opening / closing of the valves.

The plant provides 21 measurable outputs which can be used as a control feedback or for measurements of plant characteristics:

- 3 analogue signals representing level heights in the three tanks,
- 6 analogues signals representing position of the valves,
- 12 logical signals (2 for each of the 6 valves) stating that corresponding valve is fully opened / closed.

3 Initial ideal model

This chapter is focused to derivation of mathematical model of a valve. This derivation is based on ideal properties of individual components.

The ideal flow of a liquid through a pipe can be derived from Bernoulli and continuity equations for ideal liquid:

$$\left.\begin{aligned}\Delta hg &= \frac{1}{2}v^2\\ q &= S_V v\end{aligned}\right\} q = S_v\sqrt{2g}\sqrt{\Delta h} \tag{1}$$

where Δh is a difference between liquid levels on both sides of the pipe (e.g. difference between levels of tanks that are interconnected by the pipe), g is the standard gravity, v is the liquid velocity and S_V is the flow space of the pipe. The flow space S_V is controlled by the valve position p.

$$S_V = p\cdot S_{V\max} \qquad 0\le v\le 1 \tag{2}$$

where $S_{V\max}$ is the maximal flow area of the valve.

Since the flow through a valve depends only on the level difference, the valve position and constants representing pipes and cylindrical tanks, the change of water level in tank T_1 can be written as follows:

$$\frac{dh_1}{dt} = k_1\sqrt{\left|h_1-h_s\right|}\cdot\mathrm{sign}\left(h_1-h_s\right) - k_4\sqrt{h_1} \tag{3}$$

The area of all three tanks is the same and is symbolized by S_T. The k is a parameter representing valve position

$$k_i = p_i\frac{S_{V\max}\sqrt{2g}}{S_T} \qquad i=1,2,...,6 \tag{4}$$

ISBN: 978-1-61804-004-6

and q represents inflow as change of water level in time:

$$q_i = \frac{q_i'}{S_T} \qquad i = 1,2 \qquad (5)$$

Similar equations can be derived for the other two tanks. The model obtained by using ideal properties and behaviour of plant parts if further referred as "ideal model". This model of whole three tank system is successfully used in many control system studies as a demonstration example [6], [7], [8].

4. Characteristics of the valves

As stated in Section 2, each of plant's 6 valves is driven by two dedicated logical signals. These signals are used for starting valve's motor in closing or opening direction respectively. If none signal is activated the valve remains in its current position. Activation of both signals at in a particular time represents an invalid state and valve motor is stopped.

Each valve provides three output signals. The current valve position is determined by analogue signal. Higher values of signal represent closed valve and lower values represent opened valve. The other two signals are logical and state that valve is opened or closed respectively.

4.1 Valve limits and speed

Process of opening all valves at once from fully closed state to fully opened is presented in Fig. 3.

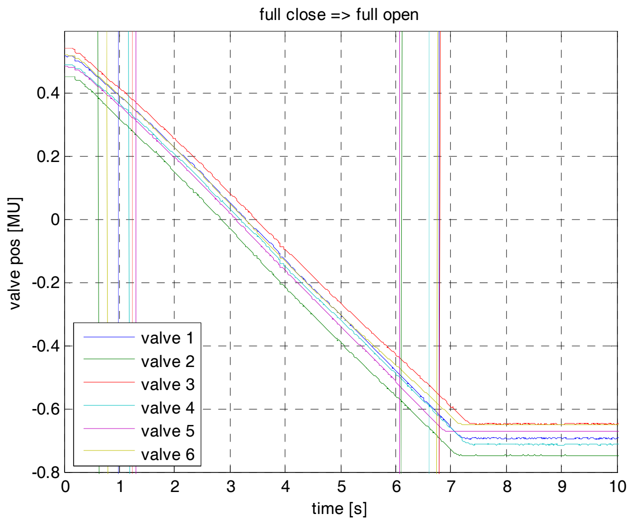

Fig. 3. Closing all valves in full range

This process represents moving of valve position in full range of its hard constraints. The vertical lines in left part of Fig. 3 represent changes the "opened" signals of individual valves. Before these signals drop down the valves are said to be opened. The vertical lines in the right part of Fig. 3 represent the changes of "closed". From these lines onward, the valves are said to be closed.

It can be observed that the initial and final positions of the valve as well as the positions corresponding to changes of "opened" and "closed" signal differ. But all the valves are moving at almost the same speed v_{valve}.

$$v_{valve} = -0.175\, MU/s \qquad (6)$$

Valve positions corresponding to hard constraints and validity of "opened" and "closed" state are summarized in Table 1.

Table 1
Valve positions for important states

Valve no.	full closed [MU]	"closed" signal change [MU]	"opened" signal change [MU]	full open [MU]
1	0.5199	0.4066	-0.6065	-0.6945
2	0.4550	0.3880	-0.5719	-0.7493
3	0.5462	0.3996	-0.5423	-0.6501
4	0.4926	0.3594	-0.5751	-0.7151
5	0.4868	0.3296	-0.5157	-0.6718
6	0.5242	0.4416	-0.5698	-0.6525

4.2 Valve flow parameter for outflow valves

Valve flow parameters k_i as appear in (3) were computed from measurements of draining through individual valves which are connected to outflow pipes (V_3, V_4, V_5 and V_6). The draining of a tank to the reservoir situated below the tanks is described by differential equation based on (3):

$$\frac{dh(t)}{dt} = -k\sqrt{h(t)} \qquad (7)$$

Integrating in an appropriate time range leads to the equation of time course of water level:

$$h(t) = \frac{k^2}{4} \cdot t^2 - k\sqrt{h(0)} \cdot t + h(0) \qquad (8)$$

where $h(0)$ is initial water level. An example of draining is presented in Fig. 4. At the beginning of the experiment, the tank was full and all valves were closed, then valve V_4 was partially opened, its position was recorded and time course of water level height was measured.

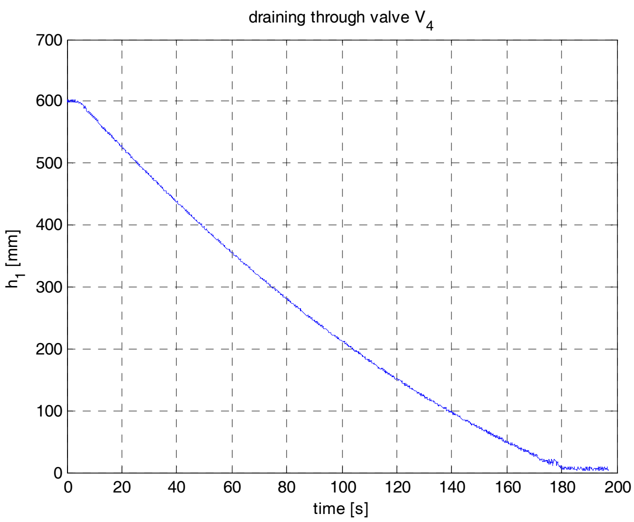

Fig. 4. Draining of tank T_1 through valve V_4

It is obvious that parabola depicted in Fig. 4 would continue below zero contrary to (8). A term corresponding to the vertical length of outflow pipe h_0 was added to the model. The vertical length h_0 is depicted in Fig. 5.

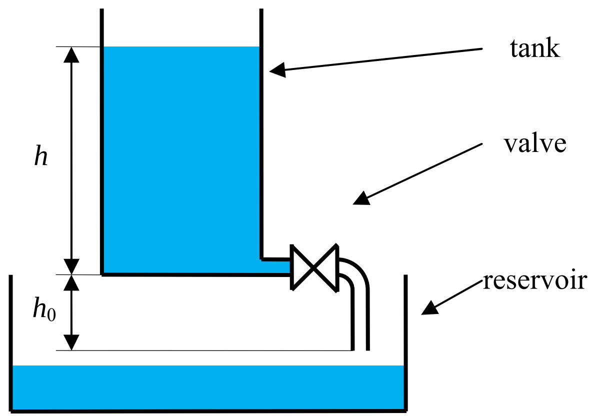

Fig. 5. Vertical length of outflow pipe (h_0)

Due to mechanical configuration of the plant, the value of h_0 for outflow valves V_3, V_4, V_5, and V_6 cannot be measured directly. But it can be identified from draining course (Fig. 4). To encapsulate h_0 into model, equations (7) and (8) were superseded:

$$\frac{dh(t)}{dt} = -k\sqrt{h(t)+h_0}$$
$$h(t) = \frac{k^2}{4}\cdot t^2 - k\sqrt{h(0)+h_0}\cdot t + h(0) \tag{9}$$

A second order polynomial (parabola) was fitted to an appropriate interval of draining data in least mean squares sense. The MATLAB function *polyfit* was used for this task. Parabola fitting is presented in Fig. 6.

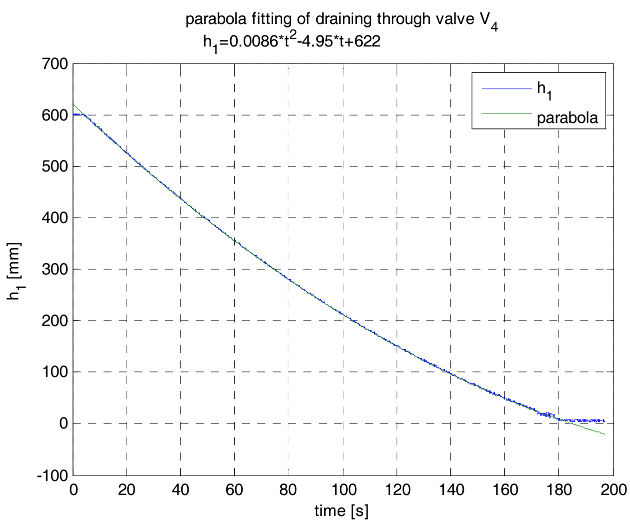

Fig. 6. Parabola fitting to the draining course

Values of k and h_0 can be easily obtained from polynomial coefficient according to (9). Valve can be closed to different positions at the beginning of draining experiment and relation between valve position and value of k can be achieved. This relation for one set of experiment on valve V_4 is presented in Fig. 7 where circles represent individual experiments. The characteristic is not strictly linear. It contains saturation of fully closed and fully opened valve. Transitions to saturation states are smooth.

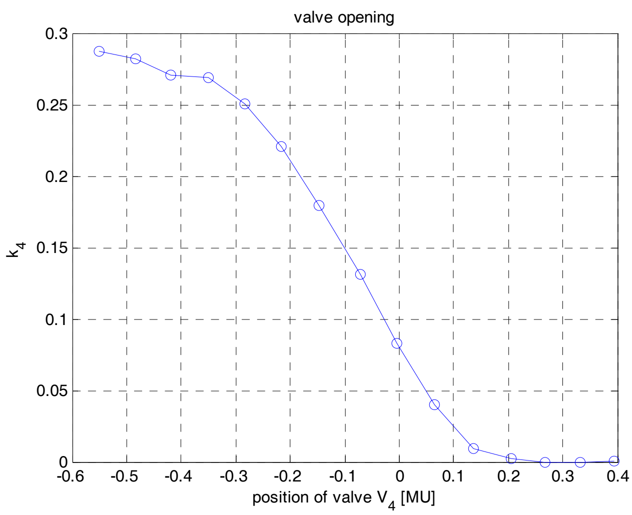

Fig. 7. Relation between valve position and k_4

4.3 Valve flow parameter for interconnection valves

Similar approach to obtaining values of k as presented in previous subsection can be used also for valves V_1 and V_2 which interconnects tanks T_1 and T_s, and T_s and T_2, respectively. Flow from the full tank T_1 to the empty tank T_s was used to measure valve constant k_1. The other valves were

ISBN: 978-1-61804-004-6

closed during the experiment. According to (3), the flow can be described by two differential equations:

$$\begin{aligned}\frac{dh_1(t)}{dt} &= -k_1\sqrt{\left|h_1(t)-h_s(t)\right|}\cdot \operatorname{sign}\left[h_1(t)-h_s(t)\right]\\ \frac{dh_s(t)}{dt} &= k_1\sqrt{\left|h_1(t)-h_s(t)\right|}\cdot \operatorname{sign}\left[h_1(t)-h_s(t)\right]\end{aligned} \quad (10)$$

Since the value of h_1 is always higher or equal to h_s, the term inside absolute values is always nonnegative. As the water flow just from T_1 to T_s and the geometry of both tanks is the same, according to mass conservation law the sum of h_1 and h_s remain the same during the experiment. Then the course of draining T_1 and filling T_s can be described by two independent differential equations.

$$\begin{aligned}\frac{dh_1(t)}{dt} &= -k_1\sqrt{2h_1(t)-h_\Sigma}\\ \frac{dh_s(t)}{dt} &= -k_1\sqrt{h_\Sigma-2h_2(t)}\end{aligned} \qquad h_\Sigma = h_1(t)+h_2(t) \quad (11)$$

Solving these equations lead to time course described by second order polynomial.

$$\begin{aligned}h_1(t) &= \frac{k^2}{2}\cdot t^2 - k\sqrt{2h_1(0)-h_\Sigma}\cdot t + h_1(0)\\ h_2(t) &= -\frac{k^2}{2}\cdot t^2 - k\sqrt{h_\Sigma-2h_2(0)}\cdot t + h_2(0)\end{aligned} \quad (12)$$

An example of courses and corresponding parabolas are depicted in Fig. 8.

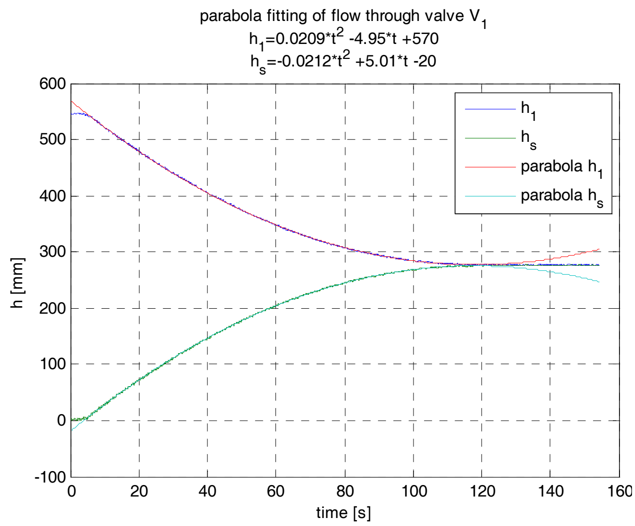

Fig. 8. Parabola fitting to the flow course through V_1

A similar approach as presented for valve V_1 was used to measure characteristics of valve V_2.

4.4 Valve hysteresis

The experiments presented in previous subsections were preformed for opening of a valve only. At the beginning, the valve was fully closed and subsequently was partially opened to a given position. In this section a problem of closing of a valve is studied. Performed experiments are similar except initial part. The experiment starts with full tank and closed valve too, but then the valve was fully opened and then partially closed to the desired position. Therefore the same valve position (value of analogue signal from a valve) was reached but from opposite direction.

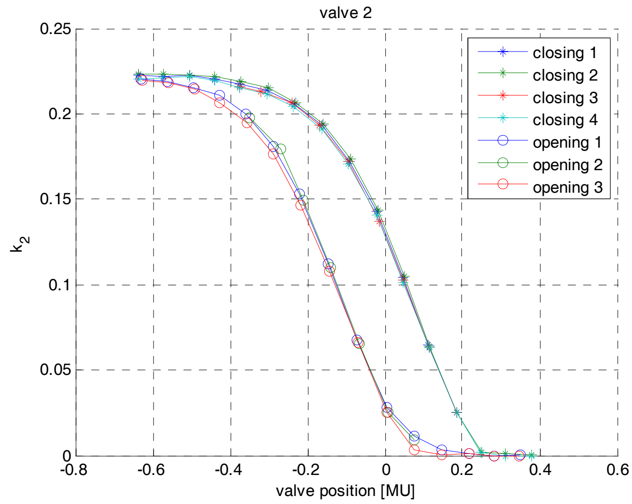

Fig. 9. Hysteresis of valve V_2

Experiments unveiled a hysteresis present in all valves. The characteristics for opening and for closing of valve V_2 is presented in Fig. 9. There are four sets of experiments for closing of the valve performed in various time in the figure. Individual experiments correspond to stars in the figure. Three set of experiment are presented for closing of the valve where individual experiments are represented by circles. The figure shows that hysteresis plays a big role in the experiments. The value of position itself does not give sufficient information about current value of parameter k_2. For example, if the position is 0 MU the value of k_2 can be anywhere in range 0.03 to 0.13. Especially in case of using the valve as an actuator the hysteresis should be taken into account. Otherwise control process can easily become unstable.

Since the shape of curves corresponding to opening and closing of the valve is similar, an average difference between them in direction of position axis can be computed. This value can be used as measure of hysteresis. Values of hysteresis, h_0 as well as maximal value of k for each valve is presented in Table 2.

ISBN: 978-1-61804-004-6

Table 2
Valve positions for important states

Valve no.	hysteresis [MU]	maximal k	h0 [mm]
1	0.0219	0.2180	-
2	0.1783	0.2237	-
3	0.0310	0.2601	126.6
4	0.0426	0.2976	127.9
5	0.1307	0.2735	121.2
6	0.0800	0.2688	97.1

4.5 Modelling of valve characteristics

The course of relation between valve position in MU and k is similar to step responses of dynamical system and therefore it was modeled in similar way. Other types of approximation functions, like sigmoids, were also tested, but did not achieve better results. A model based on transfer of 4th order aperiodic system produced satisfactory results. Thus relation between position and k was as follows:

$$pos < pos_0: \quad k = k_{max}\left[1 - \frac{1}{6} e^{\frac{-b}{a}} \frac{b^3 + 3b^2a + 6ba^2 + 6a^3}{a^3}\right]$$

$$b = pos_0 - pos \tag{13}$$

$$pos \geq pos_0: \quad k = 0$$

where pos is valve position in MU and parameters a and pos_0 were obtained by nonlinear regression. The regression for valve V_2 is presented in Fig. 10.

Behavior of system inside hysteresis area was studied as well. This task was time consuming because a performed set of experiments took more than 45 hours. The parameter k did not change its value till it reaches a border of hysteresis area, i.e. the curve of either opening or closing of the valve.

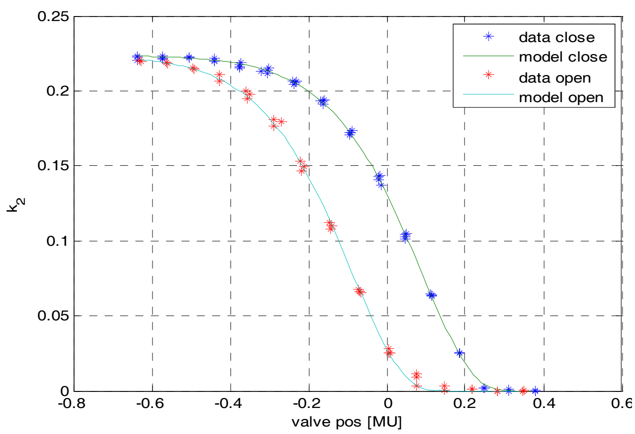

Fig. 10. Model of parameter k for valve V_2

5 Conclusion

The paper presented a development of the model of valves of a hydraulic system. The Amira DTS200 three tank system was considered but used techniques can be easily generalized to wide set of hydraulic systems. The real system contains several nonlinearities which incorporate complexity to the system. Total number of experiments concerning valves reached 433 taking altogether more than 113 hours. Resulting model includes all major nonlinearities and can be integrated into a Simulink model of whole three tank system.

Acknowledgement

This work was supported by the Ministry of Education of the Czech Republic under grant 1M0567.

References:

[1] V. Bobál, J. Böhm, J. Fessl & J. Macháček:. *Digital Self-tuning Controllers: Algorithms, Implementation and Applications* (Springer - Verlag London Ltd., 2005).

[2] G. P. Liu, Nonlinear identification and control – A neural network Approach (Springer - Verlag London Ltd., London, 2001).

[3] L. Ljung, *System identification : theory for the user* (Upper Saddle River, NJ : Prentice Hall PTR, 1999)

[4] D. M. Himmelblau, J. B. Riggs, *Basic principles and calculations in chemical engineering* (Upper Saddle River, N.J. : Prentice Hall, 2004)

[5] Amira, DTS200 Laboratory Setup Three - Tank - System (Amira GmbH, Duisburg, 2002).

[6] L.Li & D. Zhou, Fast and robust fault diagnosis for a class of nonlinear systems: detectability analysis, *Computers &Chemical Engineering, 28*, 2004, 2635-2646.

[7] D. Henry & A. Zolghadri, Norm-based design of robust FDI schemes for uncertain system under feedback control: Comparison of two approaches, *Control engineering practice, 14*, 2006, 1081-1097.

[8] S. Blazic, I. Skrjanc & D. Matko, Globally stable direct fuzzy model reference adaptive control, *Fuzzy Sets and Systems*, 2003, 3-33.