

A Novel Principle for Relay-Based Autotuning

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Abstract: - This paper presents a new simple principle for aperiodic tuning of SISO controllers used in autotuning schemes. Autotuners represent a combination of relay feedback identification and some control design method. In this contribution, models with up to three parameters are estimated by means of a single asymmetrical relay experiment. Then a stable low order transfer function is identified. Subsequently, the controller is analytically derived from general solutions of Diophantine equations in the ring of proper and stable rational functions R_{PS} . This approach enables to define a scalar positive parameter through a pole-placement root of the characteristic closed loop equation. A first order identification yields a PI-like controllers while a second order identification generates PID ones. The analytical simple rule is derived for aperiodic control response and the scalar tuning parameter $m > 0$ is then tuned according to identified time constant of an approximated transfer function.

Key-Words: Autotuning, Relay experiment, Algebraic control design, Pole-placement problem.

1 Introduction

Industrial processes are usually controlled by PID controllers, Yu in [1] refers that more than 97 % of control loops are of this type and most of them are actually under PI control. The practical advantages of PID controllers can be seen in a simple structure, in an understandable principle and in control capabilities. Moreover, PID controllers have survived changes in technology from pneumatic principles through analog and digital representation to DCS ones. It is widely known that PID controllers are quite resistant to changes in the controlled process without meaningful deterioration of the loop behavior. For 70 years, the Ziegler – Nichols tuning rule has been glorified and vilified as well. Nevertheless, the Ziegler – Nichols rule stay remains the most frequent method of PID tuning. However, there are many limitations, drawbacks and infirmities in the behavior of the Ziegler – Nichols setting. A solution for qualified choice of controller parameters can be seen in more sophisticated, proper and automatic tuning of PID controllers.

The development of various autotuning principles was started by a simple symmetrical relay feedback experiment proposed by Åström and

Hägglund in [2] in the year 1984. The ultimate gain and ultimate frequency are then used for adjusting of parameters by original Ziegler-Nichols rules. During the period of more than two decades, many studies have been reported to extend and improve autotuners principles; see e.g. [3], [4], [8], [9]. The extension in relay utilization was performed in [1], [5], [7], [14] by an asymmetry and hysteresis of a relay. Over time, the direct estimation of transfer function parameters instead of critical values began to appear. Experiments with asymmetrical and dead-zone relay feedback are reported in [10]. Also, various control design principles and rules can be investigated in mentioned references. Nowadays, almost all commercial industrial PID controllers provide the feature of autotuning.

In this paper, a new combination for autotuning method of PI and PID controllers with an aperiodic control rule is proposed and developed. The basic autotuning principle combines an asymmetrical relay identification experiment and a control design performed in the ring of proper and stable rational functions R_{PS} . The factorization approach proposed in [11] was generalized to a wide spectrum of control problems in [10], [12], [15] - [20]. A different philosophy is reported in [21]. The pole

placement problem in R_{PS} ring is formulated through a Diophantine equation and the pole is analytically tuned according to aperiodic response of the closed loop. The proposed method is compared by an equalization setting proposed in [13]. A general basic scheme of the autotuning principle can be seen in Fig.1.

2 Relay Feedback Estimation

The estimation of the process or ultimate parameters is a crucial point in all autotuning principles. The relay feedback test can utilize various types of relay for the parameter estimation procedure. The classical relay feedback test [2], was proposed for stable processes by symmetrical relay without hysteresis. Following sustained oscillation are then used for determining the critical (ultimate) values. The control parameters (PI or PID) are then generated in standard manner.

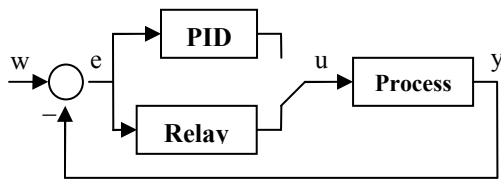


Fig.1: Block diagram of an autotuning principle

Asymmetrical relays with or without hysteresis bring further progress [1], [14]. After the relay feedback test, the estimation of process parameters can be performed. A typical data response of such relay experiment is depicted in Fig.2. The relay asymmetry is required for the process gain estimation (2) while a symmetrical relay would cause the zero division in the appropriate formula.

In this paper, an asymmetrical relay with hysteresis is used. This relay enables to estimate transfer function parameters as well as a time delay term. For the purpose of the aperiodic tuning the time delay is not exploited.

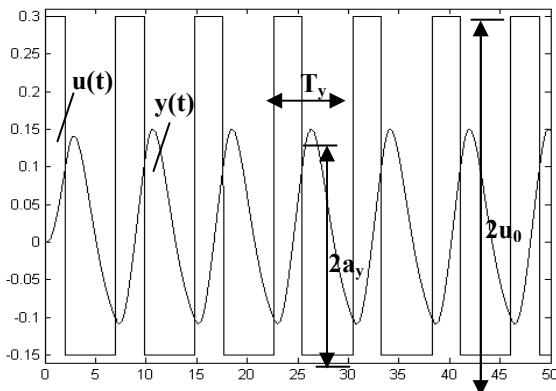


Fig.2: Asymmetrical relay oscillation of stable process

The model for first order (stable) systems plus dead time (FOPDT) is supposed in the form:

$$G(s) = \frac{K}{Ts + 1} \cdot e^{-\Theta s} \quad (1)$$

and the process gain can be computed by the relation (see [22]):

$$K = \frac{\int_0^{iT_y} y(t) dt}{\int_0^{iT_y} u(t) dt}; \quad i = 1, 2, 3, \dots \quad (2)$$

The time constant and time delay terms are given by [10]:

$$T = \frac{T_y}{2\pi} \cdot \sqrt{\frac{16 \cdot K^2 \cdot u_0^2}{\pi^2 \cdot a_y^2} - 1}$$

$$\Theta = \frac{T_y}{2\pi} \cdot \left[\pi - \arctg \frac{2\pi T}{T_y} - \arctg \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \quad (3)$$

where a_y and T_y are depicted in Fig.2 and ε is the hysteresis.

Similarly, the second order model plus dead time (SOPDT) is assumed in the form:

$$G(s) = \frac{K}{(Ts + 1)^2} \cdot e^{-\Theta s} \quad (4)$$

The gain is given by (2), the time constant and time delay term can be estimated according to [10] by the relation:

$$T = \frac{T_y}{2\pi} \cdot \sqrt{\frac{4 \cdot K \cdot u_0}{\pi \cdot a_y} - 1}$$

$$\Theta = \frac{T_y}{2\pi} \cdot \left[\pi - 2\arctg \frac{2\pi T}{T_y} - \arctg \frac{\varepsilon}{\sqrt{a_y^2 - \varepsilon^2}} \right] \quad (5)$$

3 Algebraic Control Design

The control design is based on the fractional approach; see e.g. [11], [12], [15] - [17]. Any transfer function $G(s)$ of a (continuous-time) linear system is expressed as a ratio of two elements of R_{PS} . The set R_{PS} means the ring of (Hurwitz) stable and proper rational functions. Traditional transfer functions as a ratio of two polynomials can be easily transformed into the fractional form simply by dividing, both the polynomial denominator and numerator by the same stable polynomial of the appropriate order. Then all transfer functions can be expressed by the ratio:

$$G(s) = \frac{b(s)}{a(s)} = \frac{\frac{b(s)}{(s+m)^n}}{\frac{a(s)}{(s+m)^n}} = \frac{B(s)}{A(s)} \quad (6)$$

$$n = \max(\deg(a), \deg(b)), \quad m > 0 \quad (7)$$

Then, all feedback stabilizing controllers for the feedback system depicted in Fig.3 are given by a general solution of the Diophantine equation:

$$AP + BQ = 1 \quad (8)$$

which can be expressed with Z free in R_{PS} :

$$\frac{Q}{P} = \frac{Q_0 - AZ}{P_0 + BZ} \quad (9)$$

In contrast of polynomial design, all controllers (9) are proper and can be utilized.

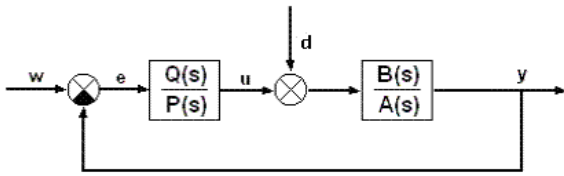


Fig.3: Feedback (1DOF) control loop

Asymptotic tracking is then ensured by the divisibility of the denominator P in (9) by the denominator of the reference $w = \frac{G_w}{F_w}$. The most frequent case is a stepwise reference with the denominator in the form:

$$F_w = \frac{s}{s+m}; \quad m > 0 \quad (10)$$

The similar conclusion is valid also for the load disturbance $d = \frac{G_d}{F_d}$. The load disturbance

attenuation is then achieved by divisibility of P by F_d . More precisely, for tracking and attenuation in the closed loop according to Fig.3 the multiple of AP must be divisible by the least common multiple of denominators of all input signals. The divisibility in R_{PS} is defined through unstable zeros and it can be achieved by a suitable choice of rational function Z in (9), see [11], [15] for details.

4 PI and PID-Like Controllers

Diophantine equation (8) for the first order systems (1) without the time delay term can be easily transformed into polynomial equation:

$$(Ts+1)p_0 + Kq_0 = s+m \quad (11)$$

with general solution:

$$P = \frac{1}{T} + \frac{K}{s+m} \cdot Z \quad (12)$$

$$Q = \frac{Tm-1}{TK} - \frac{Ts+1}{s+m} \cdot Z$$

where Z is free in the ring R_{PS} . Asymptotic tracking is achieved by the choice:

$$Z = -\frac{m}{TK} \quad (13)$$

and the resulting PI controller is in the form:

$$C(s) = \frac{Q}{P} = \frac{q_1s + q_0}{s} \quad (14)$$

where parameters q_1 and q_0 are given by:

$$q_1 = \frac{2Tm-1}{K}, \quad q_0 = \frac{Tm^2}{K} \quad (15)$$

For the SOPDT the design equation (8) takes the form:

$$(Ts+1)^2 \cdot s \cdot p_0 + K \cdot (q_2s^2 + q_1s + q_0) = (s+m)^3 \quad (16)$$

and after similar manipulations the resulting PID controller gives the transfer function:

$$C(s) = \frac{Q}{P} = \frac{q_2s^2 + q_1s + q_0}{s(s+p_0)} \quad (17)$$

with parameters:

$$p_0 = \frac{1}{T^2}; \quad q_2 = \frac{3Tm-2}{KT} \quad (18)$$

$$q_1 = \frac{3T^2m^2-1}{KT^2}; \quad q_0 = \frac{m^3}{K}$$

For both systems FOPDT and SOPDT the scalar parameter $m > 0$ seems to be a suitable „tuning knob” influencing control behavior as well as robustness properties of the closed loop system. Naturally, both derived controllers correspond to classical PI and PID ones. It is clear that (14) represents the PI controller:

$$u(t) = K_p \cdot \left(e(t) + \frac{1}{T_I} \cdot \int e(\tau) d\tau \right) \quad (19)$$

and the conversion of parameters is trivial. Relation (17) represents a PID in the standard four-parameter form [3]:

$$u(t) = K_p \cdot \left(e(t) + \frac{1}{T_I} \cdot \int e(\tau) d\tau + T_D y_f(t) \right) \quad (20)$$

$$\tau y_f'(t) + y_f(t) = y(t)$$

5 Aperiodic Tuning

Over the past 60 years after the introduction of Ziegler-Nichols rule in 1942, vehement research activity in controller tuning has been performed. More than 240 tuning rules are referred in [23], more than 100 rules for PI controllers.

A simple and attractive choice for the tuning parameter $m > 0$ can be easily obtained analytically. In the R_{PS} expression, the closed-loop transfer function K_{wy} is for (1) and PI controller (14) given in a very simple form:

$$K_{wy} = \frac{BQ}{AP + BQ} = BQ = \frac{(2Tm - 1)s + Tm^2}{(s + m)^2} \quad (22)$$

The step response of (22) can be expressed by Laplace transform:

$$h(t) = L^{-1} \left\{ \frac{K_{wy}}{s} \right\} = L^{-1} \left\{ \frac{k_1 s + k_0}{s(s + m)^2} \right\} = L^{-1} \left\{ \frac{A}{s} + \frac{B}{(s + m)} + \frac{C}{(s + m)^2} \right\}, \quad (23)$$

where A, B, C are calculated by comparing appropriate fractions in (23) and $k_1 = 2mT - 1$, $k_0 = Tm^2$. The response $h(t)$ in time domain is then

$$h(t) = A + Be^{-mt} + Cte^{-mt} \quad (24)$$

The overshoot or undershoot of this response is characterized by the first derivative condition

$$h'(t) = -mBe^{-mt} + C(e^{-mt} - tme^{-mt}) = 0 \quad (25)$$

From (25) time of the extreme of response $h(t)$ is then easily calculated by the relation:

$$t_e = \frac{C - mB}{mC} = \frac{1}{m} - \frac{B}{C} \quad (26)$$

Since the aperiodic response means that the extreme does not exist for positive t_e , it implies $t_e < 0$ and after all substitutions of A, B, C, k_1, k_0 relation (26) takes the simple form

$$1 < m \frac{B}{C} = \frac{1}{\frac{Tm}{Tm} - 1} \quad (27)$$

The denominator of (27) must be positive and less than 1 and $m > 0$ which implies the inequality:

$$\frac{1}{2T} < m < \frac{1}{T} \quad (28)$$

Any positive parameter m from (28) ensures aperiodic response. It is a question for further investigation and simulation what choice from interval (28) is the best. For autotuning philosophy time constant T is always an estimation then the middle value of (28) would be reasonable, it means the choice

$$m = \frac{3}{4 \cdot T} \quad (29)$$

Also another tuning principles for aperiodic tuning certainly exist. For the mentioned algebraic synthesis, the equalization method developed by Gorez and Klán in [13]. The idea goes out from PI controller in the form (19). The tuning rule is very simple and it leads in relations:

$$K_p = \frac{1}{2K} \quad T_I = 0.4 \cdot T_u \quad (30)$$

where K is a process gain and T_u is the ultimate period obtained from the Ziegler-Nichols experiment. However, the fulfillment of (30) by unique value of $m > 0$ is impossible, see [16]. The exact fulfillment of both relations in (30) could be

obtained in the case of two distinct roots in denominator (22), so $(s + m_1)(s + m_2)$ instead of $(s + m)^2$.

6 Examples and Analysis

The following examples illustrate the situation where the estimated model is always of the first order ones (1) without time delay and the controller has a PI structure (14).

Example 1: The first order system governed by the transfer function $G(s)$ was after the relay experiment estimated by $\tilde{G}(s)$ in the form:

$$G(s) = \frac{2}{3s + 1} \quad \tilde{G}(s) = \frac{2}{2.7s + 1} \quad (31)$$

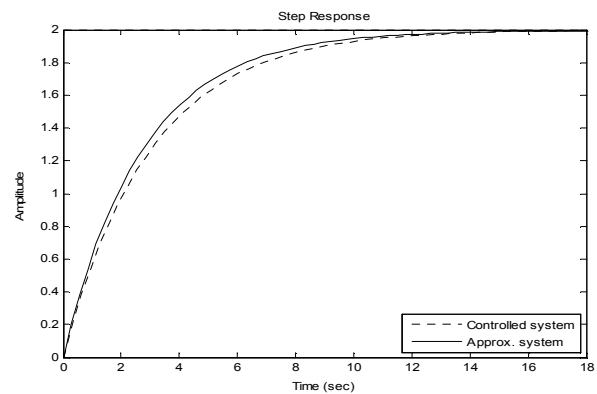


Fig. 4 Step responses of nominal and estimated systems (31)

The PI controller was then generated for three values of m within the interval given by (28), $m = 0.185$; 0.278 ; 0.370 , respectively. The responses are depicted in Fig. 5.

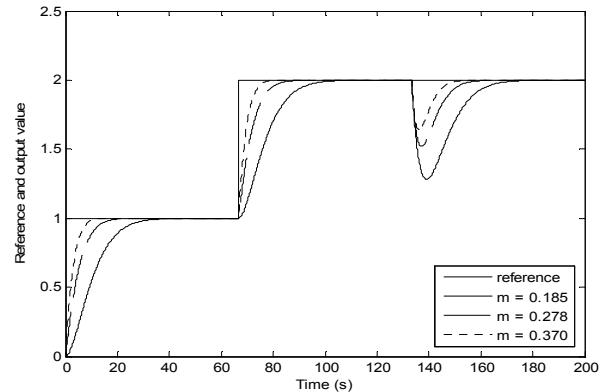


Fig. 5 Control responses (Example 1)

This choice represents the lowest, middle and upper limit values in derived interval (28). Responses in Fig. 5 demonstrate that all ones have aperiodic behavior.

Example 2: A second order (stable) system $G(s)$ without a time delay was estimated by a first order model in the above mentioned relay experiment. Both transfer functions have the form:

$$G(s) = \frac{3}{(2s+1)^2} \quad \tilde{G}(s) = \frac{3.02}{3.89s+1} \cdot e^{-1.15s} \quad (32)$$

The above mentioned relay experiments enable to estimate the original system by the first or second order transfer functions (1), (4). Step responses without time delay terms are depicted in Fig.6. The

PI controller generated from the approximated system $\tilde{G}(s)$ was designed by (17), (18) for three values of tuning parameters $m > 0$ with respect the interval given by (28). The responses are shown in Fig.7. Also, the aperiodic responses are achieved.

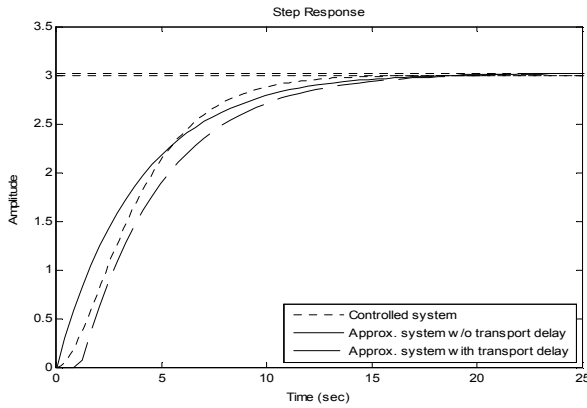


Fig. 6 Step responses of systems (32)

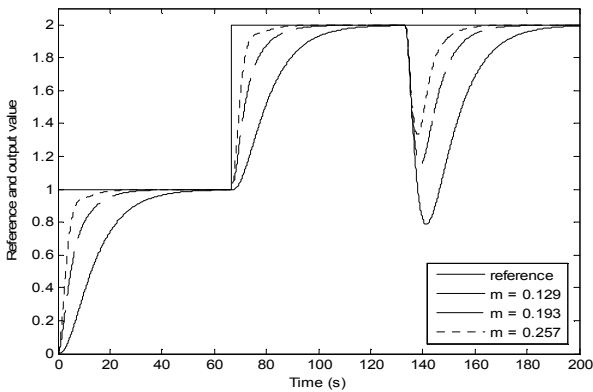


Fig. 7 Control responses (Example 2)

Example 3: A higher order system (8th order) with transfer function $G(s)$ is supposed. Again, after the relay experiment, a first order estimation $\tilde{G}(s)$ was identified, both governed by:

$$G(s) = \frac{3}{(s+1)^8} \quad \tilde{G}(s) = \frac{3}{3.52s+1} \cdot e^{-5.2s} \quad (21)$$

The step responses of systems (33) are shown in Fig. 8. Naturally, the step response of the estimated system is quite different from the nominal system $G(s)$. Again, PI controllers are generated from (17), (18) and the tuning parameter $m > 0$ can influence the control responses. Since the difference of controlled and estimated systems is considerable, it can be expected that not all values of and some of $m > 0$ represent acceptable behavior. With respect of (28), three responses are shown in Fig.9. Generally,

larger values of $m > 0$ implicate larger overshoots and oscillations. As a consequence, for inaccurate relay identifications, lower values of $m > 0$ in interval (28) can be recommended.

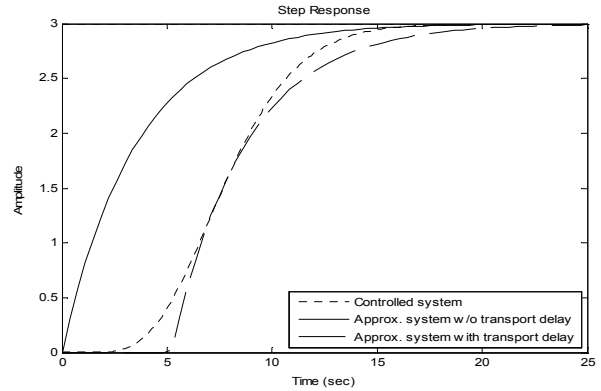


Fig.2: Step responses of systems (24)

In all control simulations, the reference value is changed in 1/3 of the simulation horizon and the load disturbance is injected in the 2/3 of the simulation horizon. All simulations were performed in Simulink environment. In the case of (33), the best response is achieved for $m = 0.142$.

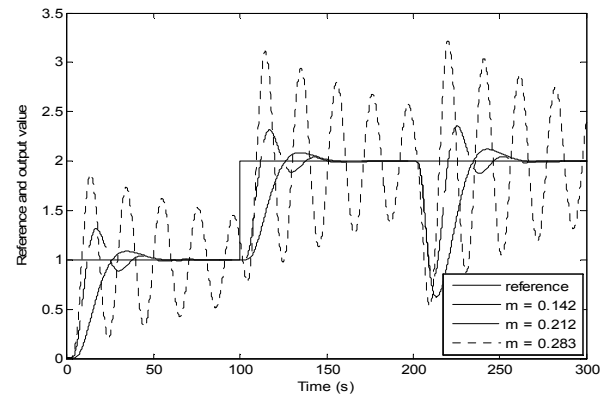


Fig.3: Control responses

7 Conclusions

This contribution gives a new combination of relay feedback identification and a control design method.

The estimation of a low order transfer function parameters is performed from asymmetric limit cycle data. The control synthesis is carried out through the solution of a linear Diophantine equation according to [11], [15], [16]. This approach brings a scalar tuning parameter which can be adjusted by various strategies. A first order estimated model generates PI-like controllers while a second order model generates a class of PID ones. The aperiodic tuning through the parameter $m > 0$ is proposed by the analytic derivation. The methodology is illustrated by several examples of various orders and dynamics.

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