One of Possible Methods of Control of Multivariable Control Loop

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Abstract: The paper deals with description and simulation verification of one of possible methods to control of multivariable controlled systems. The proposed method of control uses the so called binding members and correction members. Binding members serve to ensure of autonomy of control loop and correction members serve to ensure invariance of control loop. Simulation verifications of the control method are carried out for two-variable control loop.

Key-Words: closed loop control, control system synthesis, single-variable control loop, multi-variable control loop, simulation

1 Introduction

It is often required, at large numbers controlled systems, that their multiple outputs be controlled simultaneously. To do so, multiple inputs have to be usually subtly manipulated. The examples these controlled systems are e.g. aircraft autopilots, chemical processes, air-conditioning plants, distillation columns, turbines, etc. [1]. In these cases, it means that there is not only larger number of independent SISO (single-variable) control loop. These control loops are complex with several controlled variables where separate variables are not mutually independent. Mutual coupling of controlled variables is usually given by simultaneous action of each of input variables of controlled plant (manipulated variables and disturbance variables) to all controlled variables. These control loops are called MIMO (multi-variable) control loops and they are a complex of mutually influencing simpler control loops [1]. Special case of MIMO control loop is SISO control loop having only one input signal (manipulated variable, disturbance variable) and one output signal (controlled variable) [2].

All simulation experiments were performed in the simulation mathematical education and research software MATLAB/SIMULINK [3]. MATLAB is a widely used tool not in education but also in research; in addition to that, many researchers have produced a wide variety of educational tools based on MATLAB [4], [5].

2 MIMO Control Loop

2.1 Description of MIMO control loop

We will consider MIMO branched control loop with measurement of disturbance (see Fig.1). [1]

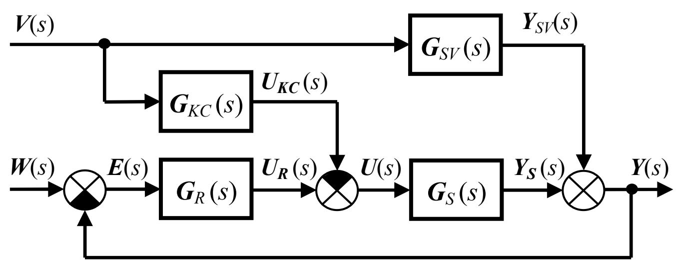

Fig.1 - MIMO branched control loop with measurement of disturbance

$\boldsymbol{G}_S(s)$, $\boldsymbol{G}_{SV}(s)$ are transfer matrixes of a controlled plant and disturbance variables and $\boldsymbol{G}_R(s)$, $\boldsymbol{G}_{KC}(s)$ are transfer matrixes of controller and correction members. Signal $\boldsymbol{Y}(s)$ $[n\times 1]$ is a vector of controlled variables, $\boldsymbol{U}(s)$ $[n\times 1]$ is a vector of manipulated variables and $\boldsymbol{V}(s)$ $[m\times 1]$ is a vector of disturbance variables; $m \le n$

Transfer matrixes of controlled plant $\boldsymbol{G}_S(s)$ and transfer matrix of disturbance variables $\boldsymbol{G}_{SV}(s)$ are considered in forms

$$\boldsymbol{G}_S(s)=\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \quad \boldsymbol{G}_{SV}(s)=\begin{bmatrix} S_{V11} & S_{V12} & \cdots & S_{V1m} \\ S_{V21} & S_{V22} & \cdots & S_{V2m} \\ \vdots & \vdots & \cdots & \vdots \\ S_{Vn1} & S_{Vn2} & \cdots & S_{Vnm} \end{bmatrix} \quad (1)$$

where

$$S_{ij}=\frac{Y_{S,i}(s)}{U_j(s)} \quad i,j=<1,\dots,n>; \quad S_{Vij}=\frac{Y_{SV,i}(s)}{V_j(s)} \quad i=<1,\dots,n>, j=<1,\dots,m>, m\le n$$

Transfer matrixes of controller $\boldsymbol{G}_R(s)$ and transfer matrix of correction member $\boldsymbol{G}_{KC}(s)$ are considered in forms

$$\boldsymbol{G}_R(s)=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix} \quad \boldsymbol{G}_{KC}(s)=\begin{bmatrix} KC_{11} & KC_{12} & \cdots & KC_{1m} \\ KC_{21} & KC_{22} & \cdots & KC_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ KC_{n1} & KC_{n2} & \cdots & KC_{nm} \end{bmatrix} \quad (2)$$

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where

$$R_{ij} = \frac{U_{R_i}(s)}{E_j(s)} \quad i,j = <1,\ldots n>; KC_{ij} = \frac{U_{KC_i}(s)}{V_j(s)} \quad i = <1,\ldots n>, \ j = <1,\ldots m>, \ m \leq n$$

2.2 Autonomy of MIMO control loop and invariance of MIMO control loop

At MIMO control it is often required in order to control loop to be autonomous and invariant.

2.1.1 Autonomy of MIMO control loop

In order to determine the condition for autonomy of control loop we start from a closed loop transfer matrix $\boldsymbol{G}_{W/Y}(s)$. i.e.

$$\boldsymbol{G}_{W/Y}(s) = \left[\boldsymbol{I} + \boldsymbol{G}_S(s)\boldsymbol{G}_R(s)\right]^{-1} \boldsymbol{G}_S(s)\boldsymbol{G}_R(s) \qquad (3)$$

For ensuring autonomy of control loop it is necessary that the matrix $\boldsymbol{G}_S(s)\,\boldsymbol{G}_R(s)$ is diagonal. On the base of this condition it is possible to derive the following relation

$$\frac{R_{ij}}{R_{kj}} = \frac{s_{ji}}{s_{jk}} \qquad i, j, k = <1,\ldots,n>, s_{jk} \neq 0 \qquad (4)$$

where R_{ij}, R_{kj} are separate members of a transfer matrix of controller $\boldsymbol{G}_R(s)$ and s_{ji}, s_{jk} are algebraic supplements of separate elements of a transfer matrix of controlled plant $\boldsymbol{G}_S(s)$.

It is considered that diagonal (main) controllers R_{11}, R_{22},…, R_{nn} are usually known already from the first design of conception of control [1]. Relation (4) is used to calculation of aside from diagonal members of a transfer matrix of controller $\boldsymbol{G}_R(s)$ (binding members), which means that above mentioned relation can be rewritten into the following form

$$\frac{R_{ij}}{R_{jj}} = \frac{s_{ji}}{s_{jj}} \qquad i, j = <1,\ldots,n>, s_{jj} \neq 0 \qquad (5)$$

2.1.2 Invariance of MIMO control loop

In order to determine the condition for invariance of control loop we start from a disturbance transfer matrix $\boldsymbol{G}_{V/Y}(s)$. i.e.

$$\boldsymbol{G}_{V/Y}(s) = \left[\boldsymbol{I} + \boldsymbol{G}_S(s)\boldsymbol{G}_R(s)\right]^{-1}\left[\boldsymbol{G}_{SV}(s) - \boldsymbol{G}_S(s)\boldsymbol{G}_{KC}(s)\right] \quad (6)$$

For ensuring absolute invariance of control loop it is necessary that the disturbance transfer matrix $\boldsymbol{G}_{V/Y}(s)$ is zero. This is possible if the following relation is valid

$$\boldsymbol{G}_{KC}(s) = \boldsymbol{G}_S^{-1}(s)\boldsymbol{G}_{SV}(s) \qquad (7)$$

Correction members KC of transfer matrixes of correction members $\boldsymbol{G}_{KC}(s)$ can be determined from the relation

$$KC_{ij} = \frac{1}{det\,\boldsymbol{G}_S}\sum_{k=1}^{n} s_{ki}\cdot S_{V,kj} \quad i, j = <1,\ldots,n>, det\,\boldsymbol{G}_S \neq 0 \quad (8)$$

where $det\,\boldsymbol{G}_S$ is a determinant of transfer matrix of controlled plant $\boldsymbol{G}_S(s)$, s_{ki} are algebraic supplements of separate elements of a transfer matrix of controlled plant $\boldsymbol{G}_S(s)$ and $S_{V,kj}$ are separate members of a transfer matrix of disturbance variables $\boldsymbol{G}_{SV}(s)$.

In case diagonal members of a transfer matrix of disturbance variables $\boldsymbol{G}_{SV}(s)$ are considered as a dominant it is possible to simplify the above mentioned relation. In this case it is considered that internal couplings are omitted at MIMO control loop and thus n SISO branched control loops with measuring of a disturbance variable are gained. Connection of all SISO branched control loops is the same and they differ only in separate transfers of controlled plants, controllers, correction members and disturbance variables (see Fig.2) [1].

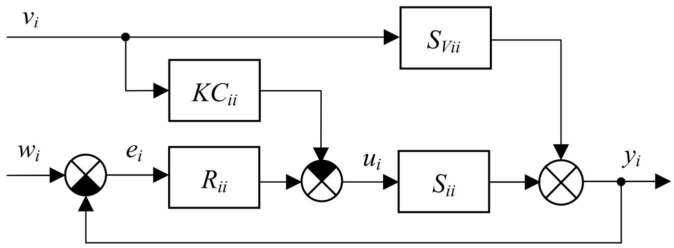

Fig.2 - Block diagram of SISO branched control loop with measuring of disturbance variable v_i

Transfer of correction members KC is then determined by using the following equation

$$KC_{ii} = \frac{S_{V,ii}}{S_{ii}} \qquad i = <1,\ldots,n>, S_{ii} \neq 0$$
$$KC_{ij} = 0 \qquad i \neq j \quad i, j = <1,\ldots,n> \qquad (9)$$

where $S_{V,ii}$ are separate members of transfer matrix of disturbance variables $\boldsymbol{G}_{SV}(s)$ and S_{ii} are separate members of transfer matrix of controlled plant $\boldsymbol{G}_S(s)$.

2.3 Control of MIMO control loop

One of the possible approaches to control of MIMO control loops is described in the following part of the paper. Generally it is possible to divide this problem into three parts

- Design of **main controllers** (diagonal controllers) by arbitrary synthesis method of SISO control loops, i.e. design of parameters of main controllers for n SISO control loops (R_{11}, R_{22}, …, R_{nn}). It is considered that original diagonal transfer functions S_{ii} ($i = 1, \ldots, n$) of transfer matrix of controlled plant $\boldsymbol{G}_S(s)$ are modified to diagonal transfer functions $S_{ii,x}$ ($i = 1, \ldots, n$). In these modified transfer functions influences of aside-from diagonal transfer functions of transfer matrix of controlled plant $\boldsymbol{G}_S(s)$, i.e. S_{ij} ($i \neq j$, $i, j = 1, \ldots, n$) on original diagonal transfer functions, i.e. S_{ii} ($i = 1, \ldots, n$) are included. Transfer functions $S_{ii,x}$, i.e $S_{11,x}$, $S_{22,x}$, $S_{33,x}$ etc. are determined from equation (10) by using relations (3) and (4)

$$S_{ii,x} = \sum_{j=1}^{n} S_{ij} \frac{s_{ij}}{s_{ii}} \qquad i = <1,\ldots,n>, s_{ii} \neq 0 \tag{10}$$

where s_{ii}, s_{ij} are algebraic supplements of separate elements of a transfer matrix of controlled plant $\boldsymbol{G_S}(s)$ and S_{ij} are separate members of a transfer matrix of controlled plant $\boldsymbol{G_S}(s)$.

- Ensuring of autonomy of control loop via **binding members** (5) of transfer matrix of controller $\boldsymbol{G_R}(s)$.
- Ensuring of invariance control loop via **correction members** *KC* by using of equations (8) or (9). Relation (9) can be used when influences of aside from diagonal elements of a transfer matrix of disturbance variables $\boldsymbol{G_{SV}}(s)$ are not dominant. In this case invariance of control loop is ensured by using n SISO branched control loops with measuring of disturbance variables.

The other strategy of control of MIMO control loop can be found e.g. in [6], [7], [8].

3 Simulation Verification of Described Method of Control of MIMO Control Loop

For simulation verification of described method of control of multivariable control loop the following two-variable control loop is considered (see Fig.1 and Fig.3).

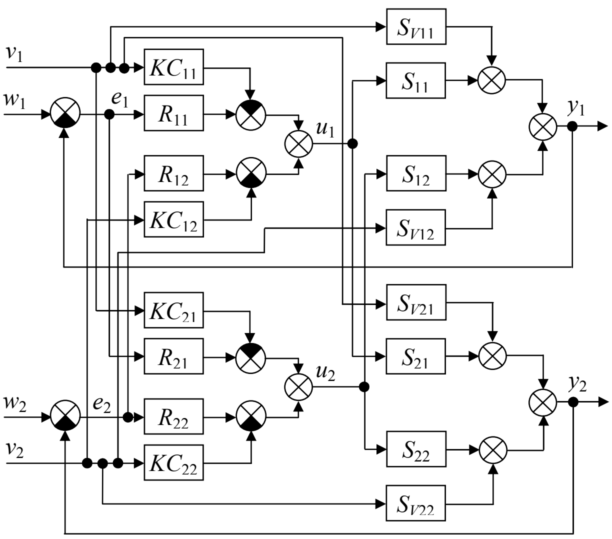

Fig.3 - Two-variable branched control loop with measurement of disturbance

3.1 Multi-variable controlled plant

It is considered two-variable controlled plant, i.e. controlled plant having two input signals and two output signals (see Fig.1 and Fig.3). The Laplace transform of an output (controlled) variable is given by the following relation

$$\boldsymbol{Y}(s) = \boldsymbol{G}_S(s)\boldsymbol{U}(s) + \boldsymbol{G}_{SV}(s)\boldsymbol{V}(s) \tag{11}$$

then

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \boldsymbol{G}_S(s)\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \boldsymbol{G}_{SV}(s)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{12}$$

where y_1, y_2 are controlled variables, u_1, u_2 are manipulated variable and v_1, v_2 are disturbance variables.

Transfer matrixes $\boldsymbol{G_S}(s)$ and $\boldsymbol{G_{SV}}(s)$ are considered in the following form

$$\boldsymbol{G}_S(s) = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{7.5}{s^2+5s+6} & \dfrac{6}{s^2+5s+6} \\ \dfrac{0.5}{s^2+3s+2} & \dfrac{1.5}{s^2+3s+2} \end{bmatrix} \tag{13}$$

$$\boldsymbol{G}_{SV}(s) = \begin{bmatrix} S_{V11} & S_{V12} \\ S_{V21} & S_{V22} \end{bmatrix} = \begin{bmatrix} \dfrac{0.8}{s^2+5s+6} & \dfrac{0.5}{s^2+5s+6} \\ \dfrac{0.95}{s^2+3s+2} & \dfrac{0.9}{s^2+3s+2} \end{bmatrix} \tag{14}$$

Step response and Nyquist diagram of transfer matrix of the controlled plant $\boldsymbol{G_S}(s)$ and of transfer matrix of disturbance variables $\boldsymbol{G_{SV}}(s)$ are presented in the following figures (see Fig. 4 - Fig.7).

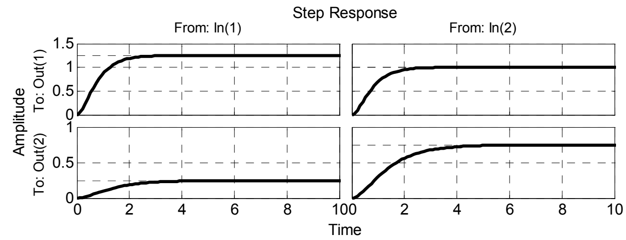

Fig.4 - Step response of transfer matrix of the controlled plant $\boldsymbol{G_S}(s)$

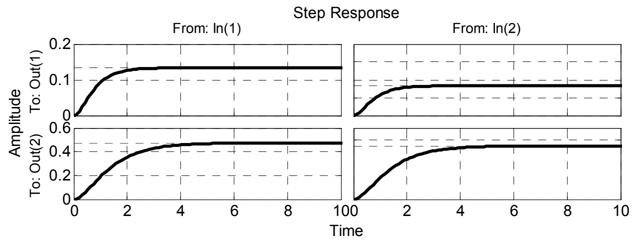

Fig.5 - Step response of transfer matrix of the disturbance variables $\boldsymbol{G_{SV}}(s)$

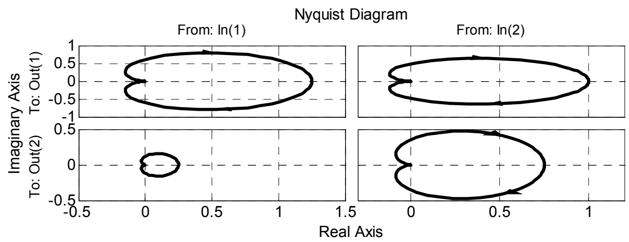

Fig. 6 - Nyquist diagram of transfer matrix of the controlled plant $\boldsymbol{G_S}(s)$

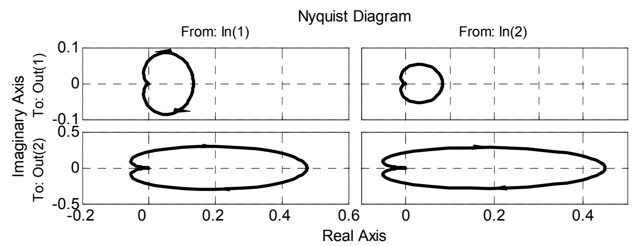

Fig. 7 - Nyquist diagram of transfer matrix of the disturbance variables $\boldsymbol{G_{SV}}(s)$

3.2 Control of two-variable control loop

In the next part of the paper the principal described in the paragraph 2.3 is used to control of two-variable control loop. First transfers of main controllers R_{11}, R_{22} are determined for modified diagonal transfer functions $S_{11,x}$ and $S_{22,x}$ (equation (10), (15)) then autonomy of control loop by using relation (4) is being solved and in the end fulfilment of the condition of invariance of control loop is ensured by using equation (8).

To calculation of transfer functions $S_{ii,x}$ was used, as mentioned above, equation (10), hence

$$\begin{aligned} S_{11,x}(s) &= S_{11}\frac{s_{11}}{s_{11}} + S_{12}\frac{s_{12}}{s_{11}} = S_{11} + S_{12}\left(-\frac{S_{21}}{S_{22}}\right) \\ S_{22,x}(s) &= S_{21}\frac{s_{21}}{s_{22}} + S_{22}\frac{s_{22}}{s_{22}} = S_{21}\left(-\frac{S_{12}}{S_{11}}\right) + S_{22} \end{aligned} \tag{15}$$

then

$$S_{11,x} = \frac{5.5}{s^2+5s+6} \quad S_{22,x} = \frac{1.1}{s^2+3s+2} \tag{16}$$

At design of main controllers parameters, which are diagonal elements of transfer matrix of controller $\boldsymbol{G}_R(s)$, the following methods were used

a) method of optimal module [9], [10]
b) pole placement method (by using polynomial approach for 1DoF configuration) [11], [12]

Beside above mentioned methods of design of parameters of main controllers (diagonal elements of transfer matrix of controller $\boldsymbol{G}_R(s)$) is possible to use also other SISO synthesis methods, e.g. Ziegler Nichols step response method, Chien, Hrones and Reswick method, the Cohen-Coon method, the method of desired model, the Naslin's method, the Whiteley method, the symmetrical optimum method, etc. [1], [9], [10], [13].

Binding members R_{ij}, which are aside-from diagonal elements of transfer matrix of controller $G_R(s)$, were calculated from relation (4). Then these members were determined from following relations

$$\begin{aligned} R_{12}(s) &= \frac{s_{21}}{s_{22}}R_{22} = -\frac{S_{12}}{S_{11}}R_{22} \\ R_{21}(s) &= \frac{s_{12}}{s_{11}}R_{11} = -\frac{S_{21}}{S_{22}}R_{11} \end{aligned} \tag{17}$$

Correction members KC_{ii} were determined from relation (8)

$$\begin{aligned} KC_{11} &= \frac{1}{det\,\boldsymbol{G}_S}\left(s_{11}S_{V11} + s_{21}S_{V21}\right) = \frac{S_{22}S_{V11} - S_{12}S_{V21}}{S_{11}S_{22} - S_{12}S_{21}} \\ KC_{12} &= \frac{1}{det\,\boldsymbol{G}_S}\left(s_{11}S_{V12} + s_{21}S_{V22}\right) = \frac{S_{22}S_{V12} - S_{12}S_{V22}}{S_{11}S_{22} - S_{12}S_{21}} \\ KC_{21} &= \frac{1}{det\,\boldsymbol{G}_S}\left(s_{12}S_{V11} + s_{22}S_{V21}\right) = \frac{S_{11}S_{V21} - S_{21}S_{V11}}{S_{11}S_{22} - S_{12}S_{21}} \\ KC_{22} &= \frac{1}{det\,\boldsymbol{G}_S}\left(s_{12}S_{V12} + s_{22}S_{V22}\right) = \frac{S_{11}S_{V22} - S_{21}S_{V12}}{S_{11}S_{22} - S_{12}S_{21}} \end{aligned} \tag{18}$$

Transfer matrix of controllers $\boldsymbol{G}_R(s)$ with utilization of chosen methods of synthesis were given by the following relation

a) method of optimal module

$$\boldsymbol{G}_R(s) = \begin{bmatrix} \dfrac{0.8182s + 1.6364}{s} & \dfrac{-1.455s - 1.4550}{s} \\ \dfrac{-0.2727s - 0.5455}{s} & \dfrac{1.8182s + 1.8182}{s} \end{bmatrix} \tag{19}$$

b) pole placement method

$$\boldsymbol{G}_R(s) = \begin{bmatrix} \dfrac{0.4109s + 0.8242}{s} & \dfrac{-0.137s - 0.2747}{s} \\ \dfrac{-0.7073s - 0.7056}{s} & \dfrac{0.8841s + 0.882}{s} \end{bmatrix} \tag{20}$$

Transfer matrix of correction members was given by the relation (8) (it was the same for all used SISO synthesis methods)

$$\boldsymbol{G}_{KC}(s) = \begin{bmatrix} -0.5455 & -0.5636 \\ 0.8152 & 0.7879 \end{bmatrix} \tag{21}$$

Mathematical programme MATLAB/SIMULINK [3] is used for simulation verification of proposed control method. Simulation scheme presented in the Fig.8 is used for these purposes.

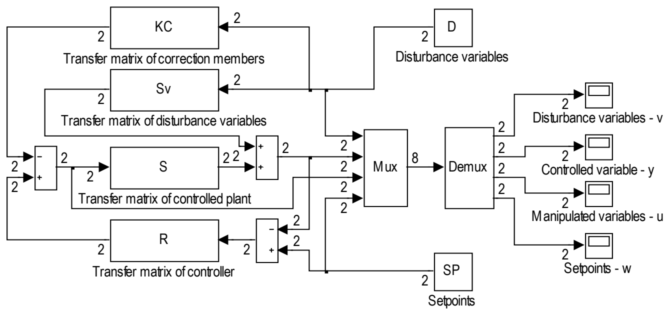

Fig.8 - Simulation scheme of two-variable control loop in the program MATLAB/SIMULINK

3.3 Simulation results and their evaluation

Simulation courses of two-variable control loop with utilization of chosen SISO synthesis methods, which are used at design of parameters of main controllers, are presented in the following figures (see Fig.9, Fig.10). The following parameters were chosen at all simulation experiments

- time vector of setpoints (t_{w1}, t_{w2}): [10, 50]
- vector of setpoints (w_1, w_2): [0.7, 0.7]
- time vector of disturbances (t_{v1}, t_{v2}): [30, 70]
- vector of disturbances (v_1, v_2): [0.5, 0.5]
- total time of simulation (t_S): 90
- time step (k): 0.02

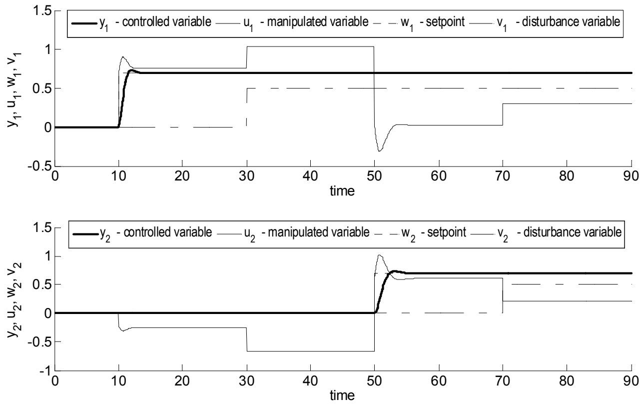

Fig.9 - Simulation course of control loop with utilization method of optimal module

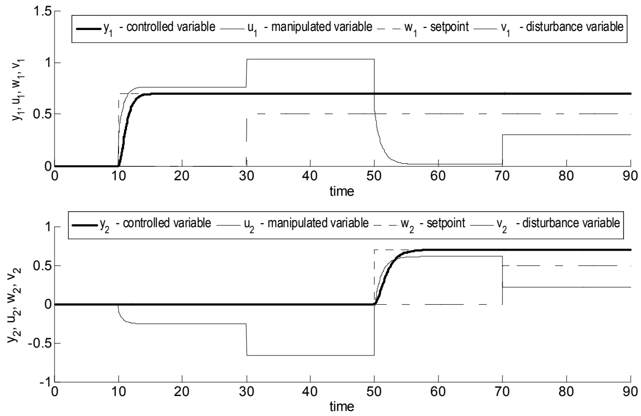

Fig.10 - Simulation course of control loop with utilization of pole placement method

It is obvious from the simulation courses of control loop shown in the Fig.9, Fig.10 and from other simulation experiments that the control loop is autonomous and also invariant. Autonomy of control loop was ensured via of aside-from-diagonal elements of transfer matrix of controller $\boldsymbol{G}_R$ (s), i.e. via binding members R_{ij}. Absolute invariance of control loop was ensured separate elements of transfer matrix of correction members $\boldsymbol{G}_{KC}$ (s), i.e. via correction members KC_{ij}.

4 Conclusion

The main aim of the paper was to describe one of the possible methods to control of MIMO control loops. The control method enables to use already known SISO synthesis method to design of main (diagonal) controllers. This method further combines classical way of ensuring of autonomy of control loop via binding members and the use of the correction members for ensuring absolute invariance of control loop. Simulation verification of proposed method of control was presented on two-variable control loop.

The proposed control method is valid under the following condition, i.e. this method can be used only for multi-variable controlled system with same number input and output signals. The future work will be focused on simulation verification of proposed method for concrete multi-variable controlled systems.

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