Adaptive Digital Smith Predictor

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Abtract: - Time-delays (dead times) are found in many processes in industry. Time-delays are mainly caused by the time required to transport mass, energy or information, but they can also be caused by processing time or accumulation. The contribution is focused on a design of algorithms for adaptive digital control for processes with time-delay. The algorithm is based on pole assignment approach. The program system MATLAB/SIMULINK was used for simulation verification of these algorithms.

Key-Words: - Time-delay; Smith predictor; Self-tuning control; ARX model; Recursive identification, Pole assignment

1 Introduction

Time-delays appear in many processes in industry and other fields, including economical and biological. They are caused by some of the following phenomena [1]:

- the time needed to transport mass, energy or information,
- the accumulation of time lags in a great numbers of low order systems connected in series,
- the required processing time for sensors, such as analyzers; controllers that need some time to implement a complicated control algorithms or process.

Consider a continuous time dynamical linear SISO (single input $u(t)$ – single output $y(t)$) system with time-delay T_d. The transfer function of a pure transportation lag is $e^{-T_d s}$ where s is complex variable. Overall transfer function with time-delay is in the form

$$G_d(s) = G(s)e^{-T_d s} \qquad (1)$$

where $G(s)$ is the transfer function without time-delay. Processes with significant time-delay are difficult to control using standard feedback controllers. When a high performance of the control process is desired or the relative time-delay is very large, a predictive control strategy must be used. The predictive control strategy includes a model of the process in the structure of the controller. The first time-delay compensation algorithm was proposed by Smith 1957 [2]. This control algorithm known as the Smith Predictor (SP) contained a dynamic model of the time-delay process and it can be considered as the first model predictive algorithm. Historically first modifications of time-delay algorithms were proposed for continuous-time (analogue) controllers. On the score of implementation problems, only the discrete versions are used in practice in this time. The adaptive digital SP based on pole assignment method is designed and verified by simulation in this paper.

2 Digital Smith Predictors

Although time-delay compensators appeared in the mid 1950s, their implementation with analogue technique was very difficult and these were not used in industry. Since 1980s digital time-delay compensators can be implemented. In spite of the fact that all these algorithms are implemented on digital platforms, most works analyze only the continuous case. The digital time-delay compensators are presented e.g. in [1], [3], [4]. The discrete versions of the SP and its modifications are suitable for time-delay compensation in industrial practice.

2.1 Structure of Digital SP

The block diagram of a digital SP is shown in Fig. 1. The function of the digital version is similar to the classical analogue version. The block $G_m(z^{-1})$ represents process dynamics without the time-delay and is used to compute an open-loop prediction. The difference between the output of the process y and the model including time-delay $\hat{y}$ is the predicted error $\hat{e}_p$ as shown is in Fig. 1 where u, w, e, e_s are the control signal, the reference signal, the error and the noise. If there are no modelling errors or disturbances, the error between the current process output and the model output will be null and the predictor output signal $\hat{y}_p$ will be the time-delay-free output of the

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process. Under these conditions, the controller $G_c(s)$ can be tuned, at least in the nominal case, as if the process had no time-delay. The primary (main) controller $G_c(z^{-1})$ can be designed by the different approaches (for example digital PID control or methods based on polynomial approach). The outward feedback-loop through the block $G_d(z^{-1})$ in Fig. 1 is used to compensate for load disturbances and modelling errors. The dash arrows indicate the tuned parts of the SP.

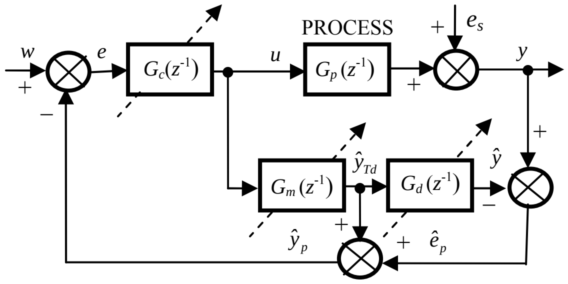

Figure 1: Block Diagram of a Digital Smith Predictor

Most industrial processes can be approximated by a reduced order model with some pure time-delay. Consider the following second order linear model with a time-delay

$$G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} z^{-d} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} z^{-d} \qquad (2)$$

for demonstration of some approaches to the design of the adaptive SP. The term z^{-d} represents the pure discrete time-delay. The time-delay is equal to dT_0 where T_0 is the sampling period. For the control of the second–order process (2) the individual parts of the controller are described by the transfer functions

$$G_m(z^{-1}) = \frac{z^{-1}\hat{B}(1)}{\hat{A}(z^{-1})}\,;\; G_d(z^{-1}) = \frac{z^{-d}\hat{B}(z^{-1})}{z^{-1}\hat{B}(1)} \qquad (3)$$

where $B(1) = \hat{B}(z^{-1})\big|_{z=1} = \hat{b}_1 + \hat{b}_2$ [5].

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Since $G_m(z^{-1})$ is the second-order transfer function, the main controller $G_c(z^{-1})$ can be a digital PID controller or suitable controller based on polynomial approach.

2.2 Digital Pole Assignment SP

The main controller $G_c(z^{-1})$ applied in this paper was designed using a polynomial approach. Polynomial control theory is based on the apparatus and methods of a linear algebra [6], [7]). The polynomials are the basic tool for a description of the transfer functions. They are expressed as the finite sequence of figures – the coefficients of a polynomial. Thus, the signals are expressed as infinite sequence of figures. The controller synthesis consists in the solving of linear polynomial (Diophantine) equations. The design of the controller algorithm is based on the general block scheme of a closed-loop with two degrees of freedom (2DOF) according to Fig. 2.

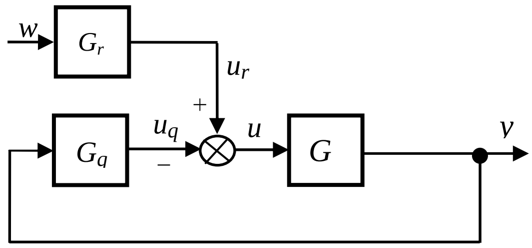

Figure 2: Block Diagram of a Closed Loop 2DOF Control System

The controlled process is given by the transfer function in the form

$$G_p(z^{-1}) = \frac{Y(z)}{U(z)} = \frac{B(z^{-1})}{A(z^{-1})} \qquad (4)$$

where A and B are the second-order polynomials. The controller contains the feedback part G_q and the feedforward part G_r. Then the digital controllers can be expressed in the form of a discrete transfer functions

$$G_r(z^{-1}) = \frac{R(z^{-1})}{P(z^{-1})} = \frac{r_0}{1 + p_1 z^{-1}} \qquad (5)$$

$$G_q(z^{-1}) = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{(1 + p_1 z^{-1})(1 - z^{-1})} \qquad (6)$$

According to the scheme presented in Fig. 2 (for $e_s = 0$), the output y can be expressed as

$$Y(z^{-1}) = \frac{G_p(z) G_r(z)}{1 + G_p(z) G_q(z)} W(z^{-1}) \qquad (7)$$

Upon substituting from Equation (4) - (6) into Equation (7) it yields

$$Y(z^{-1}) = \frac{B(z^{-1})R(z^{-1})}{A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1})} W(z^{-1}) \qquad (8)$$

where

$$A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1}) = D(z^{-1}) \qquad (9)$$

is the characteristic polynomial.

The procedure leading to determination of polynomials Q, R and P in (5) and (6) can be briefly described as follows [8]. A feedback part of the controller is given by a solution of the polynomial Diophantine equation (9). An asymptotic tracking is provided by a feedforward part of the controller given by a solution of the polynomial Diophantine equation

$$S(z^{-1})D_w\left(z^{-1}\right)+B(z^{-1})R(z^{-1})=D(z^{-1}) \qquad (10)$$

For a step-changing reference signal value $D_w\left(z^{-1}\right)=1-z^{-1}$ holds and S is an auxiliary polynomial which does not enter into controller design. A feedback controller to control a second-order system without time-delay will be derived from Equation (9). A system of linear equations can be obtained using the uncertain coefficients method

$$\begin{bmatrix} \hat{b}_1 & 0 & 0 & 1 \\ \hat{b}_2 & \hat{b}_1 & 0 & \hat{a}_1-1 \\ 0 & \hat{b}_2 & \hat{b}_1 & \hat{a}_2-\hat{a}_1 \\ 0 & 0 & b_2 & -\hat{a}_2 \end{bmatrix}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ p_1 \end{bmatrix}=\begin{bmatrix} d_1+1-\hat{a}_1 \\ d_2+\hat{a}_1-\hat{a}_2 \\ d_3+\hat{a}_2 \\ d_4 \end{bmatrix} \qquad (11)$$

where the characteristic polynomial is chosen as

$$D\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2}+d_3z^{-3}+d_4z^{-4} \qquad (12)$$

For a step-changing reference signal value it is possible to solve Equation (10) by substituting $z = 1$

$$R=r_0=\frac{D(1)}{B(1)}=\frac{1+d_1+d_2+d_3+d_4}{b_1+b_2} \qquad (13)$$

The 2DOF controller output is given by

$$\begin{aligned} u(k)=r_0w(k)-q_0y(k)-q_1y(k-1)-q_2y(k-2)+ \\ +(1+p_1)u(k-1)+p_1u(k-2) \end{aligned} \qquad (14)$$

3 Recursive Identification Procedure

The regression (ARX) model of the following form

$$y(k)=\boldsymbol{\Theta}^T(k)\boldsymbol{\Phi}(k)+e_s(k) \qquad (15)$$

is used in the identification part of the designed controller algorithms, where

$$\boldsymbol{\Theta}^T(k)=\begin{bmatrix} a_1 & a_2 & b_1 & b_2 \end{bmatrix} \qquad (16)$$

is the vector of model parameters and

$$\boldsymbol{\Phi}^T(k-1)=\begin{bmatrix} -y(k-1) & -y(k-2) & u(k-1) & u(k-2) \end{bmatrix} \qquad (17)$$

is the regression vector. The non-measurable random component $e_s(k)$ is assumed to have zero mean value $E[e_s(k)] = 0$ and constant covariance (dispersion) $R = E[e_s^2(k)]$.

All digital adaptive SP controllers use the algorithm of identification based on the Recursive Least Squares Method (RLSM) extended to include the technique of directional (adaptive) forgetting. Numerical stability is improved by means of the LD decomposition [8], [9]. This method is based on the idea of changing the influence of input-output data pairs to the current estimates. The weights are assigned according to amount of information carried by the data.

4 Simulation Verification Adaptive Digital SP Controller Algorithms

Simulation is a useful tool for the synthesis of control systems, allowing one not only to create mathematical models of a process but also to design virtual controllers in a computer. The mathematical models provided are sufficiently close to a real object that simulation can be used to verify the dynamic characteristics of control loops when the structure or parameters of the controller change. The models of the processes may also be excited by various random noise generators which can simulate the stochastic characteristics of the processes noise signals with similar properties to disturbance signals measured in the machinery.

The above mentioned SP controllers are not suitable for the control of unstable processes. Therefore, three types of processes were chosen for simulation verification of digital adaptive SP controller algorithms. Consider the following continuous-time transfer functions:

1) Stable non-oscillatory $G_1(s)=\dfrac{2}{(s+1)(4s+1)}e^{-4s}$

2) Stable oscillatory $G_2(s)=\dfrac{2}{4s^2+2s+1}e^{-4s}$

3) With non-minimum phase

$$G_3(s)=\frac{-5s+1}{(s+1)(4s+1)}e^{-4s}.$$

Let us now discretize them a sampling period $T_0 = 2\,\mathrm{s}$. The discrete forms of these transfer functions are (see Equation (2))

$$G_1\left(z^{-1}\right)=\frac{0.4728z^{-1}+0.2076z^{-2}}{1-0.7419z^{-1}+0.0821z^{-2}}z^{-2}$$

$$G_2\left(z^{-1}\right)=\frac{0.6806z^{-1}+0.4834z^{-2}}{1-0.7859z^{-1}+0.3679z^{-2}}z^{-2}$$

$$G_3\left(z^{-1}\right)=\frac{-0.5489z^{-1}+0.8897z^{-2}}{1-0.7419z^{-1}+0.0821z^{-2}}z^{-2}$$

A simulation verification of proposed design was performed in MATLAB/SIMULINK environment. A typical control scheme used is depicted in Fig. 3. This scheme is used for systems with time-delay of two sample steps. Individual blocks of the Simulink scheme correspond to blocks of the general control scheme presented in Fig. 1. Blocks Compensator 1 and Compensator 2 are parts of the SP and they correspond to $G_m\left(z^{-1}\right)$ and $G_d\left(z^{-1}\right)$ blocks of Fig. 2 respectively. The control algorithm is encapsulated in Main Pole Assignment Controller which corresponds to $G_c\left(z^{-1}\right)$ Fig. 1 block. The Identification block performs the on-line identification of controlled system and outputs the estimates of 2nd order ARX model (a1, b1, a2, b2) parameters.

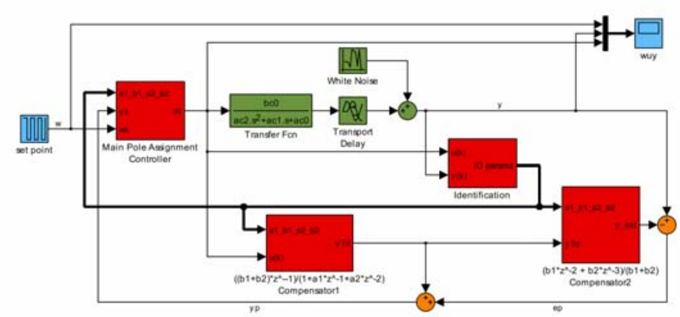

Figure 3: Simulink control scheme

The internal structure of the Main Pole Assignment Controller block is shown in Fig. 4. Block MATLAB Fcn is the heart of the controller. The inputs to this function are current ARX estimates, current and previous values of process without time-delay, reference signal as well as previous control values and sample time. The MATLAB Fcn is a standard m-function which carries out desired control algorithm as described in Section 2.

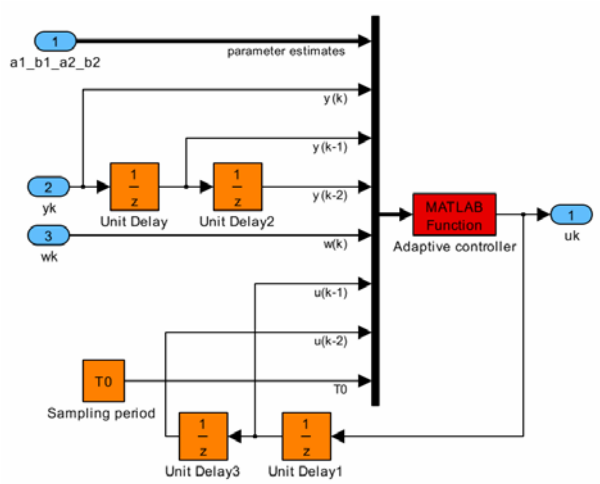

Figure 4: Internal structure of the controller

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The on-line identification part of the scheme, which is represented by block Identification block in Fig. 3, uses several parameters that are entered via standard SIMULINK dialog. This dialog is presented in Fig. 5.

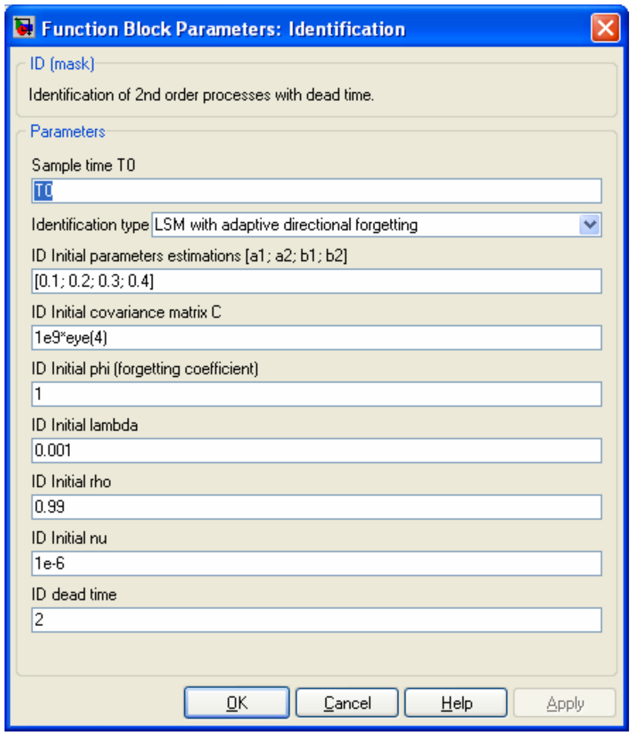

Figure 5: Dialog for setting identification parameters

The most important parameters form the point of view of the problem this papers is coping with are sample time, initial parameters estimations and dead time. The dead time is not entered in time units but in sample times. The other parameters affect the method used to compute ARX model and their detailed description can be found in Bobál et al. 2005 [7].

5 Simulation Results

The configuration for simulation verification of the designed adaptive digital SP algorithm was chosen as follows:

- The control loops were verified in the non-adaptive versions without a random noise.
- All control loops were verified in the adaptive versions with a random noise. Firstly, without a priori information (the initial values of the model parameter estimates were chosen randomly). Secondly, using a priori information (the initial estimates were chosen based on the previous experiments).

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- The outputs of the process models were influenced by White Noise Generator with mean value E = 0 and covariance $R = 10^{-4}$.

5.1 Simulation Verification (Non-adaptive Version)

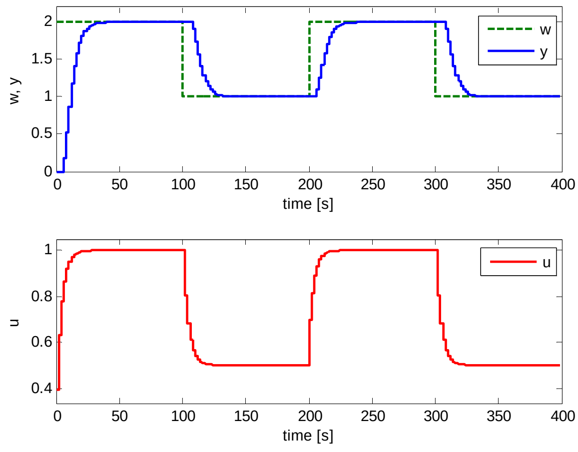

Figure 6: Control of the Model $G_1\left(z^{-1}\right)$

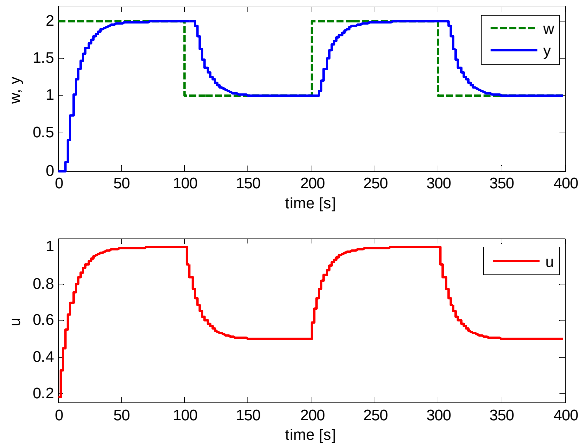

Figure 7: Control of the Model $G_2\left(z^{-1}\right)$

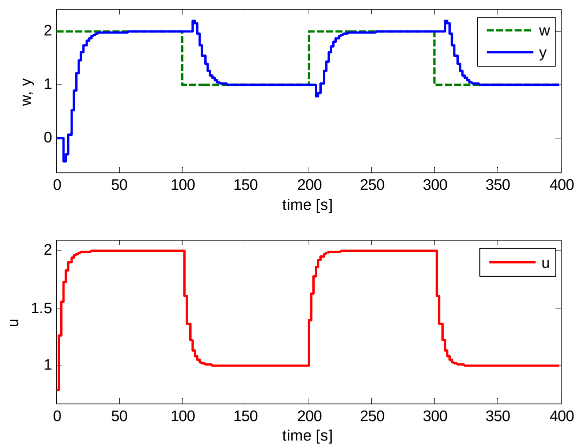

Figure 8: Control of the Model $G_3\left(z^{-1}\right)$

Figs. 6 - 8 illustrate the simulation non-adaptive control performance using Pole-Assignment controller (14). A suitable pole assignment was chosen on the basis of previous experiments. The control quality is very good (the process output y is without overshoot and controller output u has non-oscillatory course).

5.2 Simulation Verification (Adaptive Version)

Figs. 9 - 12 illustrate the simulation control performance using adaptive Pole-Assignment controller (14). From Figs. 9 and 10 (the control of the stable model $G_1\left(z^{-1}\right)$) it is obvious that the control process is not dependent on knowledge of a priori information (the control courses in both cases are practically identical).

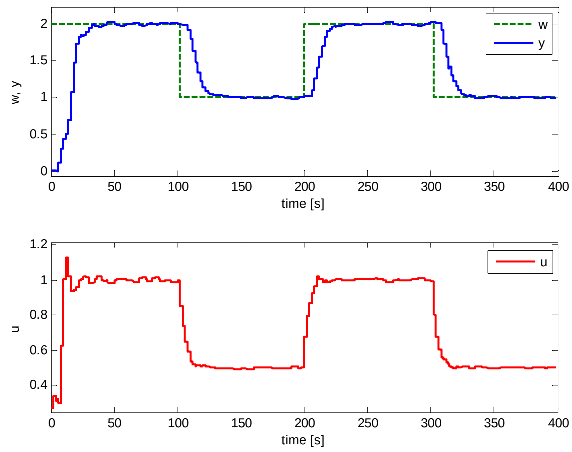

Figure 9: Control of model $G_1\left(z^{-1}\right)$ (without a priori information)

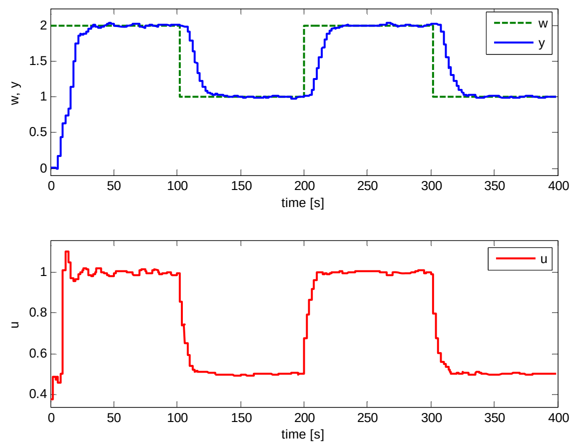

Figure 10: Control of the model $G_1\left(z^{-1}\right)$ (with a priori information)

Fig. 11 illustrates the simulation control performance of the stable oscillatory model $G_2\left(z^{-1}\right)$.

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The control process is relatively slow without overshot of *y* and *u* (it is the cautious adaptive controller).

Fig. 12 illustrates the simulation control performance of the non-minimum phase model $G_3\left(z^{-1}\right)$. The control process is good after initial part.

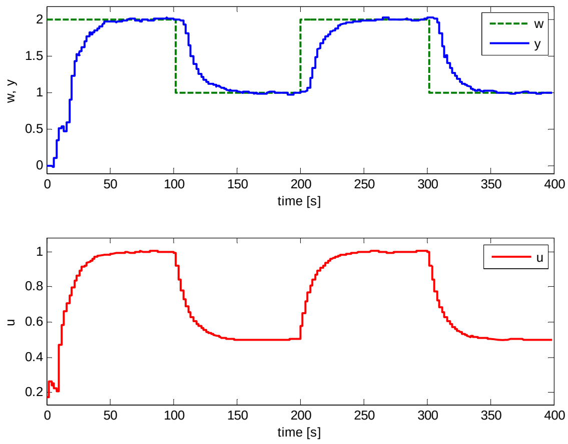

Figure 11: Control of the model $G_2\left(z^{-1}\right)$ (with a priori information)

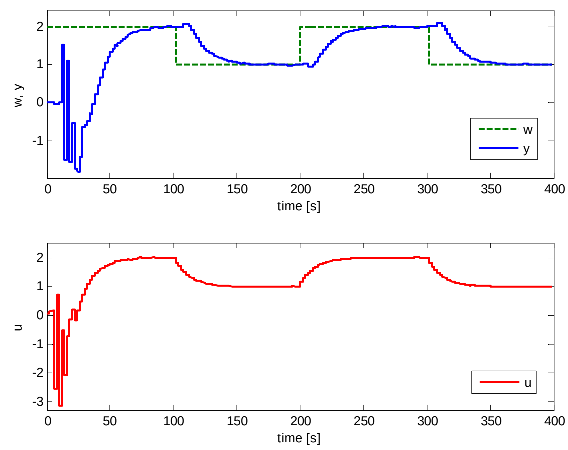

Figure 12: Control of the model $G_3\left(z^{-1}\right)$ (with a priori information)

6 Conclusion

Digital Adaptive Smith Predictor algorithms for control of processes with time-delay based on polynomial design (pole assignment) were proposed. The polynomial controllers were derived purposely by analytical way (without utilization of numerical methods) to obtain algorithms with easy implementability in industrial practice. Three models of control processes were used for simulation verification (the stable non-oscillatory, the stable oscillatory and the non-minimum phase). Achieved results of simulation verification can be regard as the first suppositions for usage of the proposed adaptive SP controllers for implementation in real time conditions.

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