The use of the Summary Graph to Finding a Formula for Current Transferring

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Abstract: - The paper deals with the symbolic solution of the switched current circuits. As is described, the full graph method of the solution can be used for finding relationships expressing current transfer, too. The summa MC-graph is constructed using two-graphs method in two-phase switching. By comparing the matrix form with results of the Mason’s formula are derived relations for current transfer.

Key-Words: - Switched current, two phases, two-graph, Mason’s formula, relations for current transfer, summary MC-graph

1 Introduction
Current memory cell can be represented by graph, too [3]. Graph methods give results in a symbolic form, which makes it can be used to finding general relations. One option is to find general relations for current transfer in switched current circuits in two-phase switching. General relations are used for the calculation method of matrix calculus of final solution. This method will be demonstrated in the example. The summa MC-graph is constructed using two-graphs method in two-phase switching, two-graphs method for switched capacitor circuits is described in [4].

2 Calculation of the transmissions
2.1 Calculation of the transmissions from the summary graph
A circuit with a switched current has got for example the schematic wiring diagram shown in Fig.1. Circuit consist of two capacitors C, and three field effect transistors T.

A solution of a switched current circuit by the two-graph method of a summary MC-graph constructed on the basis of two-graphs will be shown in Fig.3. First we draw a partial diagram for the even phase and the odd phases separately by the algorithm described in [4]. These two diagrams for individual phases are in Fig.2.

Fig. 1. Circuit consist of two capacitors C, and three field effect transistors T.

Fig. 2 Graphs for even and odd phases.
To the diagrams for individual phases, we can assign directed graphs. V-graph and I-graph of the FET (ie. for transistors T\textsubscript{1} – T\textsubscript{3} in Fig.2) are in the Table 1.

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<th>Schematic diagram</th>
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<th>Matrix</th>
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<td><img src="image3" alt="Diagram" /></td>
<td>$V_{GE}$ : $V_{CE}$ : $I_c$ : $y_{21}$</td>
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Table 1. Two-graph of the FET

For both even and odd phases it is necessary to draw a special voltage graph (ie. V-graph) and current graph (ie. I-graph). These graphs are shown in Fig.3.

![Diagram](image4)

Fig. 3 Finding common skeletons of the V-graph and I-graph

A summary MC-graph is now constructed by first finding the incomplete common skeletons of the V-graph and the I-graph in the even phase and in the odd one. In the even phase there is one incomplete common skeleton for example formed by the $y_{21}$, $y_{22}$, $y_{23}$, thus obtained loop is with the transfer $y_{21}^{(2)} + y_{22}^{(1)} + y_{23}^{(2)}$.

Thus obtained summary Mason-Coates graph is in Fig.4.
The current transfers will now be obtained from an extended graph, i.e. a graph must be extended to two branches as it is shown in Fig. 4: the first branch from the input node \( I_{\text{INP}} \) to the node \( 1E \) with transfer \( 1 \) and the second branch from the node \( 2E \) to the node \( I_{\text{OUT}} \). The transfer is equal to the transmission of its own loop at the output node. The summary graph is then evaluated by means of the Mason’s rule [1], for example transfer \( \frac{I_{2E}}{I_{1E}} \) is (1).

\[
\begin{align*}
I_{\text{OUT}} & = \frac{I_{2E}}{I_{1E}} = \sum \frac{p_{(i)}}{v_{-} - \sum_{k} s_{(k)}y_{(k)}} = \\
& = \frac{1(-y_{21}^{(1)})y_{22}^{(1)}(y_{21}^{(2)} + y_{22}^{(2)} + y_{21}^{(3)})y_{22}^{(3)}}{(y_{22}^{(1)} + y_{21}^{(2)} + y_{21}^{(3)})y_{22}^{(3)}(y_{21}^{(2)} + y_{22}^{(2)} + y_{21}^{(3)})y_{22}^{(3)} - (-z^{-1}y_{21}^{(2)})y_{22}^{(3)}y_{22}^{(3)}} \\
& = \frac{1(-y_{21}^{(1)})y_{22}^{(1)} + y_{22}^{(2)} + y_{21}^{(3)}}{(y_{22}^{(1)} + y_{21}^{(2)} + y_{21}^{(3)})y_{22}^{(3)} + y_{21}^{(2)} + y_{21}^{(3)} - z^{-1}y_{21}^{(2)}y_{21}^{(3)}}
\end{align*}
\]

(1)

**Summary Mason-Coates graph for transfer** \( \frac{I_{4O}}{I_{1E}} \)

is in Fig. 4, transfer is (2).

\[
\begin{align*}
I_{\text{OUT}} & = \frac{I_{4O}}{I_{1E}} = \sum \frac{p_{(i)}}{v_{-} - \sum_{k} s_{(k)}y_{(k)}} = \\
& = \frac{1(-z^{-1}y_{21}^{(3)})y_{22}^{(1)}(y_{21}^{(2)} + y_{22}^{(2)} + y_{21}^{(3)})y_{22}^{(3)}}{(y_{22}^{(1)} + y_{21}^{(2)} + y_{21}^{(3)})y_{22}^{(3)}(y_{21}^{(2)} + y_{22}^{(2)} + y_{21}^{(3)})y_{22}^{(3)} - (-z^{-1}y_{21}^{(2)})y_{22}^{(3)}y_{22}^{(3)}} \\
& = \frac{1(-z^{-1}y_{21}^{(3)})y_{22}^{(1)} + y_{22}^{(2)} + y_{21}^{(3)}}{(y_{22}^{(1)} + y_{21}^{(2)} + y_{21}^{(3)})y_{22}^{(3)} + y_{21}^{(2)} + y_{21}^{(3)} - z^{-1}y_{21}^{(2)}y_{21}^{(3)}}
\end{align*}
\]

(2)
The Mason-Coates graphs for transfers $I_{OE}$ (3) and $I_{IO}$ (4) are in Fig. 6 and Fig. 7.

**Fig. 6** The summa MC-graph for transfer $I_{4E}/I_{1O}$

$\frac{I_{OJ}}{I_{INP}} = \frac{I_{JE}}{I_{IO}} = \frac{\sum \Delta_{i}(1) \sum S_{i}X_{i}(k)}{V - \sum S_{i}X_{i}(k)} = \frac{1 \cdot \frac{1}{2} y_{21}^{(1)} (-\frac{1}{2} y_{21}^{(3)}) y_{22}^{(3)} y_{21}^{(3)}}{(y_{22}^{(1)} + y_{22}^{(2)}) y_{22}^{(3)} (y_{21}^{(1)} + y_{21}^{(2)} + y_{21}^{(3)}) y_{22}^{(3)} - \frac{1}{2} y_{21}^{(2)} - \frac{1}{2} y_{21}^{(1)} y_{22}^{(3)} y_{22}^{(3)} - z^{-1} y_{21}^{(3)} y_{22}^{(3)} y_{22}^{(3)}}$

$= - \frac{\frac{1}{2} y_{21}^{(1)} y_{22}^{(3)} y_{22}^{(3)}}{(y_{22}^{(1)} + y_{22}^{(2)}) (y_{21}^{(1)} + y_{21}^{(2)} + y_{21}^{(3)}) - z^{-1} y_{21}^{(3)} y_{22}^{(3)} y_{22}^{(3)}}$

**Fig. 7** The summa MC-graph for transfer $I_{4O}/I_{1O}$

$\frac{I_{OJ}}{I_{INP}} = \frac{I_{JO}}{I_{IO}} = \frac{\sum \Delta_{i}(3) \sum S_{i}X_{i}(k)}{V - \sum S_{i}X_{i}(k)} = \frac{1 \cdot \frac{1}{2} y_{21}^{(1)} (-\frac{1}{2} y_{21}^{(3)}) y_{22}^{(3)} y_{21}^{(3)}}{(y_{22}^{(1)} + y_{22}^{(2)}) y_{22}^{(3)} (y_{21}^{(1)} + y_{21}^{(2)} + y_{21}^{(3)}) y_{22}^{(3)} - \frac{1}{2} y_{21}^{(2)} - \frac{1}{2} y_{21}^{(1)} y_{22}^{(3)} y_{22}^{(3)} - z^{-1} y_{21}^{(3)} y_{22}^{(3)} y_{22}^{(3)}}$

$= - \frac{\frac{1}{2} y_{21}^{(1)} y_{22}^{(3)} y_{22}^{(3)}}{(y_{22}^{(1)} + y_{22}^{(2)}) (y_{21}^{(1)} + y_{21}^{(2)} + y_{21}^{(3)}) - z^{-1} y_{21}^{(3)} y_{22}^{(3)} y_{22}^{(3)}}$

### 2.2 Calculation of the transmission from the model of the circuit

Relations for transfers of currents (1) to (4) have been expressed as fractions in which conductivities $y_{21}^{(1)}$ and $y_{22}^{(1)}$ are found both in the numerator and in the denominator. That shows a theoretical possibility to express these transfers as ratios determinants of certain algebraic complements of matrices constructed from the mentioned conductivities $y_{21}^{(1)}$ and $y_{22}^{(1)}$. In order to find such relations, a linearized diagram of the circuit from the Fig. 1. is drawn in Fig. 8.
When the switches are on, this circuit has two nodes, so it can be described by two equations for currents $I_1$, $I_2$ constructed by means of the Kirchhoff’s law. However, as the currents can occur in even and odd phases, the total of four equations will be constructed (5). The transistors are modelled by the VCT elements with the control voltages $V_{1E}, V_{1O}$. Next, there is the voltage $V_i$ in the circuit, again in both phases, i.e. $V_{1E}$ and $V_{2O}$.

The corresponding equation system (5)

$$I_{1E} = z^{-1/2} y_{11}^{(1)} V_{1O} + y_{12}^{(1)} V_{1E} + y_{11}^{(2)} V_{1E} + y_{21}^{(2)} V_{1E}$$

$$I_{1O} = y_{11}^{(1)} V_{1O} + y_{12}^{(1)} V_{1O} + y_{22}^{(2)} V_{1E} + y_{11}^{(3)} V_{1E} + z^{-1/2} y_{21}^{(2)} V_{1E}$$

$$I_{2E} = y_{11}^{(1)} V_{1E} + y_{12}^{(1)} V_{2E}$$

$$I_{2O} = z^{-1/2} y_{21}^{(3)} V_{1E} + y_{22}^{(3)} V_{2O}$$

can be rewritten to a matrix form (6).

$$
\begin{bmatrix}
V_{1E} & V_{2E} & V_{1O} & V_{2O}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{1}{z^{1/2}} y_{21}^{(1)} & y_{22}^{(1)} & 0 & 0 \\
y_{21}^{(3)} & y_{22}^{(3)} & 0 & 0 \\
z^{-1/2} y_{21}^{(2)} & 0 & y_{22}^{(2)} + y_{21}^{(2)} & 0 \\
z^{-1/2} y_{21}^{(3)} & 0 & 0 & y_{22}^{(3)}
\end{bmatrix}
$$

By comparing with the matrix (6) it is now apparent that in the numerator there is an algebraic complement of this matrix (6) created out of this matrix by leaving out the rows 1E and 2O and the columns 2E and 2O, symbolically written $\Delta_{1E,2O:2E,2O}$. In the denominator, there is then the algebraic complement of the matrix (6) created out of this matrix by leaving out the rows 2E and 2O and the columns 2E and 2O, symbolically written $\Delta_{2E,2O:2E,2O}$.

The number of omitted odd indices, in the numerator is the algebraic complement of the matrix (6) created out of this matrix by leaving out the rows 1E and 2O and the columns 2E and 2O, symbolically written $\Delta_{1E,2O:2E,2O}$. In the denominator, there is then the algebraic complement of the matrix (6) created out of this matrix by leaving out the rows 2E and 2O and the columns 2E and 2O, symbolically written $\Delta_{2E,2O:2E,2O}$.

$$
\Delta_{1E,2O:2E,2O} = -z^{1/2} y_{21}^{(1)} y_{22}^{(1)} + y_{22}^{(2)} + y_{21}^{(2)}
$$

$$
\Delta_{2E,2O:2E,2O} = z^{-1/2} y_{21}^{(3)} y_{22}^{(3)}
$$

According to the theory of multiple algebraic complements, the sign (-1) in the numerator gives the number of omitted odd indices, in the numerator.
the row 1E is omitted, which is the first (odd index) row in the matrix, while the remaining row 2O is even in the row as well as the omitted column 2E, which is the second column in the matrix, and the column 20, which is the fourth column in the matrix.

In the denominator the omitted rows 2E and 2O and columns 2E and 2O are the second and fourth rows and columns in the matrix, i.e., they always have even indices, which then corresponds to a positive sign.

From the graph solution it is also possible to derive relations for the remaining current transfers. If the numerator of the relation (2) i.e. 

\[- z^{-\frac{1}{2}} y_{21}^{(3)} (y_{2}^{(1)} + y_{2}^{(2)} + y_{21}^{(1)}) \]

i.e. only the elements \(- z^{-\frac{1}{2}} y_{21}^{(3)}\) and \(y_{2}^{(1)} + y_{2}^{(2)} + y_{21}^{(1)}\), are written into the corresponding position in the conductivity matrix (2) the relation (9) will hold.

\[- z^{-\frac{1}{2}} y_{21}^{(3)} (y_{2}^{(1)} + y_{2}^{(2)} + y_{21}^{(1)}) =
\]

\[= \Delta_{1E,2E,3E,4E} = \begin{vmatrix}
0 & 0 \\
1 z^{-\frac{1}{2}} y_{21}^{(3)} & 0 
\end{vmatrix}
\]

For the transfer of the currents \(I_{4O} \) we can write the relation (10) if the numerator stayed the same.

\[\frac{I_{4O}}{I_{1E}} = \frac{\Delta_{1E,2E,3E,4E}}{\Delta_{2E,4O,2O,4O}}
\]

(10)

Analogically for the transfer \(I_{4O} \) \(I_{1O}\) the relation (12) from (11) holds if the numerator of the relation (11) \(z^{-\frac{1}{2}} y_{21}^{(3)} y_{21}^{(3)}\) i.e. the elements \(z^{-\frac{1}{2}} y_{21}^{(3)}\) and \(y_{21}^{(3)}\), are written into their positions given by the matrix (6),

\[= \Delta_{1O,2O,2E,2O} = \begin{vmatrix}
0 & \frac{1}{z^{-\frac{1}{2}}} y_{21}^{(3)} \\
1 z^{-\frac{1}{2}} y_{21}^{(3)} & 0 
\end{vmatrix}
\]

(11)

\[\frac{I_{4E}}{I_{1O}} = \frac{\Delta_{1O,2O,2E,2O}}{\Delta_{2E,2O,4E,4O}}
\]

(12)

and for \(I_{4O} \) relation (14) from (13).

\[\frac{I_{4O}}{I_{1O}} = \frac{\Delta_{1O,2O,2E,2O}}{\Delta_{2E,2O,4E,4O}}
\]

(14).

The numerators stay unchanged.

Another possible way of writing the above mentioned derived relations for current transfers by means of multiple algebraic complements in individual phases using the symbols stated in [1] is then (15)

\[\frac{I_{4E}}{I_{1E}} = \frac{\Delta_{1E,2E,3E,4E}}{\Delta_{2E,4E,2O,4O}} \frac{I_{4O}}{I_{1E}} = \frac{\Delta_{1E,2E,3E,4E}}{\Delta_{2E,4E,2O,4O}}
\]

\[\frac{I_{4E}}{I_{1O}} = \frac{\Delta_{1O,2O,2E,2O}}{\Delta_{2E,4E,2O,4O}} \frac{I_{4O}}{I_{1O}} = \frac{\Delta_{1O,2O,2E,2O}}{\Delta_{2E,4E,2O,4O}}
\]

where the symbol > 0 means adding a given row (or column) to the zero row (or column) of the matrix. The results (7), (10), (12), (14) are identical with the results (8.49), (8.50), (8.51), (8.52) published in [2]. But finding relations (8.49), (8.50), (8.51), (8.52) in [2] requires the use of multiple (double and triple) algebraic complements. This calculation is somewhat more difficult than described graphs method.

4 Conclusion

A unified method in analyzing switched current circuits is presented. The advantages of this approach are in its uniformity in deriving results.
from graph. The two-graph method is applied to assembly of the MC-graph. A clearly arranged set of graphs derived for different types of switching circuits can be used for finding a formula for current transferring. By comparing the results from the matrix and results obtained from the Mason's formula are derived general relations for current transfers.

It is not known that this method of graphs finding the transfer was published.

References: