Analytical Solution of a Two-dimensional Double-fin Assembly

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Abstract: This paper deals with the three-dimensional mathematical formulation of a transient problem for the heat-exchanger consisting of rectangular fins attached to both sides of a plane wall (double-fin assembly). The problem is reduced by conservative averaging method to the two-dimensional system. Analytical solution based on Green function approach is proposed. This solution is obtained in the form of the 2^{nd} kind Fredholm integral equations.

Key–Words: Transient, Three-dimensional, Heat exchange, Rectangular fin, Exact analytical solution.

1 Introduction

Extended surface is used specially to enhance the heat transfer between a solid and surrounding medium. Such an extended surface is termed a fin. Extended surfaces are widely examined in [8] - [10]. The rate of heat transfer is directly proportional to the extent of the wall surface, the heat transfer coefficient and to the temperature difference between solid and the surrounding medium. Finned surfaces are widely used in many applications such as air conditioners, aircrafts, chemical processing plants, etc. In [4] is considered performance of a heat-exchanger consisting of rectangular fins attached to both sides of plane wall. In [3, 4] works one-dimensional steady-state double-fin assembly problem is compared with the single-fin assembly. In paper [2] mathematical three-dimensional formulation of transient problem for one element with one rectangular fin is examined, reduce it by conservative averaging method [5] to the system of three heat equations with linear sink terms. In [6] was considered exact analytical solution for two-dimensional steady-state process for system with one rectangular fin by the method of Green function [1]. In [7] three-dimensional exact analytical solution for the distribution of the temperature field in the wall with one rectangular fin in the form of the 2^{nd} kind Fredholm integral equation is constructed.

2 Mathematical Formulation of 3D Problem

In this section we present mathematical three-dimensional formulation of a transient problem for one element with two rectangular fins attached to both sides. We will use following dimensionless arguments, parameters:

$$x = \frac{x'}{B+R}; y = \frac{y'}{B+R};$$
$$z = \frac{z'}{B+R}; l = \frac{L}{B+R};$$
$$l_1 = \frac{L_1}{B+R}; w = \frac{W}{B+R};$$
$$b = \frac{B}{B+R}; \delta = \frac{D}{B+R};$$
$$\beta = \frac{h(B+R)}{k}; \beta_0 = \frac{h_0(B+R)}{k}$$

and dimensionless temperatures:

$$\overline{V} = \frac{\tilde{V}(x,y,z,t) - T_a}{T_b - T_a};$$
$$\overline{V}_0 = \frac{\tilde{V}_0(x,y,z,t) - T_a}{T_b - T_a};$$
$$\overline{V}_1 = \frac{\tilde{V}_1(x,y,z,t) - T_a}{T_b - T_a};$$
$$\overline{\Theta} = \frac{\tilde{\Theta}(x,y,z,t) - T_a}{T_b - T_a};$$
$$\overline{\Theta}_0 = \frac{\tilde{\Theta}_0(x,y,z,t) - T_a}{T_b - T_a}.$$

We have introduced following dimensional thermal and geometrical parameters: k - heat conductivity coefficient for the wall and fins (the same for all components), h - heat exchange coefficient from the right side (cold side), h_0 - heat exchange coefficient

ISBN: 978-1-61804-026-8

from the left side (hot side), $2B$ - fin width (thickness), L - right fin length, L_1 - left fin length, D - thickness of the wall, W - walls' width (length), $2R$ - distance between two fins (fin spacing). Further, $\tilde{\Theta}_0$ is the surrounding (environment) temperature on the left (hot) side (the heat source side) of the wall, $\tilde{\Theta}$ - the surrounding temperature on the right (cold - the heat sink side) of the wall. Finally, $\tilde{V}_0(x,y,z,t), \tilde{V}(x,y,z,t), \tilde{V}_1(x,y,z,t)$ are the dimensional temperatures in the wall, right fin and left fin where T_a, T_b are integral averaged environment temperatures over appropriate edges which are time independent:

$$T_a = \frac{\int_0^w dz \cdot \int_b^1 \Theta(D,y,z)dy}{w(1+l)} + \frac{\int_\delta^{\delta+l} dx \cdot \int_0^w \Theta(x,B,z)dz}{w(1+l)} + \frac{\int_0^w dz \cdot \int_0^b \Theta(D+L,y,z)dy}{w(1+l)},$$

$$T_b = \frac{\int_0^w dz \cdot \int_b^1 \Theta(0,y,z)dy}{w(1+l_1)} + \frac{\int_{-l_1}^0 dx \cdot \int_0^w \Theta(x,B,z)dz}{w(1+l_1)} + \frac{\int_0^w dz \cdot \int_0^b \Theta(-L_1,y,z)dy}{w(1+l_1)}.$$

The one element of the wall (base) is placed in the domain $\{x \in [0,\delta], y \in [0,1], z \in [0,w]\}$. The rectangular right fin in dimensionless arguments occupies the domain $\{x \in [\delta, \delta+l], y \in [0,b], z \in [0,w]\}$, but the left fin occupies the domain $\{x \in [-l_1, 0], y \in [0,b], z \in [0,w]\}$.

We describe the temperature field by functions $\overline{V}_0(x,y,z,t)$, $\overline{V}(x,y,z,t)$, and $\overline{V}_1(x,y,z,t)$ in the wall and fins :

$$\frac{1}{a^2}\frac{\partial \overline{V}_0}{\partial t} = \frac{\partial^2 \overline{V}_0}{\partial x^2} + \frac{\partial^2 \overline{V}_0}{\partial y^2} + \frac{\partial^2 \overline{V}_0}{\partial z^2}, \tag{1}$$

$$\frac{1}{a^2}\frac{\partial \overline{V}}{\partial t} = \frac{\partial^2 \overline{V}}{\partial x^2} + \frac{\partial^2 \overline{V}}{\partial y^2} + \frac{\partial^2 \overline{V}}{\partial z^2}, \tag{2}$$

$$\frac{1}{a^2}\frac{\partial \overline{V}_1}{\partial t} = \frac{\partial^2 \overline{V}_1}{\partial x^2} + \frac{\partial^2 \overline{V}_1}{\partial y^2} + \frac{\partial^2 \overline{V}_1}{\partial z^2}. \tag{3}$$

We must add initial conditions for the heat equations (1) – (3):

$$\overline{V}_0|_{t=0} = \overline{V}_0^0(x,y,z), \tag{4}$$

$$\overline{V}|_{t=0} = \overline{V}^0(x,y,z), \tag{5}$$

$$\overline{V}_1|_{t=0} = \overline{V}_1^0(x,y,z). \tag{6}$$

We assume heat fluxes from the flank surfaces (edges) and from the top and the bottom edges:

$$\frac{\partial \overline{V}_0}{\partial z}|_{z=0} = Q_{0,2}(x,y,t), \tag{7}$$

$$\frac{\partial \overline{V}_0}{\partial z}|_{z=w} = Q_{0,3}(x,y,t), \tag{8}$$

$$\frac{\partial \overline{V}}{\partial z}|_{z=0} = Q_2(x,y,t), \tag{9}$$

$$\frac{\partial \overline{V}}{\partial z}|_{z=w} = Q_3(x,y,t), \tag{10}$$

$$\frac{\partial \overline{V}_1}{\partial z}|_{z=0} = Q_{1,2}(x,y,t), \tag{11}$$

$$\frac{\partial \overline{V}_1}{\partial z}|_{z=w} = Q_{1,3}(x,y,t). \tag{12}$$

3 Reducing to the 2D Model

Such type of boundary conditions (BC)(7) – (12) allows us to make the exact reducing of this three-dimensional problem to two-dimensional problem by conservative averaging method [5]. Let us introduce following integral averaged values:

$$V_0(x,y,t) = w^{-1} \cdot \int_0^w \overline{V}_0(x,y,z,t)dz, \tag{13}$$

$$\theta_0(x,y,t) = w^{-1} \cdot \int_0^w \Theta_0(x,y,z,t)dz, \tag{14}$$

$$V(x,y,t) = w^{-1} \cdot \int_0^w \overline{V}(x,y,z,t)dz, \tag{15}$$

$$\theta(x,y,t) = w^{-1} \cdot \int_0^w \Theta(x,y,z,t)dz, \tag{16}$$

$$V_1(x,y,t) = w^{-1} \cdot \int_0^w \overline{V}_1(x,y,z,t)dz. \tag{17}$$

Realizing the integration of main equations (1) – (3) by usage of the BC (7) – (12) we obtain:

$$\frac{1}{a^2}\frac{\partial V_0}{\partial t} = \frac{\partial^2 V_0}{\partial x^2} + \frac{\partial^2 V_0}{\partial y^2} + Q_0(x,y,t), \tag{18}$$

$$\frac{1}{a^2}\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + Q(x,y,t), \tag{19}$$

$$\frac{1}{a^2}\frac{\partial V_1}{\partial t} = \frac{\partial^2 V_1}{\partial x^2} + \frac{\partial^2 V_1}{\partial y^2} + Q_1(x,y,t). \tag{20}$$

Here

$$Q_0(x,y,t) = \frac{Q_{0,3}(x,y,t) - Q_{0,2}(x,y,t)}{w},$$

$$Q(x,y,t) = \frac{Q_3(x,y,t) - Q_2(x,y,t)}{w},$$

$$Q_1(x,y,t) = \frac{Q_{1,3}(x,y,t) - Q_{1,2}(x,y,t)}{w}.$$

ISBN: 978-1-61804-026-8

We add to the main partial differential equations (18) – (20) needed BC as follows:

$$\left(\frac{\partial V_0}{\partial x} + \beta_0(\theta_0(x,y,t) - V_0)\right)_{x=0} = 0, \quad y \in (b,1), \qquad (21)$$

$$\left(\frac{\partial V_0}{\partial x} + \beta(V_0 - \theta(x,y,t))\right)_{x=\delta} = 0, \quad y \in (b,1), \qquad (22)$$

$$\frac{\partial V_0}{\partial y}|_{y=0} = Q_{0,0}(x,t), \;\; x \in (0,\delta), \qquad (23)$$

$$\frac{\partial V_0}{\partial y}|_{y=1} = Q_{0,1}(x,t), \;\; x \in (0,\delta). \qquad (24)$$

We assume them as ideal thermal contact between wall and fins - there is no contact resistance:

$$V_0|_{x=\delta} = V|_{x=\delta}, \qquad (25)$$

$$\frac{\partial V_0}{\partial x}|_{x=\delta} = \frac{\partial V}{\partial x}|_{x=\delta}, \qquad (26)$$

$$V_1|_{x=0} = V_0|_{x=0}, \qquad (27)$$

$$\frac{\partial V_1}{\partial x}|_{x=0} = \frac{\partial V_0}{\partial x}|_{x=0}. \qquad (28)$$

We have following BC for the right fin:

$$\left(\frac{\partial V}{\partial x} + \beta(V - \theta(x,y,t))\right)_{x=\delta+l} = 0, \quad y \in (0,b), \qquad (29)$$

$$\left(\frac{\partial V}{\partial y} + \beta(V - \theta(x,y,t))\right)_{y=b} = 0, \quad x \in (\delta,\delta+l), \qquad (30)$$

$$\frac{\partial V}{\partial y}|_{y=0} = Q_0(x,t), x \in (\delta,\delta+l). \qquad (31)$$

We have following BC for the left fin:

$$\left(\frac{\partial V_1}{\partial x} + \beta_0(\theta_0(x,y,t) - V_1)\right)_{x=-l_1} = 0, \quad y \in (0,b), \qquad (32)$$

$$\left(\frac{\partial V_1}{\partial y} + \beta_0(V_1 - \theta_0(x,y,t))\right)_{y=b} = 0, \quad x \in (-l_1,0), \qquad (33)$$

$$\frac{\partial V_1}{\partial y}|_{y=0} = Q_{1,0}(x,t), \;\; x \in (-l_1,0). \qquad (34)$$

Finally, we introduce integral averaged values as (13) – (17)and add initial conditions for the heat equations (18) – (20):

$$V_0|_{t=0} = V_0^0(x,y), \qquad (35)$$

$$V|_{t=0} = V^0(x,y), \qquad (36)$$

$$V_1|_{t=0} = V_1^0(x,y). \qquad (37)$$

4 Exact solution of 2-D simplified problem

In this section we would explain the main idea of solution for the 2-D case of periodical system with constant dimensionless environmental temperatures $\theta_0 = 1(\Theta_0 = T_b)$ and $\theta = 0(\Theta = T_a)$. Additionally, we neglect the heat fluxes from flank edges. We consider $U(x,y,t)$ is the temperature of the right fin, $U_0(x,y,t)$ temperature of the wall and $U_1(x,y,t)$ is the temperature of the left fin. So, the main equations are:

$$\frac{1}{a^2}\frac{\partial U_0}{\partial t} = \frac{\partial^2 U_0}{\partial x^2} + \frac{\partial^2 U_0}{\partial y^2}, \qquad (38)$$

$$\frac{1}{a^2}\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}, \qquad (39)$$

$$\frac{1}{a^2}\frac{\partial U_1}{\partial t} = \frac{\partial^2 U_1}{\partial x^2} + \frac{\partial^2 U_1}{\partial y^2}. \qquad (40)$$

The BC (23), (24), (31), (34) are assumed to be homogeneous:

$$\frac{\partial U_0}{\partial y}|_{y=0} = \frac{\partial U_0}{\partial y}|_{y=1} = \frac{\partial U}{\partial y}|_{y=0} = \frac{\partial U_1}{\partial y}|_{y=0} = 0.$$

Instead of BC (21), (22), (29), (30), (32) and (33) we have:

$$\left(\frac{\partial U_0}{\partial x} + \beta_0(1 - U_0)\right)_{x=0} = 0, \;\; y \in (b,1), \quad (41)$$

$$\left(\frac{\partial U_0}{\partial x} + \beta U_0\right)_{x=\delta} = 0, \;\; y \in (b,1), \quad (42)$$

$$\left(\frac{\partial U}{\partial x} + \beta U\right)_{x=\delta+l} = 0, \;\; y \in (0,b) \quad (43)$$

$$\left(\frac{\partial U}{\partial y} + \beta U\right)_{y=b} = 0, \;\; x \in (\delta,\delta+l), \quad (44)$$

$$\left(\frac{\partial U_1}{\partial x} + \beta_0(1 - U_1)\right)_{x=-l_1} = 0, \;\; y \in (0,b), \quad (45)$$

$$\left(\frac{\partial U_1}{\partial y} + \beta_0(U_1 - 1)\right)_{y=b} = 0, \;\; x \in (-l_1,0), \quad (46)$$

Initial conditions are still standing in the form (35) – (37). The conjugations conditions on the line between the wall and the left fin are still standing in the form (27), (28) for the functions $U_0(x,y,t)$ and $U_1(x,y,t)$. The linear combination of the equations (27), (28) together with BC (41) allow us rewrite them as following BC on the left hand side of the wall:

$$\left(\frac{\partial U_0}{\partial x} - \beta_0 U_0\right)_{x=0} = \beta_0 F_1(0,y,t), \qquad (47)$$

where

$$F_1(x,y,t) = \begin{cases} \frac{1}{\beta_0}\frac{\partial U_1}{\partial x} - U_1, & 0 \le y \le b \\ -1, & b < y \le 1. \end{cases} \quad (48)$$

In the similar way using the linear combination of the equations (25), (26) together with BC (42) we rewrite following BC on the right hand side of the wall:

$$\left(\frac{\partial U_0}{\partial x} + \beta U_0\right)_{x=\delta} = \beta F_0(\delta, y, t), \quad (49)$$

where

$$F_0(x,y,t) = \begin{cases} \frac{1}{\beta_0}\frac{\partial U}{\partial x} + U, & 0 \le y \le b \\ 0, & b < y \le 1. \end{cases} \quad (50)$$

On the assumption that the functions $F_1(0,y,t)$, $F_0(\delta,y,t)$ are given we can represent solution for the wall in very well known form by the Green function:

$$\begin{aligned} U_0(x,y,t) &= \int_0^\delta \int_0^1 U_0^0(\xi,\eta) G_0(x,y,\xi,\eta,t) d\eta d\xi \\ &-a^2\beta_0 \int_0^t \int_0^1 F_1(0,\eta,\tau) G_0(x,y,0,\eta,t-\tau) d\eta d\tau \\ &+a^2\beta \int_0^t \int_0^b F_0(\delta,\eta,\tau) G_0(x,y,\delta,\eta,t-\tau) d\eta d\tau, \quad (51) \end{aligned}$$

where Green function is:

$$G_0(x,y,\xi,\eta,t) = \sum_{m,n=1}^{\infty} G_{0,m}^x(x,\xi,t) G_{0,n}^y(y,\eta,t),$$

$$G_{0,m}^x(x,\xi,t) = \frac{\phi_{0,m}(x)\phi_{0,m}(\xi)}{\|\phi_{0,m}\|^2} e^{-a^2\mu_m^2 t},$$

$$G_{0,n}^y(y,\eta,t) = e^{-a^2(\pi n)^2 t}(\cos[n\pi(y+\eta)] + \cos[n\pi(y-\eta)]),$$

$$\phi_{0,m}(x) = \cos(\mu_m x) + \frac{\beta_0}{\mu_m}\sin(\mu_m x),$$

$$\|\phi_{0,m}\|^2 = \frac{\beta_0}{2\mu_m^2} + \frac{\beta}{2\mu_m^2}\frac{\mu_m^2+\beta_0^2}{\mu_m^2+\beta^2} + \frac{\delta}{2}(1+\frac{\beta_0^2}{\mu_m^2}).$$

Here μ_m are the positive roots of the transcendental equation:

$$\tan(\mu_m\delta) = \frac{\mu_m(\beta+\beta_0)}{\mu_m^2 - \beta\beta_0}.$$

Unfortunately the representation (51) is unusable as solution for the wall because of unknown functions $F_1(0,y,t)$, $F_0(\delta,y,t)$, i.e. temperature in the fins. That is why we will pay attention to the solution for the fins now. In the same way we can rewrite the conjugations conditions (25), (26) in the form of BC on the left side of the right rectangular fin:

$$\left(\frac{\partial U}{\partial x} - \beta U\right)_{x=\delta} = \beta F(\delta,y,t), \quad (52)$$

where

$$F(x,y,t) = \frac{1}{\beta}\frac{\partial U_0}{\partial x} - U_0, \ \ 0 \le y \le b. \quad (53)$$

Then, similar as for the wall we can represent solution for the right fin in following form:

$$\begin{aligned} U(x,y,t) &= \int_\delta^{\delta+l} \int_0^1 U^0(\xi,\eta) G(x,y,\xi,\eta,t) d\eta d\xi \\ &-a^2\beta \int_0^t \int_0^b F(\delta,\eta,\tau) G(x,y,\delta,\eta,t-\tau) d\eta d\tau, \quad (54) \end{aligned}$$

where Green function is:

$$G(x,y,\xi,\eta,t) = \sum_{m,n=1}^{\infty} G_m^x(x,\xi,t) G_n^y(y,\eta,t),$$

$$G_m^x(x,\xi,t) = \frac{\phi_m(x)\phi_m(\xi)}{\|\phi_m\|^2} e^{-a^2\mu_m^2 t},$$

$$G_n^y(y,\eta,t) = e^{-a^2\lambda_m^2 t}\frac{\psi_n(y,\eta)}{2\|\psi_n\|^2},$$

$$\phi_m(x) = \cos[\mu_m(x-\delta)] + \frac{\beta}{\mu_m}\sin[\mu_m(x-\delta)],$$

$$\psi_n(y,\eta) = \cos[\lambda_n(y+\eta)] + \cos[\lambda_n(y-\eta)],$$

$$\|\phi_m\|^2 = \frac{\beta}{\mu_m^2} + \frac{l}{2}\left(1 + \frac{\beta^2}{\mu_m^2}\right),$$

$$\|\psi_n\|^2 = \frac{1}{2}\left(b + \frac{\beta}{\lambda_n^2+\beta^2}\right).$$

Here μ_m, λ_n are the positive roots of the transcendental equations:

$$\tan(\mu_n l) = \frac{2\mu_n\beta}{\mu_n^2-\beta^2},$$

$$\tan(\lambda_n b) = \frac{\beta}{\lambda_n}.$$

Finally, we rewrite the conjugations conditions in the form of BC on the right side of the left rectangular fin:

$$\left(\frac{\partial U_1}{\partial x} + \beta_0 U_1\right)_{x=0} = \beta_0 F_2(0,y,t), \quad (55)$$

where

$$F_2(x,y,t) = \frac{1}{\beta_0}\frac{\partial U_0}{\partial x} + U_0, \ \ 0 \le y \le b \quad (56)$$

ISBN: 978-1-61804-026-8

So, solution for the left fin we can represent in following form:

$$U_1(x,y,t)=\int_{-l_1}^{0}\int_0^b U_1^0(\xi,\eta)G_1(x,y,\xi,\eta,t)d\eta d\xi$$
$$+a^2\beta_0\int_0^t\int_0^b G_1(x,y,-l_1,\eta,t-\tau)d\eta d\tau$$
$$+a^2\beta_0\int_0^t\int_0^b F_2(0,\eta,\tau)G_1(x,y,0,\eta,t-\tau)d\eta d\tau$$
$$+a^2\beta_0\int_0^t\int_{-l_1}^0 G_1(x,y,\xi,b,t-\tau)d\xi d\tau, \quad (57)$$

where Green function is:

$$G_1(x,y,\xi,\eta,t)=\sum_{m,n=1}^{\infty}G_{1,m}^x(x,\xi,t)G_{1,n}^y(y,\eta,t),$$
$$G_{1,m}^x(x,\xi,t)=\frac{\phi_{1,m}(x)\phi_{1,m}(\xi)}{\|\phi_{1,m}\|^2}e^{-a^2\mu_m^2 t},$$
$$G_{1,n}^y(y,\eta,t)=e^{-a^2\lambda_m^2 t}\frac{\psi_{1,n}(y,\eta)}{2\|\psi_{1,n}\|^2},$$
$$\phi_{1,m}(x)=\cos[\mu_m(x+l_1)]+\frac{\beta}{\mu_m}\sin[\mu_m(x+l_1)],$$
$$\psi_{1,n}(y,\eta)=\cos[\lambda_n(y+\eta)]+\cos[\lambda_n(y-\eta)],$$
$$\|\phi_{1,m}\|^2=\frac{\beta_0}{\mu_m^2}+\frac{l_1}{2}\left(1+\frac{\beta_0^2}{\mu_m^2}\right),$$
$$\|\psi_{1,n}\|^2=\frac{1}{2}\left(b+\frac{\beta_0}{\lambda_n^2+\beta_0^2}\right).$$

Here μ_m ,λ_n are the positive roots of the transcendental equations:

$$\tan(\mu_n l_1)=\frac{2\mu_n\beta_0}{\mu_n^2-\beta_0^2},$$

$$\tan(\lambda_n b)=\frac{\beta_0}{\lambda_n}.$$

Using notation (54) and representation (50) we can easy obtain the following equation:
$F_0(\delta,y,t)=$

$$-a^2\int_0^t\int_0^b F(\delta,\eta,\tau)\Gamma(\delta,y,\delta,\eta,t-\tau)d\eta d\tau$$
$$+C_0(y,t), \quad (58)$$

where

$$\Gamma(x,y,\xi,\eta,t)=\left(\frac{\partial}{\partial x}+\beta\right)G(x,y,\xi,\eta,t),$$
$$C_0(y,t)=\frac{1}{\beta}\int_\delta^{\delta+l}\int_0^1 U^0(\xi,\eta)\Gamma(\delta,y,\xi,\eta,t)d\eta d\xi.$$

In the similar way we find equation for $F_1(0,y,t)$, using (48) and (57):
$F_1(0,y,t)=$

$$=a^2\int_0^t\int_0^b F_2(0,\eta,\tau)\Gamma_1(0,y,0,\eta,t-\tau)d\eta d\tau$$
$$+C_1(y,t), \quad (59)$$

where

$$\Gamma_1(x,y,\xi,\eta,t)=\left(\frac{\partial}{\partial x}-\beta_0\right)G_1(x,y,\xi,\eta,t),$$
$C_1(y,t)=$
$$=\frac{1}{\beta_0}\int_{-l_1}^0\int_0^b U_1^0(\xi,\eta)\Gamma_1(0,y,\xi,\eta,t)d\eta d\xi$$
$$+a^2\int_0^t\int_0^b \Gamma_1(0,y,-l_1,\eta,t-\tau)d\eta d\tau$$
$$+a^2\int_0^t\int_{-l_1}^0 \Gamma_1(0,y,\xi,b,t-\tau)d\xi d\tau.$$

Next,we find equation for $F(\delta,y,t)$ using (53) and (51):
$F(\delta,y,t)=$

$$-a^2\frac{\beta_0}{\beta}\int_0^t\int_0^l F_1(0,\eta,\tau)\Gamma_0(\delta,y,0,\eta,t-\tau)d\eta d\tau$$
$$+a^2\int_0^t\int_0^b F_0(\delta,\eta,\tau)\Gamma_0(\delta,y,\delta,\eta,t-\tau)d\eta d\tau$$
$$+C(y,t), \quad (60)$$

where

$$\Gamma_0(x,y,\xi,\eta,t)=\left(\frac{\partial}{\partial x}-\beta\right)G_0(x,y,\xi,\eta,t),$$
$$C(y,t)=\frac{1}{\beta}\int_0^\delta\int_0^l U_0^0(\xi,\eta)\Gamma_0(\delta,y,\xi,\eta,t)d\eta d\xi.$$

Finally, using (56) and (51) we get equation for $F_2(0,y,t)$:
$F_2(0,y,t)=$

$$=-a^2\int_0^t\int_0^l F_1(0,\eta,\tau)\Gamma_2(0,y,0,\eta,t-\tau)d\eta d\tau$$
$$+a^2\frac{\beta}{\beta_0}\int_0^t\int_0^b F_0(\delta,\eta,\tau)\Gamma_2(0,y,\delta,\eta,t-\tau)d\eta d\tau$$
$$+C_2(y,t), \quad (61)$$

where

$$\Gamma_2(x,y,\xi,\eta,t)=\left(\frac{\partial}{\partial x}+\beta_0\right)G_0(x,y,\xi,\eta,t),$$
$$C_2(y,t)=\frac{1}{\beta_0}\int_0^\delta\int_0^l U_0^0(\xi,\eta)\Gamma_2(0,y,\xi,\eta,t)d\eta d\xi.$$

When solved system of integral equations (58) – (61) we can obtain the temperatures fields in the wall (51), left fin (57) and right fin (54).

5 Conclusion

We have constructed exact solution of a system of heat transfer equations for 2D T-shape domain. This solution is obtained in the form of the 2^{nd} kind Fredholm integral equations. If wall's and fin's materials differ, coefficients a^2 in differential equations (1) – (3) also differ. In case of non-homogeneous environment (surrounding temperatures $\tilde{\Theta}, \tilde{\Theta}_0 \neq 0, 1$), known integrals (Green function multiplication by non-homogeneous term) in solutions (51),(54), (57) appear. This means that in (58) – (61) non-homogeneous term changes, but described algorithm still is applicable.

Acknowledgements: The research was supported by the University of Latvia, ESF research project No. 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 and in the case of the second author, it was supported by the Latvian Council of Science (grant No. 09.1572).

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