Single and Double Layer Spiral Planar Inductors Optimisation with the Aid of Self-Organising Migrating Algorithm

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Abstract: - Planar inductors made on a printed circuit boards are rather commonly employed today and there are various software applications to help the designer with their proposal. In this paper we describe the optimisation of a single and double layer spiral inductors made by the help of evolutionary Self-Organizing Migrating Algorithm in order to achieve the required inductance while the resistance of the inductor's conductor was as low as possible. Primarily, these inductors are of large extent, supposed to be utilized at low-frequency applications such as proximity sensors or metal detectors, but the results stated in this paper can be generalized for various applications. A simple Maple algorithm was also created in order the results gained from the evolutionary algorithm could be verified.

Key-Words: - SOMA Algorithm, Flat spiral inductor, Inductor on PCB, Optimisation, Maple, Inductance

1 Introduction

There are several models and approximations estimating the inductance of planar inductors. These models usually show some uncertainty as the inductance expression is quite complex, involving the self inductance of the conductor, mutual inductance among particular current-turns and the influence of the inductor surroundings.

The models are usually employed at numerical calculations but as it is demonstrated in this paper, they can also be utilized by evolutional algorithms. The list of methods developed in the past is, according to [10], rather wealthy; let us mention for example the Expanded Grover method, Bryan method, Terman method, all mentioned in [4] for rectangle-shaped inductors, or Wheeler, Gleason or Olivei approximations for spiral-shaped inductors. Seeking for a simple method suitable for being solved by the resources of the evolutionary algorithm an expression based on current sheet approximation has been chosen. This expression for the inductance of a planar spiral can be obtained by approximating the sides of the spirals by symmetrical current sheets of equivalent current densities [6]. The detailed description of this method can be found in [8].

2 Problem Formulation

A spiral coil is a geometrical spiral-shaped formation made of a copper film on the printed circuit board. This copper spiral is electrically conductive shoving an electrical resistance for the direct current. On multi-layer printed circuit boards there is a possibility to create the spiral coils in several levels, which can be, in some cases, more convenient. For the alternating current the inductor shows, except the electrical resistance, also a property called inductance, which describes the capability of producing a magnetic field around the inductor. From the electrical point of view the inductance acts as complex impedance being proportional to the frequency of the alternating current. The ideal inductor shows only the inductance while its electrical resistance is negligible. In consideration of this fact the optimisation introduced in this paper is aimed for the large planar inductor design in order the required inductance was achieved with the minimal electrical resistance of the inductor [10]. Primarily, such a large spiral coil inductor is intended to be employed in a metal detector, but this method is also suitable for designing smaller loop antennas, Tesla coils, proximity sensors etc. Because the inductor is supposed to operate at low frequencies, parasitic parameters as self-capacitance, skin-effect influence etc., were neglected.

2.1 Spiral coil geometry

In the figure 1 a single-layer spiral inductor is depicted. From the figure it is obvious that the

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inductor is physically described by the parameters as follows: D_{OUT} is the outer diameter of the inductor [m], D_{IN} expresses its inner diameter [m], w is the width of the conductor [m] and s is the width of the insulating gap [m]. For the purposes of this paper it is better to know the number of current-turns n rather the inner diameter D_{IN}. The inner diameter D_{IN} can be then expressed as follows:

$$D_{IN} = D_{OUT} - (n+1)\cdot w - n\cdot s \quad [m] \tag{1}$$

For further calculations let us consider there are only quite weak requirements on the boundaries of the parameter D_{IN}:

$$0 \le D_{IN} \le D_{OUT} \quad [m] \tag{2}$$

The double layer spiral planar inductor, which is also discussed in this paper, is of the same geometry as depicted in figure 1, but it consists of two such spirals, one placed right above the other with a small insulation gap. Such inductor can be created on two-layer printed circuit board, using the base material as the insulation.

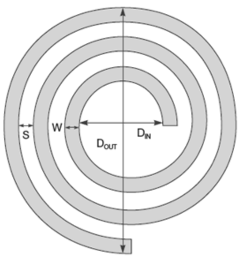

Fig. 1 Spiral coil inductor geometry [6].

2.2 Single layer spiral coil inductance

The approximation of the single-layer spiral inductor is described in [6] and in [7]. The basic expression is:

$$L = \frac{\mu \cdot n^2 \cdot D_{avg} \cdot C_1}{2} \cdot \left(ln\left(\frac{C_2}{\sigma}\right) + C_3 \cdot \sigma + C_4 \cdot \sigma^2 \right) \quad [H] \tag{3}$$

The meanings of the parameters are as follows: μ - material permeability [$H.m^{-1}$], for copper film can be considered as a constant μ = 1,26 $H.m^{-1}$, n – number of turns, D_{avg} – average diameter of the coil [m], σ – coil fill ratio saying how much the area of the inductor is filled with current-turns, C_1 to C_4 shape-dependent constants [7]. For the purposes of this paper we need to describe parameters D_{avg} and σ by the following equations, employing parameters described above:

$$\sigma = \frac{D_{OUT} - D_{IN}}{D_{OUT} + D_{IN}} = \frac{n\cdot(w+s)+w}{2\cdot D_{OUT} - (n\cdot(w+s)+w)} \quad [-] \tag{4}$$

$$D_{avg} = \frac{D_{OUT} + D_{IN}}{2} = D_{OUT} - \frac{w\cdot(n+1)+n\cdot s}{2} \quad [m] \tag{5}$$

For the single-layer spiral coil the constants C1 to C4 are, according to [3], as follows: C1 = 1.00, C2 = 2.46, C3 = 0.00, C4 = 0.20. Now, employing the equation (3), we can yield the following expression to be used by the evolutionary algorithm:

$$L = \frac{\mu \cdot n^2 \cdot \left(D_{OUT} - \frac{w\cdot(n+1)+n\cdot s}{2}\right)\cdot C_1}{2} \cdot \left(ln\left(\frac{C_2}{\frac{n\cdot(w+s)+w}{2\cdot D_{OUT} - (n\cdot(w+s)+w)}} \right) + C_4 \cdot \left(\frac{n\cdot(w+s)+w}{2\cdot D_{OUT} - (n\cdot(w+s)+w)} \right)^2 \right) \quad [m] \tag{6}$$

All the parameters used in the expression (6) are described in the text above. The inductance is now defined as a function of parameters n, w, s and D_{OUT}.

2.3 Double layer spiral coil inductance

The inductance of double layer spiral planar coil is described as:

$$L = L_1 + L_2 \pm 2M \quad [H] \tag{7}$$

The meanings of the parameters are as follows: L_1 and L_2 are the self-inductances of the first and second spiral coil [H] and M is the mutual inductance between those two coils [H]. This inductance is quite difficult to determine as it depends on the mutual geometry of the particular conductors, the gap between the coils and many other factors. Depending on the current directions, the mutual inductance increases or decreases the total inductance of the inductor. For our purposes, both coils must be series-connected in such way the current passed through them in the same direction. According to [7], mutual inductance can be expressed as:

$$M = K_C \sqrt{L_1 L_2} \quad [H], \tag{8}$$

where K_C is a coupling factor that must lie between 0 and 1. According to [7], the coupling factor can be expressed as:

$$K_C = \frac{n^2}{0.64\cdot(Ax^3 + Bx^2 + Cx + D)\cdot(1.67n^2 - 5.84n + 65)} \quad [-] \tag{9}$$

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The meanings of the parameters are as follows: n is the number of the current turns, x is the mutual distance of both coils [m] and A, B, C and D are constants that are, according to [7], as follows:
A=0.000184, B=-0.000525, C=0.001038,
D=0.001001.

Combining the appropriate equations, one can express the inductance of double layer planar spiral coil by the following equation:

$$L = 2\cdot\left(1+\frac{n^2}{0.64\cdot\left(Ax^3+Bx^2+Cx+D\right)\cdot(1.67n^2-5.84n+65)}\right) \cdot\frac{\mu n^2\cdot\left(d_{out}-\frac{w\cdot(n+1)+ns}{2}\right)\cdot C_1}{2} \cdot\left(ln\left(\frac{C_2}{\frac{n\cdot(w+s)+w}{2\cdot D_{OUT}-\left(n\cdot(w+s)+w\right)}}\right)+C_4\cdot\left(\frac{n\cdot(w+s)+w}{2\cdot d_{out}-\left(n\cdot(w+s)+w\right)}\right)^2\right)[H] \quad (10)$$

2.4 Single layer spiral coil resistance

There is also a need of retrieving an expression describing how the electrical resistance of the inductor depends on the parameters n, w, s and D_{OUT}. Basically, the conductor resistance can be described as follows:

$$R = \rho\cdot\frac{l}{S} \quad [\Omega] \quad (11)$$

Where: R – electrical resistance [Ω], l – conductor length [m], S – conductor cross section [m^2], ρ – conductor specific electrical resistance, for copper ρ = $1.75\cdot10^{-8}$ [Ω.m]. The cross section of the copper conductor can be expressed according to its height and width. The height was considered to be as high as 18 µm, which represents one of the standards for printed circuit boards copper clad. Now the electrical resistance of the copper conductor can be expressed as a function of its length l and width w:

$$R = 1,75\cdot10^{-8}\cdot\frac{l}{w\cdot1,8\cdot10^{-5}} = \frac{35}{36}\cdot\frac{l}{w}\cdot10^{-3} \quad [\Omega] \quad (12)$$

The width of the conductor is one of the parameters to be limited by the technological process restrictions at the printed circuit board production and can be expressed easily. More difficult is to retrieve the length of the spiral-shaped conductor. An approximation of the spiral-shaped conductor by an infinitely thin line described in polar coordinates has been employed, considering the diameter of the spiral decreases by (w + s) in each current-turn. The diameter of the current-turns can be described as a function of the angle of rotation, measured from the beginning of the spiral:

$$r = \frac{D_{OUT}}{2} - \frac{\alpha}{2\pi}\left(w+s\right); \alpha \in 0, 2\pi n \quad [m] \quad (13)$$

Where: α – radius vector angle [rad], w – conductor width [m], s – insulation gap width [m]. According to [9] the length of such spiral can be expressed as:

$$l = \int_0^{2\pi n}\sqrt{\left(r(\alpha)\right)^2+\left(\frac{d}{d\alpha}r(\alpha)\right)^2}\,d\alpha \quad [m] \quad (14)$$

Provided the derivation of the function r (expressed in (13)) is continuous throughout the whole interval <0, 2πn>. Now the electrical resistance of the inductor can be expressed as a function of current-turns number n, isolation gap width s, conductor width w and the outer diameter of the inductor D_{OUT}:

$$R = \frac{35}{36}\cdot\frac{1}{w}\cdot10^{-3}\cdot\int_{\alpha=0}^{\alpha=2\pi n}\sqrt{\left(\left(\frac{D_{OUT}}{2}\right)-\frac{\alpha}{2\pi}(w+s)\right)^2+\left(\frac{w+s}{2\pi}\right)^2}\,d\alpha \quad [\Omega] \quad (15)$$

2.5 Double layer spiral coil resistance

As stated above, in double layer spiral coil both coils are series-connected. Because one supposes both coils to be identical, the resistance of double layer spiral coil can be expressed as follows:

$$R = \frac{35}{18}\cdot\frac{1}{w}\cdot10^{-3}\cdot\int_{\alpha=0}^{\alpha=2\pi n}\sqrt{\left(\left(\frac{D_{OUT}}{2}\right)-\frac{\alpha}{2\pi}(w+s)\right)^2+\left(\frac{w+s}{2\pi}\right)^2}\,d\alpha \quad [\Omega] \quad (16)$$

3 Methods

The optimisation of the coil was processed according to appropriate above mentioned equations by two methods. First of all, a simple Maple algorithm was created to process the optimisation and then the same task with the same parameters was run on the Self-Organizing Migrating Algorithm. The results are discussed in the following text.

3.1 Maple algorithm

The flowchart of the algorithm is depicted in figure 3. This algorithm can be modified for both, the single and the double layer coil. The algorithm creates graphs that display how the optimal parameters depend on the required inductance of the coil. First of all an optimisation of a single-layer coil

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for metal detector was processed with the following parameter restrictions:

Table 1. Parameter limits for single-layer coil optimisation by Maple algorithm

Parameter	Value	Description
L_{MIN}	50 µH	Lower inductance limit
L_{MAX}	1 mH	Upper inductance limit
L_{STEP}	25 µH	Inductance step between L_{MIN} and L_{MAX}
D_{OUT_MIN}	180 mm	The minimum diameter of the coil
D_{OUT_MAX}	205 mm	The maximum diameter of the coil
D_{OUT_STEP}	5 mm	Coil diameter change step
W_{MIN}	1 mm	The minimum trace width
W_{MAX}	1,5 mm	The maximum trace width
W_{STEP}	100 µm	Trace width change step
S_{MIN}	250 µm	The minimum insulation gap width
S_{MAX}	1,5 mm	The maximum insulation gap width
S_{STEP}	50 µm	Insulation gap width change step
TOL	1 %	Tolerance of the achieved inductance
N_{MIN}	1	The minimum current-turns number
N_{MAX}	100	The maximum current-turns number
N_{STEP}	1	Current turns number change step (may be a real number)
D_{IN}	$D_{IN} > 0.5D_{OUT}$	Inner coil diameter

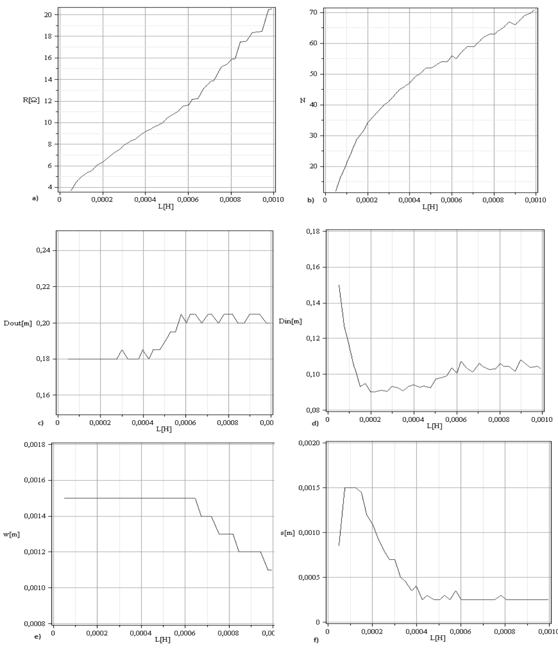

Fig. 2. Maple algorithm optimisation results – dependences on the required inductance for: a) resistance, b) number of current-turns, c) outer diameter, d) inned diameter, e) conductor trace width, f) insulation gap width

The brief description of the algorithm is as follows. It works in a cycle for the inductances from L_{MIN} to L_{MAX}. In each iteration it generates the best combination of the parameters D_{OUT}, D_{IN}, w, s and n for which the proper inductance L is achieved with the pre-set tolerance TOL and the resistance is as small as possible. The best parameters combination is chosen from the list of all parameter combinations for which the proper inductance L is achieved.

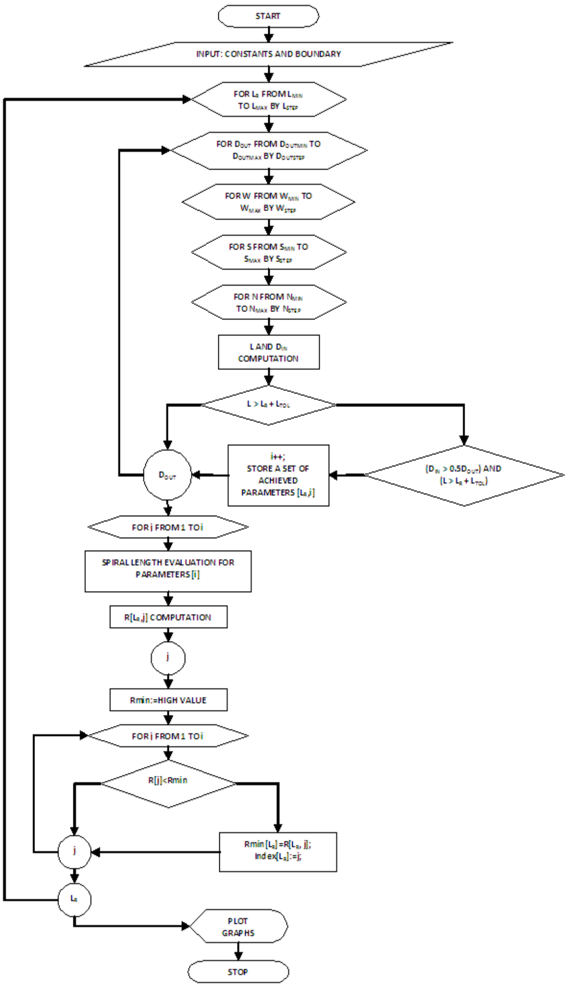

Fig. 3. Maple algorithm flowchart

3.2 Self-Organizing Migrating Algorithm

The optimisation of the parameters of the inductor is not as simple as it appears. There are four parameters intended for optimisation: outer diameter of the inductor, width of the inductor, width of isolation gap and number of current-turns. It can be said we are looking for combination of mentioned parameters so the calculated inductance is 250 µH. Calculation is ensured by (6) in the case of single layer inductors and (10) in the case of multi-layer inductors. Also the inequality for D_{IN} mentioned in Table 1 must be accomplished in both cases as well as the equations (15) and (16) that describe the coil

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resistance. The resistance should be as low as possible.

Since the task of optimisation is not elementary, there was considered application of artificial intelligence. The Self-Organizing Migrating Algorithm (SOMA) [1-4] was chosen as a form of artificial intelligence. SOMA belongs to the category of memetic or evolutionary (as it is inaccurately frequently mentioned in literature) algorithms. In the case of memetic algorithms, individuals divert to specific directions [4] in opposite to evolutionary algorithms where new individuals are created. Detailed description of SOMA is published e.g. [1-4] thus only short introduction to SOMA follows.

SOMA proceeds optimisation while migration loops. During migration loops, the individuals representing provisory solution of optimisation problem migrate toward the best-evaluated individual. This process can be represented as cooperation among members of wildlife shoals in the nature while hunting, finding food etc. If there is intended to reach collective intentions, the cooperation becomes evident. Individuals of SOMA are vectors containing parameters which have to be optimised. It can be said that these individuals have collective intention to reach the global minimum or maximum. Since the shoal is leaded by the best member (the strongest, fastest etc.), there is necessary to establish the best individual in the case of SOMA too. The selection of the best individual is provided by evaluation in according to fitness function. There are calculated the fitness values of each individual. The fitness value represents the evaluation of individuals. The fitness values are re-calculated within each migration loop thus the leader of the individuals can be different every migration loop. Other individuals subsequently migrate toward the leader thus the optimised solution is searched.

In the case of SOMA, the individual is a vector containing parameters for optimisation. The form of vector is defined by specimen. The specimen specifies the number of parameters and boundaries of those parameters. In according to the specimen, the individuals are subsequently created. While finding parameters of the inductor, the specimen has following parts:

$D_{out}, D_{out} \in \langle 0.18; 0.205 \rangle$ [m],

$w, w \in \langle 0.2 \cdot 10^{-3}; 1.5 \cdot 10^{-3} \rangle$ [m],

$s, s \in \langle 0.2 \cdot 10^{-3}; 1.5 \cdot 10^{-3} \rangle$ [m],

$n, n \in \langle 1; 1000 \rangle$

At the beginning there is necessary to create the first migration. This migration contains individuals with parameters randomly set according to specimen. SOMA subsequently optimises individuals in this migration. All individuals are evaluated thus their fitness values are calculated. The best evaluated individual becomes a leader. Other individuals start migrating toward the best one. The calculation of fitness values proceeds within fitness function which represents optimisation mechanism. The optimisation consists of substitution of parameters in equations (6), (15) in the case of single layer inductor and equations (10), (16) in the case of double layer inductor.

SOMA has following parameters:

- *Population size* - The amount of individuals contained in one migration
- *Migrations* - The amount of migration loops
- *Step* - The step-size of migrating individuals
- *Path length* - The length of migration path
- *PRT* - Perturbation of migration process
- *Minimal diversity* - Difference between the fitness of the best and the worst individual

These parameters were set according to the following table:

Table 2. Soma parameters

SOMA Parameter	Value
Population size	100
Migrations	100
Step	0.11
Path length	3
PRT	0.1
Minimal diversity	10

There are some variances of SOMA which differ in a way of migration of individuals. In this research SOMA_All_To_One was used. All individuals migrate toward the best one.

4 Results discussion

The optimisation was processed for 250 μH planar spiral coil with both single or double layers. The results generated by the Maple algorithm are as follows: If one requires a planar single-layer spiral 250 μH coil the resistance of which is as low as possible and the parameters of which must comply to the limits listed in the table 1, the best possible results to be achieved are as mentioned in the following table.

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Table 3. Achieved parameters for the single-layer coil

Parameter value	Maple algorithm	SOMA algorithm[1]
Resistance R	7 Ω	8.6 Ω
Outer diameter D_{OUT}	180 mm	195.9 mm
Trace width w	1.5 mm	1.5 mm
Insulation spacing s	0.85 mm	0.201 mm
Number of turns n	37	28.7
Coil inductance	250 µH	249.6 µH

[1] More results were achieved by the SOMA, only the best set of the results is shown.

If one requires a planar double-layer spiral 250 µH coil the resistance of which is as low as possible, the parameters of which must comply with the limits listed in the table 1 and the distance between the layers is 1.5 mm, the best possible results to be achieved are as mentioned in the following table.

Table 4. Achieved parameters for the double-layer coil

Parameter value	Maple algorithm	SOMA algorithm[2]
Resistance R	9 Ω	< 10 Ω
Outer diameter D_{OUT}	180 mm	194 mm
Trace width w	1.5 mm	0.59 mm
Insulation spacing s	1.4 mm	0.21 mm
Number of turns n	2 x 18	2 x 5
Coil inductance	250 µH	Approx. 60 µH

[2] Satisfactory results were not reached; this is an example of wrong results.

As can be seen from the results, the outputs of Maple algorithm and SOMA algorithm are consistent with respect to small tolerances for a single layer coil, but in the case of the double layer one the solution was obviously not achieved. This is in accordance with the graphs of history of the evolution that are shown in the figure below.

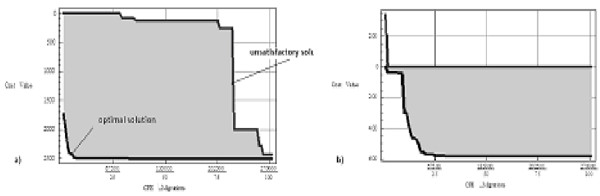

Fig. 4. The history of SOMA evolution for optimising the single layer spiral inductor (a) and the double layer spiral inductor not reaching the result (b).

It is obvious that in the figure 4b the optimal solution curve did not achieve the value of – 2.500 (the required coil inductance in µH multiplied by -10) and the solution was not found. On the other hand the solution for a single layer spiral inductor, represented by figure 4a, was found quite quickly. It is considered that Population size and Migration parameters must be increased in order the proper solution could be found for the double-layer inductor because the equations to be solved are more complicated. This will, indeed, considerably extend the time needed to find the solution.

4 Conclusion

The paper deals with the possibility of utilizing SOMA algorithm at large single and double layer spiral coils optimisation. Comparing with the results gained by the algorithm created in Maple environment it was proved the results obtained by SOMA algorithm are applicable although there is a need of further SOMA parameters setting revision, especially in the case of double layer inductors.

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