The Kinematic and Dynamic Analysis of the Crank Mechanism with SolidWorks Motion

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Abstract: - The objective of this application is to analyze the kinematics and dynamics of the crank mechanism and compare the theoretical with SolidWorks Motion results. The kinematic parameter of the mechanism are: the linear displacements $X_M$, $X_{Mo}$, the linear velocity $V_M$ and the linear acceleration $a_M$ of the piston.

Key-Words: - Driving rod, crank, displacements, velocity, acceleration, SolidWorks Motion

1 Introduction

The crank mechanism convert alternative translational movement to a continues rotating movement, or vice versa, fig. 1. The piston has a translational motion between the two extreme points: interior „$M_i$” and exterior „$M_e$”. The stroke „$c$” is the distance between two points and is calculated by equation $c=2\cdot r$, where „$r$” is the crank radius.

$X_u = r \cdot \left[1 - \cos(\phi) + \frac{\lambda}{2} \sin^2(\phi)\right]$ (1)

$\lambda = \frac{r}{L_5}$ (2)

$X_{M_o} = L_5 + r - X_M$ (3)

$V_M = r \cdot \omega \cdot \left[\sin(\phi) + \frac{\lambda}{2} \sin(2 \cdot \phi)\right]$ (4)

$a_M = r \cdot \omega^2 \cdot \left[\cos(\phi) + \lambda \cos(2 \cdot \phi)\right]$ (5)

2 Problem Formulation

The maximal velocity is obtained for a position where the crank and the driving rod are perpendicular and is calculated by relation (6). The accelerations are zero for the points of the maximum piston velocity. The accelerations are maximal for the two end stroke points:

• interior point „$M_i$” (for $\phi = 0^\circ$) calculated with relation (7);
• exterior point „$M_e$” (for $\phi = 180^\circ$) calculated with relation (8).
\[
V_{\text{max}} = r \cdot \omega \cdot \left(1 + \frac{\lambda}{2}ight) \quad (6)
\]
\[
a_{\text{max}} = r \cdot \omega^2 \cdot \left(1 + \lambda \right) \quad (7)
\]
\[
a_{\text{max}} = -r \cdot \omega^2 \cdot \left(1 - \lambda \right) \quad (8)
\]

The crank mechanism is loaded with active forces produced by the pressure of the working of cylinder and inertial forces due to acceleration of the masses in motion.

The resultant of these forces \( P_{\text{rec}} \) acts upon the cylinder axis and can be decomposed according to the driving rod direction \( P_{\phi} \), relation (9) and perpendicular to the cylinder axis \( P_{\psi} \), relation (10).

The force of the driving rod \( P_{\phi} \) can be decomposed, in the crank knob, according to the crank direction \( P_{\phi} \), relation (11) and perpendicular to this direction \( P_{\psi} \), relation (12). The rotary moment of the crank shaft \( M \) is given by relation (13).

\[
P_{\phi} = \frac{P_{\text{rec}}}{\cos(\psi)} \quad (9)
\]
\[
P_{\psi} = P_{\text{rec}} \cdot \tan(\psi) \quad (10)
\]
\[
P_{\phi} = P_{\phi} \cdot \sin(\phi + \psi) = \frac{P_{\text{rec}}}{\cos(\psi)} \cdot \sin(\phi + \psi) \quad (11)
\]
\[
P_{\psi} = P_{\psi} \cdot \cos(\phi + \psi) = \frac{P_{\text{rec}}}{\cos(\psi)} \cdot \cos(\phi + \psi) \quad (12)
\]
\[
M = P_{\phi} \cdot r = \frac{P_{\text{rec}}}{\cos(\psi)} \cdot r \cdot \sin(\phi + \psi) \quad (13)
\]

3 The stages of application
- Creation of the parts geometry;
- Creation of the assembly mechanism;
- Activation of the SolidWorks Motion module [4];
- Creation and specification of the study’s options;
- Specify Rotary Motor;
- Specify Force;
- Running the design study;
- Simulation and theoretical results comparison.

3.1 The mechanism geometry
The crank mechanism geometry is presented in fig. 2 and fig. 3. The assembly is composed from 6 connected components. The support and the cylinder components are fixed, and the crank, the driving rod, the piston and the bolts are mobile. The crank and driving rod length are \( r = 35 \text{ mm} \) and \( L_{b} = 75 \text{ mm} \) respectively. Both parts have 10 mm width and 5 mm thickness. The pins diameter is 4 mm. The mechanism stroke is \( c = 70 \text{ mm} \).

3.1.1 The support geometry
The support, fig. 4, has a box base 40x40x5, over which is superimposed the \( \Phi 8 \times 30 \) cylinder, centered on support face.

3.1.2 The crank geometry
The crank, fig. 5, has an elliptical shape \( 35 \times 10 \times \Phi 8 \times \Phi 4 \), extruded on 5 mm distance.

3.1.3 The driving rod geometry
The driving rod, fig. 6, has an elliptical shape \( 75 \times 10 \times \Phi 4 \times \Phi 4 \), extruded on 5 mm distance.

3.1.4 The pin geometry
The pin, fig. 7, has a cylindrical shape \( \Phi 4 \times 10 \).
3.1.5 The piston geometry
The piston, fig. 8, is a union of the \( \Phi 30 \times 40 \) cylinder and \( 5 \times 5 \times 35 \) box.

3.1.6 The cylinder geometry
The cylinder dimensions, fig. 9, are \( \Phi 50 \times \Phi 30 \times 62.5 \times R2 \).

3.2 Create the assembly mechanism
Create a new assembly document and save it as Ansamblu. First step require to insert the parts into the assembly and then to apply constraints between them.

3.2.1 Insert components
We must activate Insert Components command from Assembly toolbar. Then select a part or assembly from the Part/Assembly to Insert list or click Browse to open an existing document. Next click in the graphics area to place the component or click \( \checkmark \) to place the component origin coincident with the assembly origin.

Only the Suport component will be placed with the origine coincident with the assembly origin. By default, the first part placed in an assembly is fixed and has a (f) before its name in the FeatureManager design tree. The other components: Crank, Driving rod, Pin, Piston, Cylinder will be placed without this restriction. The Pin part will be inserted twice, receiving the name Pin1 and Pin2, like in fig. 10.

3.2.2 Adding mating relationships
To position the Cylinder part from the fixed Suport part, the following mates must be applied: Distance1, Distance2, Parallel2, fig. 11, fig. 12, fig. 13.

The following mates will be applied between the rest of the components:
- Concentric1 Suport Crank
- Coincident5 Suport Crank
- Concentric2 Crank Pin1
- Coincident6 Crank Pin1
- Concentric3 Driving rod Pin1
- Coincident7 Driving rod Pin1
- Concentric4 Driving rod Pin2
- Coincident8 Driving rod Pin2
- Concentric5 Piston Pin2
- Coincident9 Piston Driving rod
- Concentric6 Piston Cylinder

3.3 Activation of the SolidWorks Motion module and specify study’s options
A motion study can be created from Tools → Add-Ins menu and accessed from a Motion Study tab at the lower portion of the graphics area.
To specify the study’s options, we must to:
- Click Properties (MotionManager toolbar).
  - Set 25 value in Frames per second field.
  - Activate Animate during simulation and Show all Motion Analysis messages options.
- Under Motion Analysis, click Advanced Options.
  - Select GSTIFF solver.
  - Click OK and \( \checkmark \)
To specify the initial position and the study’s time:
• Position the mechanism in the initial position, fig. 14;
• Right-click the final key point and click Edit Key Point Time;
• Select Exact Time to enter the exact key point time: 1 sec;
• Click ↪;
• From the lower right corner of the MotionManager, click Zoom to Fit to rescale the timeline view.

3.3.1 Specify Rotary Motor
• Click Motor to create a new rotary motor;
• Select the face of the crank;
• Select Constant speed from Motor Type list;
• Set 60 value in the Constant speed motor field;
• Click ✓; the Rotary Motor branch will be created in the MotionManager design tree.

3.3.2 Specify Force
• Click Force to create and apply a force;
• Select the face of the piston;
• Select Constant from Force Function list;
• Set 100 N value in the Constant value F1 field;
• Click ✓; the Force branch will be created in the MotionManager design tree;

3.3.3 Running design study
• Click Calculate to run the study.
During computing period, the Motion Analysis Messages window will display specific informations; at the end of the computing period, the following options will be available:
• the button - to stop the animation;
• the slider - to set the animation into a current point;
• the list – to select the speed of the animation;
• the list – to select the type on animation: Normal (display at normal speed), Loop (display in a continuous loop), Reciprocate display forward, the reverse in a continuous loop);
• the button – to save the animation as avi or other file type;
• the button – to insert a view rotation or explode/collapse at the current time bar location.

4 Simulation and theoretical results comparison
Click Results and Plots to view the displacement curve as a function of time, fig. 15. Define the plot by:
• Displacement/Velocity/Acceleration;
• Linear Displacement;
• X component;
• Select the face of the Support and Pin2 elements for Simulation element; the selection will define the Pin2 displacement relative to the cylinder of the Support, where the global origin point is positioned.

Fig. 15 shows the maximal/minimal linear displacement: \(X_{Mi}=110\) mm and \(X_{Me}=40\) mm, the difference between these values is the stroke \(c=70\) mm. For the rotary motor speed of 60 RPM, 1 second corresponds to a complete mechanism rotation. The start point of this cycle is the maximal displacement \(X_{Mi}=110\) mm (point \(M_i\) fig. 1); at a halfway distance the minimal displacement \(X_{Me}=40\) mm is obtained (point \(M_e\) fig. 1); finally the mechanism returns to point \(M_i\).
Click Results and Plots to view the linear velocity curve $V_M$ as a function of time, fig. 16. Define the plot by:

- Displacement/Velocity/Acceleration;
- Linear Velocity - X component;
- Select the face of the Pin2 element for Simulation element; the selection will define the Pin2 velocity relative to the global coordinate system.

Fig. 16 show the maximal / minimal velocity values: 243 mm/sec and -243 mm/sec. The start and finally velocity value of the 1 second cycle is 0 mm/sec. The maximal velocity value calculated through rel. (6) is 243.86 mm/sec.

The $X_{Mo}$ and $V_M$ results are presented graphically in fig. 17, where the SolidWorks Motion points are placed over the theoretical linear displacement and velocity curves.

Click Results and Plots to view the acceleration curve $a_M$ as a function of time, fig. 18. Define the plot by:

- Displacement/Velocity/Acceleration;
- Linear Acceleration - X component;
- Select the face of the Pin2 element for Simulation element; the selection will define the Pin2 acceleration relative to the global coordinate system.

Fig. 18 shows the maximal/minimal acceleration values: 2026.5 mm/sec$^2$ and -1013.8 mm/sec$^2$. The start and finally acceleration value of the 1 second cycle is 2026.5 mm/sec$^2$.

Click Results and Plots to view the motor torque curve $M$ as a function of time, fig. 19. Define the plot by:

- Forces;
- Motor Torque - Y Component;
- Under Select rotational motor to create results, select the motor created for the application.

Fig. 19 shows the maximal/minimal motor torque values: 3866 N-mm and -3863 N-mm. The start and finally motor torque value of the 1 second cycle is 0 N-mm.

The results are presented graphically in fig. 20, where the SolidWorks Motion points are placed over the theoretical curve of the acceleration curve and motor torque.
Click Results and Plots to view the force curve as a function of time. Define by:

- **Forces**;
- **Reaction Force**;
- **X component** – for $P_b$ force, fig. 21; select the mate Concentric4 between Driving rod and Pin2 in the field Simulation element; select the Driving rod part in the field Component to define XYZ directions (optional); after selection, the three vectors attached to the Driving rod part, will define the local coordinate system in which the results will be displayed.
- **X component** – for $P_n$ force, fig. 22; select the mate Concentric5 between Piston and Pin2 in the field Simulation element; because the field Component to define XYZ directions (optional) remain empty, the results will be displayed according to the global coordinate system.
- **X component** – for $P_R$ force, fig. 23; select the mate Concentric2 between Crank and Pin1 in the field Simulation element; select the Crank part in the field Component to define XYZ directions (optional).
- **Z component** – for $P_T$ force, fig. 24; select the Crank part in the field Component to define XYZ directions (optional).

The results are presented graphically in fig. 25 where the SolidWorks Motion points are placed over the theoretical force curves.

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