Education of Economics with Maple

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Abstract: - The paper introduces experiences with education of economics of bachelor and master study at the Faculty of Business and Management of Brno University of Technology, which can be shared in national and international cooperation of economic universities. Particularly, teaching economical/financial modeling with Maple where students learn mathematics and mathematical modeling as an important interactive support for understanding and presenting solved real economic problems. In this paper are demonstrated example of exercise of microeconomics and two master theses applied in practice with the support of Maple.

Key-Words: - Education; Economics; Economic modeling; Maple system; Mathematics; Mathematical model; E-learning

1 Introduction

The requirements for education of economics and development in the whole world are closely linked with the needs of the current student generation, the communication and technical achievements and the various forms of studies (present, lifelong, e-learning etc.). In the age of the so-called "Net Generation" and widespread use of social networking tools, representing a significant part of the modern web technology (Web 2.0).

It issued from Person-Centred Approach (PCA) since its invention by Carl Rogers in the fifties of the last century has proven over the years as being practically applicable not solely in psychology but also in learning. It has been gradually extended by Rogers, his daughter Nathalie Rogers, and others [1] as a fully-fledged pedagogical approach suitable, among others, also for university education.

Motschnig and Derntl [2] have proposed its extension in terms of technology enhanced learning in the early 2000s and coined the term Person-Centred Technology-enhanced Learning (PCTeL).

We have shown to be an appropriate methodology reflecting the needs and expectations of the coming generation of students. It corresponds to what Stephen Downes [3] understands as “E-learning 2.0”.

Students of the Faculty of Business and Management of Brno University of Technology (FBM BUT) are led to study of economics and development as the understanding of the economic theory as its applications in real practice. This requires to use current Information and Communication Technology (ICT) for the qualitative analysis of real economic phenomena.

Simon and Blume [4] in 1994 wrote: “Within the last 30 years, mathematics has emerged as the language of economics. Today economists view mathematics as an invaluable tool at all levels of study, ranging from the statistical expression of real-world trends to the development of fully abstract economic system.”

Therefore, the education form and the content of courses of economics could be connected with mathematics and mathematical modeling courses that have to be continuously innovated [5].

Education of economics and its development at the Faculty of Business and Management (FBM) of Brno University of Technology (BUT) plays an important role also in the context of national and international cooperation and relations. Students’ internships abroad at FMB BUT is encouraged as
well as the teachers’ mobility (teaching abroad, study visits, international projects) supported by the BUT itself, the Ministry of Education, Youth and Sports of the Czech Republic and by international institutions and projects.

The paper introduces experiences with education of economics and development of the FBM BUT, which can be shared in national and international cooperation of economic universities.

2 Education of modeling in economics with Maple

In the last years, the FBM BUT followed above ideas [1], [2], [3], [5] and solved the project No. 3186/2010 [6] with the financial support of the Ministry of Education, Youth and Sports of the Czech Republic.

It was based on Clickable Mathematics of Maplesoft [7], where computer algebra system Maple [8] was chosen for the project No. 3186/2010 because BUT has used Maple multi licenses for its computer laboratory since 1996 and included some experiences of Geoff Renshaw from Oxford University for students of economics [5].

The concept of Online Clickable Mathematics includes [7]:

- Smart, context-sensitive right-click menus for instant access to Maple solvers and other command-free operations;
- Extensive range of palettes for visual editing of mathematical expressions;
- Interactive plotting and animation controlled by the mouse and not by endless parameters and attributes in a command;
- “Drag and drop” operations on plots, expressions, text, and more;
- Interactive assistants that provide easy mechanisms to solve and explore advanced topics for economical modeling such as differential equation-solving, optimization, and advanced visualization;
- A Maple Portal for Students, which acts as a guide for hundreds of common tasks from mathematics courses;
- Built-in selection of interactive tutors that offer visual e-learning environments for many important mathematical topics in pre-calculus, calculus, linear algebra, and more for economics;
- Handwriting recognition of mathematical symbols and equations;
- WYSIWYG document processing features that let you create complex documents of economics courses more quickly and easily than in a word processor or LaTeX;
- An Exploration Assistant that allows students and teachers to instantly create interactive mini economic applications and examples to explore the parameters of expressions.

Maple environment is user friendly for students and teachers. It offers complex solutions to problems through an interactive mathematical modeling step by step (i.e. construction, analysis and modification of simple or nontrivial models of real economic phenomena, or specific models which need high performance computing, downloading large database sets etc. and their solutions). Maple provides symbolic and numeric computations, advanced graphical two dimensional or three dimensional visualization, simulation and animation, [9].

Mathematic tools of Maple for students of economics including calculus, linear algebra, solving linear, algebraic and differential equations, optimization together with their applications in economic modeling and simulation, which traditionally understood as a hard and tedious discipline, create at the same time a great opportunity to show the real strength of PCtel [8].

Due to its symbolic power, Maple can be used in solving a tedious algebraic model, and even if a close form solution is not obtained, sensitivity analysis of the results and its visualization with respect to certain parameters can be executed at ease relative to other languages. This is very important for economic modeling in practice.

Therefore, the main idea of the project No. 3186/2010 was to strengthen the basic knowledge of mathematics and used this by students in economics on the basis of friendly interpretation of solved economic examples with Maple [6]. It was followed rules in teaching economic modeling [2], [3]:

- support for both teamwork and autonomy of students and their independence on teacher, particularly time and space independence;
- on-line learning, increased interactivity;
- provide ICT background (Maple and Internet) of virtualization and communication with social networking;
- interdisciplinary character of resolving problems in micro and macroeconomics with mathematics;
- active participation of students in projects, including research and in resolving specific problems of practice;
- cooperation with academic sphere (national and international) in economic universities.
The developed multimedia guideline [6] was based on interactivity and capabilities of Maple in the mathematical modeling of economic phenomena to support a decision making of CEO, company managers etc. It is underpinned by Maple which provides a broad base of computational tools individual and teamwork for the future use in practice [10].

Students could also download the Maple manual for economic modeling (in Czech) that is located at the web page of the *Czech Maple User Group* [11].

On the Maplesoft web page is possible to find special Centers, where students and teachers can find appropriate Maple programs and solved examples for education and assessment of knowledge in economics:

- the *Student Help Center* [12] offers a Maple student forum, on-line mathematic OraclEs, training videos and a mathematics homework resource guide;
- the *Teacher Resource Center* [13] offers teachers to get the most out of their Maple teaching experience. It provides sample applications, course material, training videos, white papers, e-books, podcasts, and tips, see [14];
- the *Maple T.A. Content Center* [16] offers hundreds of Maple T.A. Course Modules and Question Banks. Maple T.A. Technology is the potential for training content of the curriculum and for determination of clear rules for testing students’ knowledge. It offers high-quality means for classical teaching as well as for e-learning, training of knowledge and creation of tests. It is a real contribution for the students, instructors, tutors, guarantors and administrators (on-line training, testing, evaluation incl. creation of statistics of drawbacks or success, etc.).

### 2.1 Finance modeling tools in Maple

Maple has also a few packages that are very useful in Financial Engineering including partial differential equation solver, the statistical package and linear algebra. As well there are some recent toolboxes like the financial toolbox that can be utilized in simulation of price process etc. The Finance package is new package in the new release 15 of Maple which can be used in education of economics. It contains many tools for advanced financial modeling, as well as accessible tools for personal finance. On the personal finance side, there are tools that can be used for computing with mortgages or retirement packages.

The financial modeling tools include a wide range of stochastic processes that can be used to model option prices, such as Brownian motion, Ito processes, a stochastic volatility process, and more.

It also includes tools to compose complex processes out of these building blocks. Users can also create, manipulate, and analyze many types of financial instruments, such as American, Bermudan, and European options and swap ions and several types of bonds; short rate models; term structures of interest rates; and cash flows. The instruments can then be priced using analytic methods, lattice methods or Monte Carlo simulation - all using one of many date arithmetic conventions. Finally, the processes occurring in the package can be visualized in several ways.

### 3 Economically motivated examples

Application of calculus and linear algebra in economics are difficult for students of bachelor or master study of economics. Simple “mathematization” and comprehensible clarification of common problem from practice with computer support of Maple is a possibility to motivate at the beginning of subject matter students, in order to understand, that the mentioned mathematical objects have a sense to study just for their application in practice.

We show this on the example of economic exercise of definite integral its geometric interpretation from [6].

*Example 1:* Mr. Brown supposed, that in the first three years the oil consumption \( y \) in dependence on time \( x \) will grow exponentially \( y = e^x \) in his newly based trucking company. He was surprised, when he found out, that after three years, the oil consumption in the first year really grew exponentially, however the second and the third year has grown quadratic i.e. \( y = x^2 \). *How large is the amount of saved oil?*

Students solved this problem interactively with Maple. The depiction of curve and the delimitation of area, whose content matches in demand different oil consumption, (Fig. 1), is possible easily perform the clicking calculus and the simple Maple commands:

```maple
with(plots):
p:=plot(x,exp(x),x=0..5,y=0..25,thickness=2):
l:=line([3,0],[3,23],color=black,linestyle=3):
k:=line([1,0],[1,1.5],color=black,linespace=3):
f:=transform((x,y)->[x,y+exp(x)]):
p2:=plot(exp(x)-x2,x=1..3,filled=true,color=cyan):
```

...
display(l,k,f(p2));

\[
\int_1^3 (\exp(x) - x^2) \, dx; \text{evalf}(\%) \\
- e - \frac{26}{3} + e^3 \quad 8.7005884;
\]

The conclusion of this simple example solved interactively by students is clear: Mr. Brown will save cca 8.7 units of oil capacity in the second and the third year.

4 Master theses for practice
The FBM is interested in the good employment of its graduated students in practice. Therefore, it supports the close connection of education economics with practice of all forms of collaboration. The best results are reached in collaboration with companies on processing of final student works – master thesis. Students use their knowledge from bachelor and master study of economics e.g. in an economic modeling, optimization of company expenses and profit, regression analysis of time series, econometrics applications and other quantitative methods with an use of appropriate Maple programs.

Example 2: Student Tomas Bartulec in his master thesis [17] used Maple program “A Feedforward Neural Network Forecasting Exercise from Application Centre” [18] and applied this into the mathematical models of business efficiency of given company. After input analysis he divided input data of company for Maple program into three groups (teaching, testing, verifying). The output of Maple program was the value computed by using of neural network, whose process copying the process of values of EVA indicator [19], [20].

The basic architecture of the neural network was chosen a six layer preceptor working in the area of real numbers with linear rating of neurons. The network connection was absolute. The network was created by the Maple and it was composed of three coherent modules.

The neural network used Maple statements for storage of necessary data and intermediate calculated vectors and matrix. The matrix and vector form of data enables a simple use of cycles in Maple program. At learning of the neural network for EVA indicator was made as a whole 20 223 study epochs, [17]. The junction graph (Fig. 2 - output of Maple shows, that neural network is copying the trend of EVA indicator calculated classical way in [19], [20].

This master thesis introduced to company management a modern way of the use of neural network that automatically learnt how to transform “raw” input values from the balance on the concrete important EVA indicator.

Example 2: Student Žigárdy in his master thesis [21] used the polynomial regression analysis of Maple for data downloading from the Stock Exchange and developed interactive Maplet (Fig. 3) which also contains the visualization of fitting data by regression models. The Maplet required the automatic choice of the degree of regression polynomial with the use of statistical methods, which must reflect numerous conditions based on the experience of economics, finance or business.
Company managers can easily use this interactive Maplet for the prognosis of a behavior of the Stock Exchange. Its advantage is that it can easily upload time series in two ways: as the already downloaded data and stored data in the Maplet (Internal Data) or as the external sources (External data) by parsing data from the Stock Exchange.

Note: The Maplet is designed to process the large collection of data, too. That is typical for example in a financial environment (such as Stock Exchange etc.). We set a time period (in prescribed format xx.yy.zz) in the Maplet for the internal data (Start Date and End Date). We can set for the external data (Time period) their names of dependent and independent variables, which are used for labeling axes. The last toolbar on the left panel of the Maplet chooses output which we are interested in.

4 Conclusion
In our experiences with using Maple in bachelor and master study courses of economics (mathematics, microeconomics, macroeconomics etc) we could see that Maple has yielded four substantial and meaningful pedagogical improvements:

• As just mentioned, Maple has made the notion of ordinal utility much easier to get across to students.
• The graphical capabilities of Maple have allowed students to explore economic relationships in much more detail than previously possible.
• Using Maple to strip out the algebraic tedium of constrained optimization problems has allowed my students to not “lose the forest for the trees”. That is, students have a better appreciation for the mathematical foundation of microeconomics, and are able to spend more time thinking about the economic intuition built into their solutions, rather than concentrating disproportionately on the calculation of the solutions themselves.
• Using the power of Maple to solve constrained optimization problems should allow economists to begin to explore a larger variety of utility functions, and other economic relationships as well, including production functions.

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References: