Mathematical Algorithm for Heart Rate Variability Analysis

ALBERTS ALDERSONS, ANDRIS BUIKIS
University of Latvia
Raina bulv. 29, Riga, LV1459
LATVIA
aldalb@inbox.lv, buikis@latnet.lv, http://www.lza.lv/scientists/buikis.htm

Abstract: - Paper is destined for use in medicine, psychology and psychophysiology, in man’s self-development training, breathing technique’s training, in the field of stress resistance, health promotion, strengthening of the capacity for work; and it relates to the apparatus and methods for detection of the heart rate variability and it’s use in providing biofeedback during training sessions of organism’s vegetative balance and coherence.

Key-Words: Heart, Heart rate variability, Biofeedback, Method for detection, Self-development training, Stress resistance, Coherence.

1 Introduction
Heart Rate Variability (HRV) has occupied considerable and leading state as a non–invasive method for investigation of the activity levels and dynamics of interaction of the sympathetic and parasympathetic branches of the Vegetative Nervous System (VNS). During last two decades investigation of the physiological mechanisms of HRV, development of new apparatus and new computer programs for its registration, interpretation and practical utilization are considerably progressed.

2 Formulations of Problem and Nowadays Statement
Contrary to the view that under optimal conditions the heart beats sequence should remind the metronome, this is definitely not so. Due to influence of Autonomic Nervous System, affecting the sinus node (nervous center, located in the heart, which activates each of the next cardiac cycle starting after a pause), pulse beats followed each other at different time intervals, and, as a result, the time span between two consecutive heart beats can vary over a wide range – from 400 to 1500 msec. Plotting these following time intervals graphically, we get a wavy line. It is called the Heart Rate Variability (HRV) line. It turned out that this curve is very informative.

2.1. HRV is very easy to register
Any technique, that allows you to record an electrocardiogram (ECG), is valid. As recording equipment the conditions does not play important role (as it is for many other so called psycho physiological methods, for example, galvanic skin response), the technical details of the HRV record is no longer even object for serious discussions in the scientific literature.

2.2. HRV curve is very informative
When registering a heart beat, we get a simple series of numbers (intervals between every two consecutive heart beats in milliseconds, in average 70 numbers during 1 minute, such as, for example, 721, 743, 824, 801, 789, etc.) from which we may derive many different indicators of the activity of vegetative nervous system, and, in addition, these numbers are each characterized by strongly different physiological or psychological conditions. That is why, due to the wide availability of hardware, we see a rapid increase of searches of new algorithms, approbation of new mathematical models.

2.3. HRV application is very convenient and of practical importance
We should start with the fact that the HRV was one of the chief methods used for evaluation of physiological state in aerospace medicine and psychology (the period around years 1950-1980, mainly in Russia). There are many studies that indicate the relationship between emotions and...
changed SRV indicators. HRV may be used as a indicator of risk prediction after myocardial infarction [3], for assessing of Autonomic dysfunction in patients with fibromyalgia [4], [5], as an Indicator of Mental Workload [6], of state anxiety in healthy individuals [7], as indicator of sleep disorders [8], as indicator of validity of different Breathing styles [9], as tool for analysis of different emotional states [10].

2.4. Mathematical aspects of the SRV methodology and measurement standards.

Time Domain Methods

The variations in heart rate may be evaluated by a number of methods. Perhaps the simplest to perform are the time domain measures. In these methods, either the heart rate at any point in time or the intervals between successive normal complexes are determined. In a continuous ECG record, each QRS complex is detected, and the so-called normal-to-normal (NN) intervals (that is, all intervals between adjacent QRS complexes resulting from sinus node depolarisations) or the instantaneous heart rate is determined. The simplest variable to calculate is the standard deviation of the NN intervals (SDNN), that is, the square root of variance. Other commonly used statistical variables calculated from segments of the total monitoring period include SDANN, the standard deviation of the average NN intervals calculated over short periods, usually 5 minutes, which is an estimate of the changes in heart rate due to cycles longer than 5 minutes, and the SDNN index, the mean of the 5-minute standard deviations of NN intervals calculated over 24 hours, which measures the variability due to cycles shorter than 5 minutes.

The most commonly used measures derived from interval differences include RMSSD, the square root of the mean squared differences of successive NN intervals, NN50, the number of interval differences of successive NN intervals greater than 50 ms, and pNN50, the proportion derived by dividing NN50 by the total number of NN intervals. All of these measurements of short-term variation estimate high-frequency variations in heart rate and thus are highly correlated. Since many of the measures correlate closely with others, the following four measures are recommended for time domain HRV assessment (1) SDNN (estimate of overall HRV), (2) HRV triangular index (estimate of overall HRV), (3) SDANN (estimate of long-term components of HRV), and (4) RMSSD (estimate of short-term components of HRV).

2.5. Frequency Domain Methods

The analysis of the tachogram has been applied since the late 1960s. Power spectral density (PSD) analysis provides the basic information of how power (variance) distributes as a function of frequency. Independent of the method used, only an estimate of the true PSD of the signal can be obtained by proper mathematical algorithms. Methods for the calculation of PSD may be generally classified as nonparametric and parametric. In most instances, both methods provide comparable results. The advantages of the nonparametric methods are (1) the simplicity of the algorithm used (fast Fourier transform [FFT] in most of the cases) and (2) the high processing speed, while the advantages of parametric methods are (1) smoother spectral components that can be distinguished independent of preselected frequency bands, (2) easy post processing of the spectrum with an automatic calculation of low- and high-frequency power components with an easy identification of the central frequency of each component, and (3) an accurate estimation of PSD even on a small number of samples on which the signal is supposed to maintain stationery. The basic disadvantage of parametric methods is the need of verification of the suitability of the chosen model and of its complexity (that is, the order of the model).

3 Training Method for Promotion of Emotional Stress Reduction,
**Psychological Coherence and Vegetative Balance** [11]

### 3.1. Algorithm

Investigation is destined for use in medicine, psychology and psychophysiology, in man’s self-development training, breathing technique’s training, in the field of stress resistance, health promotion, strengthening of the capacity for work; and it relates to the apparatus and methods for detection of the heart rate variability and its use in providing biofeedback during training sessions of organism’s vegetative balance and coherence [11].

According to offered method, the evaluation of the heart rhythm’s variability parameters is performed during the three time intervals between four consecutive heart beats; and, when the arithmetical difference between two consecutive intervals has identical sign with the sign of arithmetical difference between previous two consecutive intervals (previous set of intervals), a positive biofeedback signal (PBFS) is generated, but if the differences are with different sign, a negative biofeedback signal (NBFS) is generated; during time interval, greater than 20 sec, a “Central Index (CI)” is calculated, according to formula:

\[
\text{PBFS, NBFS and CI are calculated in direct time regime, and are demonstrated on the screen of mobile training device or personal computer.}
\]

They are used for ruling various systems (physical medicine apparatus, learning systems, different light – sound stimulation devices).

The invention relates to breathing technique training, stress resistance, health and capacity building areas, in particular, to apparatus and methodologies for determining heart rate variability and its application in biofeedback of various vegetative body's balance and consistency conditions during training.

It is well known that the autonomic nervous system (ANS) regulatory mechanisms are affecting many organs and systems of human body. The system works as a very complex physiological oscillator unit. One of the main functions of ANS is provision of the optimal balance between the oscillators. Any protracted regulatory balance disorder may lead to functional disturbances that can damage the human body. One of the main creators of this imbalance is emotional stress. There are many methodologies that apply for funds to reduce emotional stress, its positive impact on ANS regulatory function. This technology is designed to switch the ANS to specific "resonant" state, characterized by many physiological parameters synchronous oscillator, coherent, sine wave-like characteristic, such as heart rate (SD), BP (blood pressure), respiration rate (ER), and so on.

Good representing of resonant states is a heart rate variability (HRV), or respiratory sinus arrhythmia where the heart rhythm (HR) oscillates synchronously with the respiratory cycle. Such synchronization automatically regulates many other body functions, including certain brain rhythms and metabolic processes. It is also known that the HRV is the represent not only of his own rhythm, but also of the size of the state of harmony of many other autonomic nervous system reactions, so that could be used as the body's overall coherence and the coincidence index. Since the aim of the stress-reduction techniques is to create a resonance between the oscillating physiological parameters, it is important to be able to assess the dynamics of this resonance significance and stability in quantitative values, and generate feedback to reinforce positive changes.

To make SRV analysis and evaluation of the SRA parameters electrocardiographic (ECG), signal is commonly used. Interbeat intervals are derived from the ECG as the intervals between two adjacent R-waves. It is very accurate and promising method, although quite inconvenient and relatively expensive. Photoplethysmography is used as an alternative method, applying the small finger position sensor. The sensor emits infrared light into the skin. The emitted light in part, linked by blood flow. Light absorption / reflection coefficient is proportional to blood flow changes. Pletismogramm signal contains periodic rapid elevations showing vascular pulsations. They can be used to determine heart interbeat intervals, characterized by the distances between two pletizmogrammas peaks. In all we know the source of information where the feedback is used for the realization of HRV,
the calculation algorithm is characterized by any of the similar characteristics - the time interval, necessary for heart rate records to allow quality and standards of appropriate HRV calculations. Classical case, by international agreement, this amount of time should be 5 minutes long [1]. After the prototype and analogues, the most modest calculating the required number of pulse beats, which can calculate the approximate data SRV - 100 pulse beats (1950-up to a 80-year Soviet scientific data, which occurred in the Soviet cosmonaut psycho physiological monitoring of the condition). Absolutely the smallest number of pulse beats needed to make something quite inaccurate, but calculate the average heart rate, pulse rate is 10-15 beats, but the average pulse is not the pulse rate variability. This means that when working on a prototype or analogue techniques, 
a) To collect data before the feedback signal for generation must wait at least 1-6 minutes; 
b) Each subsequent result can be obtained again only after 1-6 minutes; 
c) the work in "moving average" mode is in better shape, because the waiting for "dead" period is necessary only in the beginning, and then we are able to be calculate for each pulse all SRV figures for the previous 1.5-6-minute period, but the improvement is somewhat apparent (illusory), because in essence we do not obtain a HRV description of each concrete short time moment, but we have the result of previous period average. All these methods are satisfactory, if we are interested in slow, tens of minutes and hours changing physiological responses. But usually all the physiological and psychological processes that we want to adjust to the reverse link are not so long. Even breathing, is used to improve the pulse variability. Very rarely, including yoga exercises, one respiratory cycle lasts longer than one minute. So, with the prototype and its analogues, we only very conditionally can hope that we as a feedback signal truly use peculiar HRV indicators. But any stress during minutes may generate enormous changes, and that is why adequate HRV record figures are essential.

3.2. Practical application of the proposal in detail
The first part is exactly the same for all pulse variability detecting methods. It is as follows. By means of any standard EKG or pulse beat recording device time intervals between each successive electrocardiogram QRS complex or pulse wave must be fixed with an accuracy of at least 1 ms. It is generally recognized series of numbers (768, 753, 728, 764, 801, 853, etc.), which means that, for example, between the first and second QRS complex onset elapses 768 ms or 0.768 seconds; respectively, during 1 minute exist 60/0,768=78,12 heart beats, and that is so called instantaneous heart rate or instantaneous pulse. The time elapsed between the second and the third QRS complex or the second and the third pulse beat is 79.68, the following instantaneous pulse is 82.43 per minute and so on. But the momentary heart cycle length (or instantaneous pulse) rates in practice are not used as pulse feedback indicators. They are very changeable, and physiological benefits lies precisely in its ability to characterize this variability mathematically, i.e., to find an algorithm that is the best and more practical to characterize a specific physiological condition or situation, and can be used as feedback indicators. Standard HRV indicators are appropriate for scientific purposes, but are less adequate for practical purpose as biofeedback elements. Our work allowed us to see the possibility to derive a special mathematical algorithm for processing the instantaneous pulse data with the aim to generate a realistic indication of the vegetative n.s. regulatory processes, and, thus, create a parameter, which could serve as a convenient indicator used biofeedback. Usually offered calculations, used for biofeedback signal, need at least 100 -500 test points, located before the instantaneous time point recorded. For realisation of our algorithm only 2 points before the instantaneous time point recorded are necessary, and, more importantly, the conclusions we make with only one, the most recent time interval, with his relationship to the previous interval, which is just a report. The result therefore does not
describe the average physiological state of several minutes, but about 1 second. Thus, feedback signal can be shown immediately after the fourth heart beat.

3.3. Our algorithm is implemented as follows

The following convention is adopted:

\[ P(n) = \text{the time moment of the current heart beat (fourth, if the calculations carried out at the fourth heart percussion)}; \]
\[ P(n-1) = \text{(third, if the calculations carried out at the fourth heart percussion)}; \]
\[ P(n-2) = \text{(second, if the calculations carried out at the fourth heart percussion)}; \]
\[ P(n-3) = \text{(first, if the calculations carried out at the fourth heart percussion)}; \]
\[ T(n) = \text{time interval between } P(n) \text{ and } P(n-1); \]
\[ T(n-1) = \text{time interval between } P(n-1) \text{ and } P(n-2); \]
\[ T(n-2) = \text{time interval between } P(n-2) \text{ and } P(n-3); \]

Beginning from 4-th pulse beat and forth, each pulse beat is granted with following designation (Parameter \( V(x) = + \) or \( V(x) = - \)) according to following algorithm:

\[ V(n) = "+", \text{ if } T(n) > T(n-1); \]
\[ V(n) = "-", \text{ if } T(n) < T(n-1); \]
\[ V(n) = "0", \text{ if } T(n) = T(n-1); \]
\[ V(n-1) = "+", \text{ if } T(n-1) > T(n-2); \]
\[ V(n-1) = "-", \text{ if } T(n-1) < T(n-2); \]
\[ V(n-1) = "0", \text{ if } T(n-1) = T(n-2). \]

On each pulse beat beginning from the fourth beat, the following calculations are made:

- If \( V(n) = "+" \) and \( V(n-1) = "+" \) then PBFS (Positive Biofeedback Signal) = PBFS + 1
- If \( V(n) = "-" \) and \( V(n-1) = "-" \) then PBFS = PBFS + 1
- If \( V(n) = "+" \) and \( V(n-1) = "-" \) then NBFS (Negative Biofeedback Signal) = NBFS + 1
- If \( V(n) = "-" \) and \( V(n-1) = "+" \) then NBFS = NBFS + 1
- If \( V(n) = "0" \) and \( V(n-1) = "-" \) then NBFS = NBFS + 1
- If \( V(n) = "+" \) and \( V(n-1) = "0" \) then NBFS = NBFS + 1
- If \( V(n) = "0" \) and \( V(n-1) = "0" \) then NBFS = NBFS + 1

For time period of 20 sec or longer the “Central Index (CI)” is calculated as follows:

\[ \text{Central Index (CI)} = \frac{\text{amount of all PBFS} \times \text{amount of all NBFS}}{\text{PBFS amount} \times \text{NBFS amount}} \times 100 \text{ (percent)} \]

This calculation can be performed in both ways: immediately after each heart beat, or as a retrospective analysis of a particular situation, particular time interval may be made. Since CI is always within the range between 0 and 100%, and the value estimate is unchangeable, it may well be used as comparative indicator and benchmark for the one and the same person at different life, work and health situations, as well as various human condition inter-comparisons.

CI is very dynamic, if you use it as a moving average, for example, from 20 heart beats, and, at the same time, it is also a solid, stable indicator, that can be used to compare averages for different people, or one and the same human figures on different days, months or years, if the expense of a 5-minute long recorded is performed in standardized conditions. Many of our measurements allow introducing approximate boundaries of this index - readings above 50% usually mean good health and high performance. Specially trained people, who manage yoga and deep breathing techniques, are able to increase this figure, and long to hold 70-90% range. To emotional stress, the figure falls below 30%, and in some situations, it is only 3-6%.

4 Conclusions

Training technique to reduce the emotional stress, physiological coherence and autonomic balance, through a special breathing movements, body postures, psychological, spiritual and other practices that comply with heart rate variability feedback control, in which the parameter estimate can be made and the feedback signal is generated at each pulse of percussion, the calculations are made for direct-time, and are played on portable devices or PC screen, and the parameters used in various systems management (physical medicine equipment, stress management, training and work on capacity-building, light - sound stimulation systems) varies with the fact that, in order to improve the efficiency of the method 1st than 20 seconds over a longer period,
calculate the "Central Index (CI)," according to the formula: amount of all PASS / (all PASS and NASS amount) * 100 (percent); 2nd CI is calculated as a single value for a selected period of time, both as a "rolling average" of at least 5 previous pulse intervals between adjacent pulse beats; 3rd PASS, NASS and CI will be calculated direct-time, and are played on portable devices or a PC screen; 4th PASS, NASS and CI using different management systems (physical medicine equipment, training systems, different light - sound stimulation systems)

Acknowledgements: Research was supported by Council of Sciences of Latvia (Project 09.1572).

References: