Motor Voltage High Harmonics Influence to Efficient Energy Usage

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Abstracts: Analysis of the effect of non-sinusoidal voltages on the three-phase induction motor is presented in the paper. When the induction motors are supplied by a rectangular shape of the voltage inverter with high levels of harmonic voltage ($U_{h,i} = 1/h_i$), increase of the power losses in the windings of stator and rotor is: close to 20%$P_{Cu,N}$, for the lower power motors (3-10 kW), and a little bit over 30% $P_{Cu,N}$, for high-power motors (> 100 kW). The latter figure corresponds to the values that are found in literature, while the value of 20%$P_{Cu,N}$ when it comes to lower power motors, is much higher than the figure (about 5-10%) which is quoted in the literature. The reason for this is in the fact that it is (wrongly) believed that the resistance of the rotor does not change for higher harmonics frequencies, ie. that is the identical for all harmonics ($R_{r,h} = R_{r,1} = R_r = Const.$).

Key words: Induction motor, Non-sinusoidal voltage, Harmonics, Power losses, Energy efficiency.

1 Introduction

Electrical energy savings in the drive could be realized by improvements of power quality in the consumer network. Electrical energy consumption in the plant can be reduced by improving the quality of AC power consumers. Namely, power losses and reactive load are further increased due to unbalanced voltage and (or) the presence of harmonics in supply voltage. Unbalanced voltage can occur due to the presence of larger single-phase consumers or asymmetrical capacitor banks with damaged or capacitors switched off due to the fuse burning only in one phase. Nowadays, the presence of higher harmonics in the supply voltage is even more frequent due to the growth of consumers who are supplied through the rectifiers and inverters: regulated actuators, electrothermal consumers and consumers alike.

The term power quality [1] mainly involves the quality of the supply voltage, which should meet the prescribed criteria in respect of:

– voltage (tolerances are within $U_N \pm 5\%$),

– allowed harmonic distortion $THD_U \leq 3\text{-}8\%$ (higher value refers to the network of lower voltage), and

– permissible voltage unbalance is 3%, and has a greater impact on the proper and economical operation of the engine.

Long ago, the presence of third harmonic in voltage waveform has been observed, but since it practically does not affect the operation of the engine (except when the engine is connected through a neutral conductor), the effect of voltage distortion on the operation of induction motors was not further considered. Voltage distortion in electric power systems and networks is more pronounced, and the higher voltage harmonics on the motor connection may exist for two reasons:

– motor is supplied by a variable frequency converter to regulate speed [2], and

– higher harmonics exist in the network voltage and they are consequence of the existence of other nonlinear loads and (or) converters which are supplied from the network, [3].

Namely, while in plants with the frequency and speed regulators are taking steps to reduce power losses, motors which are designed to operate at a fixed frequency are exposed to an increased power loss when there are higher harmonics in the voltage waveform. For these reasons, you should first consider the latter case, ie. discus the impact of higher voltage harmonics on the parameters and characteristics of the motor. It should be started from the equivalent scheme with appropriate parameters for the higher harmonics. Based on the value resistance ($R_{M,h}$), reactance ($X_{M,h}$) and motor impedance ($Z_{M,h}$) for the higher harmonics, [4] and [5], it is possible to determine:

a) the impact of voltage harmonics on the increase in losses and reduced efficiency of induction motors, as well as

b) the impacts of motor resistance ($R_{M,h}$), reactance ($X_{M,h}$) and impedance ($Z_{M,h}$) to the total resistance, reactance and impedance of the network for higher harmonics.

The latter is particularly important in a network with capacitors for reactive power compensation when there is a possibility of resonance, because in these regimes increases the influence of impedance on the overall impedance of the motor network, so that it can sometimes be decisive for the emergence of resonant phenomena.

Equivalent circuit of induction motor for higher harmonics is similar to the one of the induction motor in short circuit with the locked rotor. Motor slip, compared to harmonic rotating fields, is given by (l):

$$s_h = 1 \pm (1 - s_1)/h \qquad (1)$$

where h is the order of harmonic ($h = f_h/f_1$) and s_1 is the motor slip in relation to the first harmonic field. Thus, at normal motor operation with slip $s = 0.01$-0.06, slip for higher harmonics is $s_h = 1 \pm 1/h \approx 1$ (approximately equal to 1), while in the starting mode $s_h = 1$ (exactly equal to one). Skin effect causes an increase of the rotor conductor resistance and reduction of the rotor conductor inductance (Fig. 2). The effect is similar to the short circuit motor mode (with the locked rotor), but the increase of the resistance and decrease of the inductance are even greater since the frequencies of the induced currents in the rotor chamber are several times larger, $f_2 = (h \pm 1)f_1 >> 1$, for $h \geq 5$.

In order to determine the resistance and reactance of motor equivalent scheme for higher harmonic it is best to start with the appropriate scheme for induction motor in the short circuit regime. But, in literature and in practice it is often done differently:

a) When it comes to calculations of power losses, it is usually started from the characteristics and motor parameters for the nominal regime, probably because they are more accessible.
b) When it comes to modeling of an induction motor in order to calculate higher harmonics in power networks:
   – resistance of the motor is modeled within the modeling of the total active power of all consumers in the network, though it is apparent that it does not correspond with the impact of motor resistance on the suppression of higher harmonics, and
   – equivalent reactance is estimated based on the number and (installed) motor power in operation and the motor inverse reactance, which is multiplied by the number $h = h/f_1$ (h-order of the harmonic).

As the frequency of induced currents in the rotor is h times higher, compared to the frequency of induced currents in the rotor of the motor in short circuit, it is logical that it should be started from the parameters and equivalent schemes for induction motor in the regime of short circuit. Besides the obvious increase in accuracy, the procedure will not be more complicated, because in the paper [6] developed method is presented for calculating the parameters of motor equivalent schemes in short circuit on the basis of the catalog data, which will be shortly presented and used to determine the parameters of the equivalent scheme for the higher harmonics.

2 Equivalent Circuit and Motor Parameters for High Harmonics

2.1 Equivalent circuit of induction motor for high harmonics

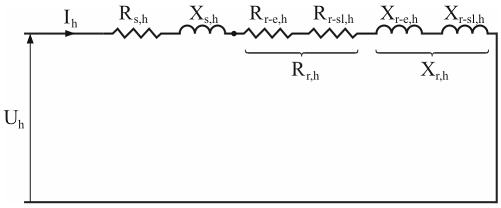
Equivalent circuit for the higher harmonics is identical to the corresponding equivalent circuit for short circuit mode for primary frequency f_1. Only, instead of rotor short circuit resistances ($R_{r-e,sc}$, $R_{r-sl,sc}$) and rotor slot short circuit reactance (X_{r-e}, $X_{r-sl,sc}$), the corresponding values rotor resistances ($R_{r-e,h}$, $R_{r-sl,h}$) and rotor reactances ($X_{r-e,h}$, $X_{r-sl,h}$) for the higher harmonics are present (Fig. 1a).

Increasing the order of harmonic ($h = f_h/f_1$) leads the frequency of currents inducted in rotor conductors to increase h times, compared to the one in the regime of short circuit voltage when there is only the first harmonic frequency f_1. Skin effect is practically expressed only on the part of the conductor in the slot of the rotor, ie. leads only to increase of rotor slot resistance ($R_{r-sl,h}$) and reduction of slot rotor slot inductance ($L_{r-sl,h}$). As a result, values of rotor slot resistance and rotor slot reactance become equal, starting from relative penetration depth $\zeta = H/\partial \geq 2$, [4]-[6]. As the depth of penetration is, already for the fifth harmonic, $\partial_{Al,h=5} = 4.5$ mm, $R_{r-sl,h}$ is always equal to $X_{r-sl,h}$ ($R_{r-sl,h} = X_{r-sl,h}$), Fig. 2. For this reason, similar to the corresponding scheme for short-circuit mode, the rotor reactance ($X_{rl,h}$) and the rotor resistance ($R_{r,h}$) are separated into two components in the equivalent circuit for the higher harmonics [4], Fig.1a, ie:

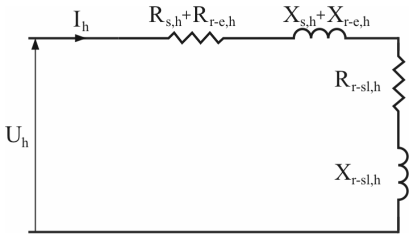
$$R_{r,h} = R_{r-sl,h} + R_{r-e,h} \qquad (2)$$

$$X_{r,h} = X_{r-sl,h} + X_{r-e,h} \qquad (3)$$

where $R_{r\text{-}e,h}$ and $X_{r\text{-}e,h}$ are rotor end resistance and rotor end reactance, successively ("e" in the index comes from the English abbreviation of the word "end").



(a)



(b)

Fig. 1: Motor equivalent circuit for higher harmonics: a) with separate rotor resistances ($R_{r\text{-}e,h}$, $R_{r\text{-}sl,h}$) and reactances ($X_{r\text{-}e,h}$, $X_{r\text{-}sl,h}$) and b) with grouped resistances ($R_{s,h} + R_{r\text{-}e,h}$) and reactances ($X_{s,h} + X_{r\text{-}e,h}$)

Finally, resistance and reactance of stator windings, and resistance and reactance of rotor conductors outside of slots are grouped (Fig. 1b), for which the influence of skin effects can be neglected, and whose values are not changing in all modes from the nominal regime to the regime of short circuit. These are:

- collective resistance $R_{s,h} + R_{r\text{-}e,h}$ and
- summary reactance, $X_{s,h} + X_{r\text{-}e,h}$.

The remaining resistance $R_{r\text{-}sl,h}$ and reactance $X_{r\text{-}sl,h}$, were separated on the equivalent circuit, Fig. 1b.

2.2 Parameters of Equivalent Circuit for High Harmonics

Current frequencies of individual harmonics in the rotor winding are h times higher ($f_{r,h} \approx h \cdot f_1 = h \cdot f_{r,SC}$), so the theoretical depth of penetration (∂_h) of these currents is $\sqrt{h}$ times lower. The explanation is based on the fact that, for motors with power above 3 kW (or with relative depth of penetration $\zeta_{ScC} = H/\partial_{SC} \geq$ ≥ 1.2), not the entire section (or height H) of rotor bar is being used, so the actual depth of penetration of individual harmonic currents (∂_h) is $\sqrt{h}$ times lower. From that it is concluded that the corresponding section of rotor conductor is $\sqrt{h}$ times lower, so the values of rotor slot resistance are $\sqrt{h}$ times higher and values of rotor slot inductance are $\sqrt{h}$ times lower as compared to those values for the fundamental harmonic in short circuit regime. In general, rotor slot resistance ($R_{r\text{-}sl,h}$), rotor slot inductance ($L_{r\text{-}sl,h}$) and rotor slot reactance ($X_{r\text{-}sl,h}$) in function of harmonic order $h = f/f_1$ (ie. the relative depth of penetration $\zeta_h = H/\partial_h$) are shown in Fig.2.

If the relative penetration depth equals $\xi = H/\delta_1 \geq 1.5$, then equality $X_{r\text{-}sl,sc} = R_{r\text{-}sl,sc}$ is already true for the fundamental harmonic. As for the harmonics of order $h \geq 5$ the relative depth of penetration is equal to $\xi_h = H/\delta_h \geq 2$, then for motors of all powers is always:

$$R_{r,h} = R_{r-sl,h} + R_{r-e,h} \tag{4}$$

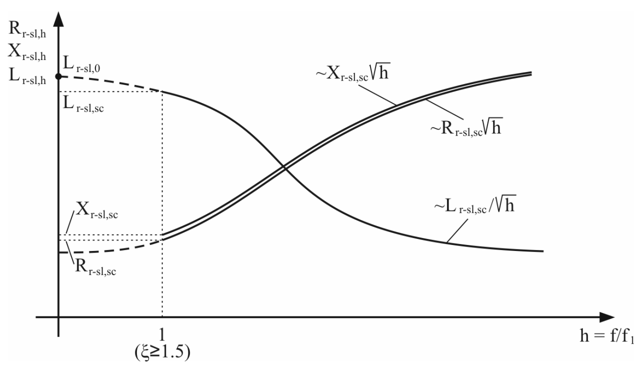


Fig. 2: Rotor slot resistance ($R_{r\text{-}sl,h}$), slot inductance ($L_{r\text{-}sl,h}$) and rotor slot reactance ($X_{r\text{-}sl,h}$) depedencies of harmonic order $h = f/f_1$, expresed by the values for short circuit parameters: $R_{r\text{-}sl,sc}$, $L_{r\text{-}sl,sc}$ and $X_{r\text{-}sl,sc}$

Begining from the value of relative penetration value $\zeta = H/\partial \geq 1.5$, or for the height of rotor alumunium bars $H_{Al} \geq 15$ mm, the following relations are valid (Fig. 2):

$$R_{r-sl}(hf_1)/R_{r-sl,sc}(f_1) = \sqrt{h} \tag{5}$$

$$L_{r-sl}(hf_1)/L_{r-sl,sc}(f_1) = 1/\sqrt{h} \tag{6}$$

According to this, it is being concluded that, for motors with power above 5 kW ($H_{Al} \geq 15$ mm), the following equations could be written:

$$R_{r-sl,h} = R_{r-sl,sc} \cdot \sqrt{h} \tag{7}$$

$$L_{r-sl,h} = L_{r-sl,sc} / \sqrt{h} \tag{8}$$

$$X_{r-sl,h} = X_{r-sl,sc} \cdot \sqrt{h} \tag{9}$$

This means that, based on given values of rotor slot resistance ($R_{r\text{-}sl,sc}$), rotor slot inductance ($L_{r\text{-}sl,sc}$) and rotor slot reactance ($X_{r\text{-}sl,sc}$) for primary frequency f_1 in short circuit mode, the corresponding parameter values for higher harmonics $h = f_h/f_1$ can be calculated, ie. values: resistance ($R_{r\text{-}sl,h}$), inductance ($L_{r\text{-}sl,h}$) and reactance ($X_{r\text{-}sl,h}$).

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The values of penetration depth in the copper conductors in stator winding are $\delta_{Cu,} \geq 1.6$ mm for frequencies $f \leq 2000$ Hz. As a rule, since the diameter of the stator winding conductors is $d_{Cu} \leq 2$ mm, it can be assumed that the value of stator windings resistance keeps almost the same value for all harmonics of order $h \leq 40$ (or 2000 Hz/50 Hz), ie. the following equality is valid:

$$R_{s,h} \approx R_{s,1} = R_s \text{, for } h \leq 40 \tag{10}$$

The same assumption approximately applies to the rotor end resistance ($R_{r\text{-}e,h}$) and to the rotor end inductance ($L_{r\text{-}e,h}$). On this basis, the following equations could be written:

$$L_{r-e,h} \approx L_{r,e} = R_s \tag{11}$$

$$X_{s,h} + X_{r-e,h} = h\left(X_s + X_{r-e}\right) \tag{12}$$

$$R_{s,h} + R_{r-e,h} = R_s + R_{r-e} \tag{13}$$

The total value of motor resistance ($R_{M,h}$), motor reactance ($X_{M,h}$) and motor impedance ($Z_{M,h}$), for higher current harmonics, are given in the following expressions:

$$R_{M,h} = \left(R_s + R_{r-e}\right) + R_{r-sl,sc} \cdot \sqrt{h} \tag{14}$$

$$X_{M,h} = \left(X_s + X_{r-e}\right) \cdot h + X_{r-sl,sc} \cdot \sqrt{h} \tag{15}$$

$$Z_{M,h} = \sqrt{R_{M,h}^2 + X_{M,h}^2} \tag{16}$$

Higher harmonics currents ($I_{M,h}$), due to the existence of the corresponding higher harmonics voltages ($U_{M,h}$), are being calculated using the formula (as a percentage of nominal current $\%I_N$):

$$I_{M,h} = 100 \cdot U_{M,h} / Z_{M,h} \left(\%I_N\right) \tag{17}$$

Power losses in the motor, which are the consequence of harmonic currents through the windings of stator and rotor, are being calculated using the formula (as a percentage of nominal motor power $\%P_N$):

$$P_{Cu,h} = 100 \cdot \frac{R_{M,h} \cdot I_{M,h}^2}{\eta \cdot cos\,\varphi} \left(\%P_N\right) \tag{18}$$

Commonly, power losses which are the consequence of harmonic currents through the windings of stator and rotor, are being calculated as a percentage of nominal power losses in motor windings ($\%P_{CuN}$). Thus, assuming that losses P_{CuN} make up one half of the total power losses in motor, their value can be determined from the formula:

$$P_{Cu,h} = 100 \cdot \frac{R_{M,h} \cdot I_{M,h}^2}{\eta \cdot cos\,\varphi} \cdot \frac{2\eta}{1-\eta} \left(\%P_{CuN}\right) \tag{19}$$

In the paper of this author [7] is shown that:

- values of rotor slot resistances ($R_{r,sc}$) and rotor slot reactances ($X_{r,sc}$) in the short circuit mode are approximetely the same for motors of all powers in a serie, and they are approximetely equal to each other, ie:

$$R_{r-sl,sc} = R_{r,sc} - R_{r-e,sc} \approx Const. \quad 20)$$

$$X_{r-sl,sc} = X_{r,sc} - X_{r-e,sc} \approx Const. \tag{21}$$

  Their values are in narrow limits $X = R_{r\text{-}e,sc} \approx \approx 0.025 \div 0.030$ pu, respectively for the motors of large (>100 kW), medium (11-50 kW) and low power (1-7.5 kW);
- the values of the remaining part of the rotor resistance and reactance corresponding to part of the conductor outside the slots, are approx. $R_{r\text{-}e} \approx 0.33R_r$, $X_{r\text{-}e} \approx 0.33X_r$,
- the summary value of resistance $(R_s + R_{r\text{-}e}) \approx \approx Const.$, retains approximately the same value in all modes, operating regime, the regime of short circuit and for regimes with higher harmonics, and that applies to summary value of the related inductance $(L_s + L_{r\text{-}e}) \approx Const.$

3 Analysis of Motor Performance at Non Sinusoidal Voltage

3.1 Basic Terms

Motor performance at non sinusoidal voltage will be considered on the example when the motor is powered with a voltage of rectangular shape, containing harmonics of order $h = 1, 5, 7, 11, 13, 17, ..., 35$ and 37, whose amplitudes, respectively, have value of U_1/h. The analysis is usually performed by dissolving a given wave voltage curve into the harmonic series and specifically examining (calculating) the impact of each of the harmonics present. In a rectangular shaped voltage, harmonic series does not contain odd harmonics, ie:

$$u = \frac{4}{\pi} \cdot U \cdot \left[sin\,\omega t + \frac{sin\,3\omega t}{3} + \frac{sin\,5\omega t}{5} + ... \right] \tag{22}$$

Each harmonic voltage creates a magnetic field that rotates with the speed (n_h) which is significantly higher than the speed of rotation of the first harmonic (n_1), ie:

$$n_h = \frac{60 f_h}{p} = \frac{60h \cdot f_1}{p} = h \cdot n_1 \tag{23}$$

where f_h is frequency of harmonic of order h.

As the phase shift of first harmonic is 120^0, for higher harmonics that shift is $\alpha_h = 120^0 \cdot h$. Therefore, the third and higher harmonics divisible by three are in-phase and do not produce a magnetic field. Because of different phase shifts (α_h), the phase sequence of harmonics of different order is also different. Thus, 5th harmonic creates a field that rotates in the opposite direction in relation to the fundamental harmonic. For the 7th harmonic, the direction of rotation fields coincides with the fundamental harmonic. In motor regime with slip equal to $s = 0.01\text{-}0.06$, in relation to the rotating fields of higher harmonics the slip is approx. equal to 1 ie. $s_h = 1 \pm 1/h \approx 1$.

3.2 High Harmonic Losses when Motor Operates at Rectangular Shaped Voltage from Converter

When motor is supplied by rectangular shaped voltage, which contains harmonics of order $h = 1, 5, 7, 11, 13, 17, 19, ...35$ and 37, and whith amplitudes of $U_h = U_1/h$, it is required to calculate the correspondent approximate values of power losses for motors with nominal powers from 5 kW to 400 kW (ie. for the values of stator resistance from $R_s = 0.045Z_N$ to $R_s = 0.015Z_N$ and correspondent approximate values in short circuit mode, $R_{r,SC} \approx 0.03$, for each motor.

For motors with power 3 kW - 400 kW, in the same order, the ranges of values are given for:

- efficency factor $\eta = 0.85\text{-}0.95\%$ and power factor $cos\varphi = 0.85\text{-}0.92$, ie. $\eta \cdot cos\varphi = 0.72\text{-}0.875$,
- stator resistance (R_s), for example from $R_s = 0.045Z_N$ to $R_s = 0.015Z_N$,

and the corresponding values of resistance and reactance of short circuit, which are calculated in [5], [7]:

- the corresponding value of stator resistance $R_{s,h} = 0.015Z_N \div 0.050Z_N = Const.$,
- the corresponding value of rotor resistance $R_{r,h} = 0.03 \cdot \sqrt{h}$,
- the corresponding values of motor resistance $R_{M,h}$, according to expression (14),
- the corresponding values of motor reactance $X_{M,h}$, according to expression (15),
- the corresponding values of motor impedance $Z_{M,h}$, according to expression (16),
- higher harmonic currents, according to expression (17), and
- the corresponding value of the power losses due to higher harmonics, as a percentage of nominal motor power $P_{M,h}$ [$\%P_N$], according to expression (18), and as a percentage of nominal power losses in the windings $P_{M,h}$ [$\%P_{Cu,N}$] according to expression (19).

The given results in Table 1, show that, in the specified harmonic content ($U_{h,i} = 1/h_i$, $h_i = 1\text{-}37$) in the supply voltage of rectangular shape which is obtained from the frequency converter, the percentage of additional power losses, $P_{M,h}$ [$\%P_N$], is not relatively high:

- when it comes to motors of greater power (>100 kW), an increase of losses is for about $0.94\%P_N$, so a decrease in efficiency is for 1%,
- when it comes to motors of lower power (3-10 kW), an increase of losses is for about $1.68\%P_N$, so a decrease in efficiency is for 1.7%.

The literature [4], [7], often states the percentage of increase of power losses in the windings of stator and rotor, ie. in $P_{M,h}$ [$\%P_{Cu,N}$], due to the higher harmonics. Data from Table 1, column $P_{M,h}$ [$\%P_{Cu,N}$], show that:

- an increase of losses is for $19.05\%P_{Cu,N}$, when it comes to motors of power (3-10 kW),
- an increase of losses is for $34.92\%P_{Cu,N}$,, when it comes to motors of greater power (>100 kW).

This last figure corresponds to the values that are found in literature, while the value of $19.05\%P_{Cu,N}$, when it comes to motors of lower power (3-10 kW), is much higher than the figure (about 5-10%) which is referred to in literature (about 5-10%). The reason for this is in the fact that it is (wrongly) believed that the resistance of the rotor does not change for higher harmonics frequencies, ie. that is the identical for all harmonics ($R_{r,h} = R_{r,1} = R_r = Const.$), which brings the difference mentioned above – and error. Things are different, because the rotor resistance is variable: $R_{r,h} > R_{r,SC} > R_{r,1}$. The explanation is the following: for motors with powers higher than 5 kW (or with relative penetration depth of $\zeta_{ScC} = H/\partial_{SC} \geq 1.5$) in short circuit mode not the entire section (or height H) of rotor bars is being used, [6]. On this basis, the conclusion is made, that the corresponding section of rotor conductor is $\sqrt{h}$ times lower.

Consequently, the values of rotor slot resistance are higher and values of rotor slot inductance are $\sqrt{h}$ times lower as compared to those values for the fundamental harmonic in short circuit mode.

Some examples from the literature can be used as a proof of the view that the rotor resistance changes when it comes to low-power motors. Specifically in [8], the influence of harmonics on the motor of low power (1.6 kW) was tested. The calculation results, which were carried out assuming that $R_r = Const.$, gave increase in power losses of 12.6%, while the experimental measurements showed that the actual increase in losses was 18.5%. Our calculations, give rise to losses of 19%, which slightly deviates from the measured values. The accuracy of our calculations has been increased with respect to the fact that slot reactance of the rotor increases $\sqrt{h}$ times, for the higher harmonics of order *h*.

In [9] it is proved that values of rotor inverse resistance are higher and values of rotor inverse inductance are lower $\sqrt{2}$ times than the corresponding resistance and reactance for short circuit regime.

Table 1: Values of resistances ($R_{M,h}$), reactances ($X_{M,h}$) and impedances ($Z_{M,h}$) and corresponding currents and power losses for motors with power > 100 kW (left) and lower power, 3-10 kW (right), when the motor is powered by the rectangular voltage from converter, which contains harmonics h = 1, 5, 7, 11, 13, ...35 and 37

$h = f/f_1$	U_h	R_s	$R_{r,h}$	$R_{M,h}$	$X_{M,h}$	$Z_{M,h}$	$I_{M,h}$ [$\%I_N$]	$P_{M,h}$ [$\%P_N$]	$P_{M,h}$ [$\%P_{CuN}$]
1	1	0.015-0.050	0.03	0.045-0.080	0.161	0.167-0.180			
5	0.2	0.015-0.050	0.067	0.072-0.117	0.735	0.739-0.744	26.99	**0.618-1.184**	**23.447-13.418**
7	0.143	0.015-0.050	0.079	0.094-0.129	1.018	1.022-1.053	13.79	**0.213-0.341**	**7.668-3.864**
11	0.111	0.015-0.050	0.099	0.114-0.144	1.579	1.583-1.586	7.01	**0.069-0.099**	**2.504-1.122**
13	0.077	0.015-0.050	0.108	0.123-0.158	2.416	2.419-2.421	3.18	**0.015-0.022**	**0.556-0.252**
17	0.059	0.015-0.050	0.124	0.129-0.174	2.694	2.643-2.646	2.23	**0.007-0.012**	**0.244-0.136**
19	0.052	0.015-0.050	0.131	0.146-0.181	3.249	3.252-3.254	1.6	**0.005-0.007**	**0.180-0.079**
23	0.043	0.015-0.050	0.144	0.159-0.194	3.526	3.534-3.536	1.22	**0.003-0.004**	**0.106-0.046**
25	0.04	0.015-0.050	0.15	0.165-0.200	3.833	3.836-3.838	1.04	**0.002-0.003**	**0.075-0.036**
29	0.034	0.015-0.050	0.162	0.177-0.212	4.08	4.084-4.086	0.83	**0.001-0.002**	**0.036-0.025**
31	0.032	0.015-0.050	0.167	0.182-0.217	4.357	4.361-4.362	0.73	**0.001-0.002**	**0.036-0.025**
35	0.029	0.015-0.050	0.177	0.192-0.227	4.91	4.919-4.920	0.59	**0.001-0.002**	**0.036-0.025**
37	0.027	0.015-0.050	0.182	0.197-0.232	5.187	5.191-5.192	0.52	**0.001-0.002**	**0.036-0.025**
Total	***THDu* =30.3%**						***THDi* =31.5%**	**$\Sigma P_{M,h}$ [$\%P_N$] 0.94-1.68%**	**$\Sigma P_{M,h}$ [$\%P_{CuN}$] 34.92 -19.05%**

4 Conclusion

The most important conclusions regarding motor operation with non sinusoidal voltage are presented. When the induction motors are supplied by rectangular shaped voltage from frequency converter with high levels of harmonic voltages ($U_{h,i} = 1/h_i$), increase of power losses in stator and rotor windings amounts to:

- around 19% $P_{Cu,N}$, when it comes to lower power motors (3-5 kW), a
- around 34% $P_{Cu,N}$, when it comes to high-power motors (> 100 kW).

The latter figure corresponds to the values that are found in literature, while the value of 20%$P_{Cu,N}$ when it comes to lower power motors (3-10 kW), is much higher than the figure (about 5-10%) which is quoted in the literature. The reason for this is in the fact that it is (wrongly) believed that the resistance of the rotor does not change for higher harmonics frequencies, ie. that is the identical for all harmonics ($R_{r,h} = R_{r,1} = R_r = Const.$), which brings the difference mentioned above – and error.

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