The influence of modeling approach for the MIMO closed-loop systems on the results of simulation in Simulink

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Abstract: - The paper reports results of comparison of the three approaches for the simulation of the continuous time closed-loop multi input multi output (MIMO) system. In this case the system is described by two first-order differential equations and represents the real system with two inputs and two outputs. The synthesis of MIMO controller is based on the pole placement method. The controller is tuned by the settings of two adjustable parameters. Simulations are realized in Matlab/Simulink for three cases of representation of the closed-loop system for the same values of parameters in every case. The first one is based on modeling controlled system and the controller in the form of particular transfer functions from each input to an appropriate outputs. Second approach is modification of the first one. The particular transfer functions are modeled in the form of differential equations by the blocks from Simulink libraries. The last simulation approach is also performed by Simulink blocks, for the system modeled directly from the two differential equations description, and controller represented also as two differential equations fulfilling law of control.

Key-Words: - MIMO system, MIMO controller, Synthesis, Simulation, System description, Matlab/Simulink

1 Introduction

Simulation of systems is the important part of successful design and application in many fields. Systems can be based on various physical cores, but fortunately, they can be described the same form in many cases. A basic description of the continuous-time system is in the form of differential equations which are derived from fundamental laws of physics Kirchhoff laws and Newton laws for translational or rotational movement, for example [1]. In the control theory those systems are usually described by transfer functions (TF) that are defined as the quotient of Laplace images of output and input value under the zero initial conditions. Systems with one input and one input value (SISO) are described by one transfer function, but systems with multiple inputs and outputs (MIMO) are described by transfer matrix. It is obvious that for MIMO systems there rises problem in the TF description. The main problem relates to the noncommutativity of matrix multiplication [2]. Not every system operates as we want it to do. But with right acting on the system we can reach desired behavior. Another system that purposely acts on the given system is called controller. Controller also can be described the same way as the system. Control theory offers various methods for determining structure and/or coefficients of the regulator. One method for synthesis of the controller was used in this work and is called pole placement method [3]. For the simulation of response of these systems in the one degree of freedom (1DOF) configuration, various approaches can be used. Simulink offers modeling of numerous systems by elementary blocks as integrators, gains, sums and others. It also consists of predefined blocks like the transfer function block. The question is, whether the approach of the systems modeling has any influence on simulation results.

2 Problem Formulation

2.1 The system and controller description

Let the system be described by two first order differential equations in the monic form with constant coefficients (1). This system can represent any real interconnected system with two inputs and two outputs, having two accumulators of energy in it.

$$y_1' + a_{11}y_1 + a_{12}y_2 = b_{11}u_1 + b_{12}u_2$$
$$y_2' + a_{21}y_2 + a_{22}y_1 = b_{21}u_1 + b_{22}u_2 \qquad (1)$$

These equations can be expressed by the matrix form,

$$\mathbf{A_L} * \mathbf{Y} = \mathbf{B_L} * \mathbf{U} \qquad (2)$$

After taking Laplace transformation on (1) and rewriting it into (2),

$$\begin{bmatrix} s + a_{11} & a_{12} \\ a_{21} & s + a_{22} \end{bmatrix} * \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} * \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \qquad (3)$$

By rearranging (3), representation of the system can be obtained in the form,

$$Y(s) = G(s) * U(s) \qquad (4)$$

where

$$G(s) = A_L^{-1} * B_L \qquad (5)$$

is called left polynomial matrix fraction description (LPMFD) and is equal to right polynomial matrix fraction description (RPMFD),

$$G(s) = B_R * A_R^{-1} \qquad (6)$$

Where

$$A_R^{-1} = \frac{adj(A_R)}{\det(A_R)} \qquad (7)$$

Transfer matrix ***G(s)*** with parts that represents partial transfer functions of each input to appropriate output,

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \qquad (8)$$

Thus the system can be modeled by configuration that is represented by transfer function blocks like,

$$Y_1 = G_{11} * U_1 + G_{12} * U_2$$
$$Y_2 = G_{21} * U_1 + G_{22} * U_2 \qquad (9)$$

Now, let the controller of the given system be described also by LPMFD resp. RPMFD. Then the transfer matrix of the controller can be determined as,

$$G_R = P_L^{-1} * Q_L = Q_R * P_R^{-1} \qquad (10)$$

The transfer matrix (12) can be also disassembled into partial transfer functions. Then the controller can be implemented as,

$$U_1 = G_{R11} * E_1 + G_{R12} * E_2$$
$$U_2 = G_{R21} * E_1 + G_{R22} * E_2 \qquad (11)$$

$$G_R(s) = \begin{bmatrix} G_{R11}(s) & G_{R12}(s) \\ G_{R21}(s) & G_{R22}(s) \end{bmatrix} \qquad (12)$$

2.2 1DOF MIMO configuration

2.2.1 Signals in the closed loop MIMO system

One degree of freedom (1DOF) MIMO configuration is a basic closed-loop regulation circuit as shown in Fig.1. All signals are considered in vector form, where partial signals are defined as polynomial fractions. The transfer functions of the controller and the system are in the form of LPMFD.

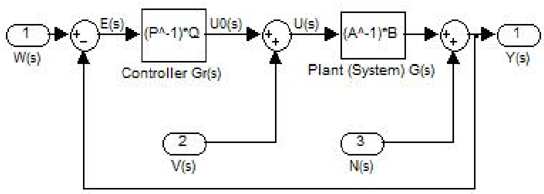

Fig.1 1DOF closed-loop regulation circuit

Let define signals in Fig. 1 as,

$$w(s) = \begin{bmatrix} \frac{h_{w1}(s)}{f_{w1}(s)} & \frac{h_{w2}(s)}{f_{w2}(s)} \end{bmatrix}^T \qquad (13)$$

$$v(s) = \begin{bmatrix} \frac{h_{v1}(s)}{f_{v1}(s)} & \frac{h_{v2}(s)}{f_{v2}(s)} \end{bmatrix}^T \qquad (14)$$

$$n(s) = \begin{bmatrix} \frac{h_{n1}(s)}{f_{n1}(s)} & \frac{h_{n2}(s)}{f_{n2}(s)} \end{bmatrix}^T \qquad (15)$$

then (13,14,15) can be expressed in the convolution form,

$$w(s) = F_W^{-1}(s) * h_w(s) \qquad (16)$$

$$v(s) = F_V^{-1}(s) * h_v(s) \qquad (17)$$

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$$n(s) = F_n^{-1}(s) * h_n(s) \tag{18}$$

According to Fig.1 equations for the signals can be rewritten as,

$$y(s) = A_L^{-1} * B_L * u(s) + n(s) \tag{19}$$

$$u(s) = u_0(s) + v(s) \tag{20}$$

$$u_0(s) = P_L^{-1} * Q_L * e(s) \tag{21}$$

$$e(s) = w(s) - y(s) \tag{22}$$

Is possible to show that after few algebraic operations and by using equality of the polynomial matrix fraction descriptions, every signal consists of inverse of the matrix $\boldsymbol{D_r}$, where according to (7),

$$D_r^{-1} = \frac{adj(D_r)}{\det(D_r)} \tag{23}$$

Where det($\mathbf{D_r}$) is characteristic polynomial, and figures in denominator of every partial transfer function in transfer matrix of MIMO system. So it is obvious that it has major influence on stability of whole system. The need of asymptotic tracking of reference signal and compensation of disturbances is fulfilled when,

$$\lim_{t \to \infty}[e(t)] = \lim_{s \to 0}[s * e(s)] = 0 \tag{24}$$

To (24) become true, every denominator in transfer functions of signals have to be eliminated. So there must exist matrix $\boldsymbol{F}$ (compensator) such that,

$$F = I * f \tag{25}$$

and

$$P_R = F * \tilde{P}_R \tag{26}$$

Where, the f is the least common multiple of every denominator in the signals transfer functions.

2.2.2 Synthesis of the controller

According to stability of the system the controller is given by solving matrix Diophantine equation with diagonal matrix $\boldsymbol{D}$ on the right side with stable polynomials on the main diagonal,

$$A_L * F * \tilde{P}_R + B_L * Q_R = D \tag{27}$$

Can be proven that Diophantine equation has a solution if $\boldsymbol{D}$=RGCD($\boldsymbol{A_L}$,$\boldsymbol{B_L}$). Solution of (27) can be found by method of uncertain coefficients, for example. As another very useful method for solving such types of equations it has shown to be the generalized Euclidean algorithm. This algorithm can determine both structure and coefficients of the regulator as well as it can compute RPMFD. Equality of fraction descriptions can be expressed as,

$$A_L * (-B_R) + B_L * A_R = 0 \tag{28}$$

Eqs. (27) and (28) can be rewriten into the form,

$$[A_L \quad B_L] * \begin{bmatrix} F * \tilde{P}_R & -B_R \\ Q_R & A_R \end{bmatrix} = [(D \quad 0)] \tag{29}$$

Augmented matrices can be expressed such that one can become another one by elementary column modifications.

$$\left[\begin{array}{cc} A_L * F & B_L \\ \hline 1 & 0 \\ 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc} D & 0 \\ \hline \tilde{P}_R & -B_R \\ Q_R & A_R \end{array}\right] \tag{30}$$

One approach for choosing a form of polynomial matrix $\boldsymbol{D}$ can be,

$$D = \begin{bmatrix} (s+\alpha)^{2 * \deg(A)} & 0 \\ 0 & (s+\beta)^{2 * \deg(A)} \end{bmatrix} \tag{31}$$

So by solving (30) controller that can be adjusted by two adjustable parameters α and β is derived. The law of control via RPMFD is then,

$$F * U = Q * P_R^{-1} * E \tag{32}$$

After reduction, convolution and transformation to time domain another form of regulator is,

$$U(t) = \int Q * E(t)dt \tag{33}$$

3 Problem Solution

The main purpose of this work is simulation of the regulation of closed-loop 1DOF MIMO system by

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the three different methods, each implemented for the same parameters. First method is based on the connection of partial transfer functions to the appropriate signals and on creating closed-loop circuit in simulink in the form of (9) and (11) by transfer function blocks. Fig. 2 shows such simulation circuit.

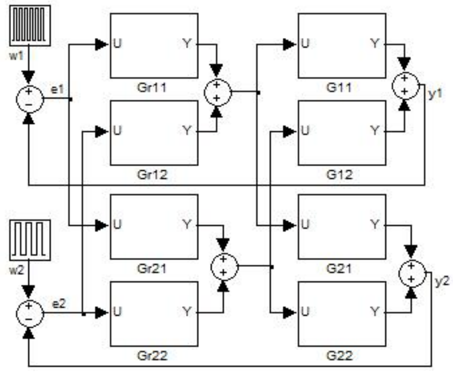

Fig.2 Simulink block configuration for simulation of the first two approaches.

For the first approach the blocks G_{11}, G_{12}, G_{21}, G_{22}, G_{r11}, G_{r12}, G_{r21}, G_{r22}, were determined by created matlab m-file in the form shown in Fig. 3.

```
[b11 s + b11 a22 - a12 b21       b12 s + b12 a22 - a12 b22 ]
[-------------------------       ------------------------- ]
[           %1                              %1             ]
[                                                          ]
[-a21 b11 + b21 s + b21 a11      -a21 b12 + b22 s + b22 a11]
[--------------------------      --------------------------]
[           %1                              %1             ]

                 2
        %1 := s   + s a22 + a11 s + a11 a22 - a12 a21
```

Fig.3 Transfer matrix G(s) as output from Matlab

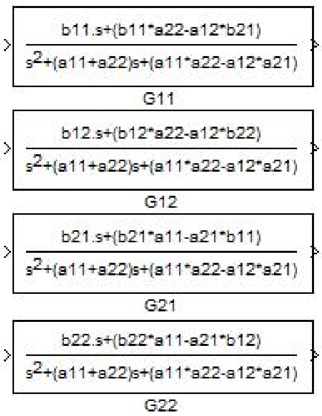

Fig.4 Partial TF representation of the system

Similar form as in Fig.3 was obtained for the controller, but there is not enough space to show result from Matlab. Fig.4 represents implementation of partial transfer functions from Fig.3 into Simulink TF blocks. And Fig.5 shows implementation of regulator into blocks,

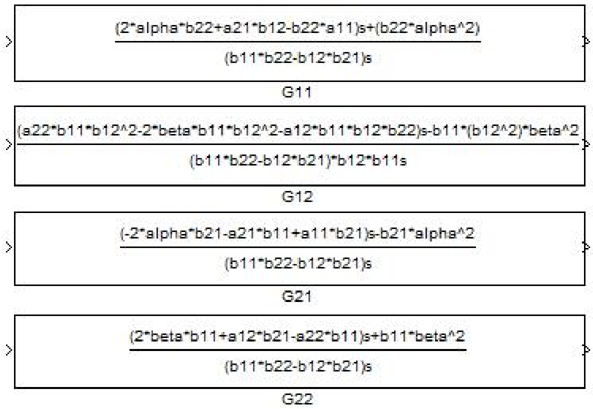

Fig.5 Partial representation of the controller

All TF blocks in Fig.4 and Fig.5 were substitued by subsystem blocks. Each subsystem was created the same way as G_{11} shown in Fig. 6, for example.

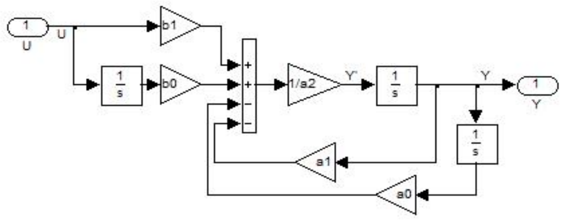

Fig.6 Subsystem block representing partial transfer function G_{11}

Final approach is based on modeling the system the same method as presented in Fig.6, but modeled equations were (1). The schematic is shown in Fig.7.

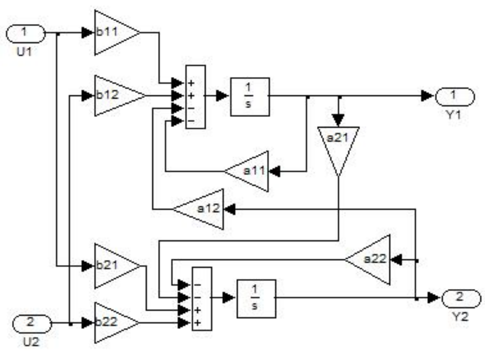

Fig.7 Schematic for modeling system by the third approach

Controller is modeled the same way and it is based on equation (33) as shown in Fig.8.

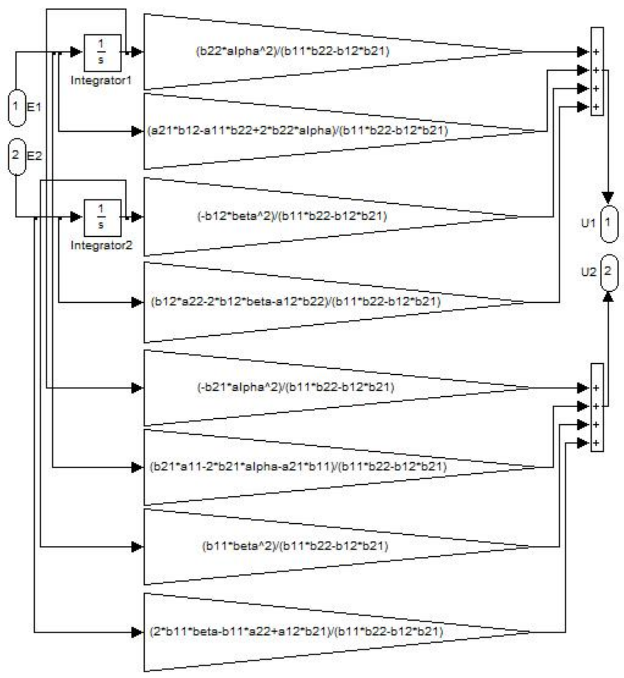

Fig.8 Schematic for modeling of the controller by the third approach

For the simulation of each approach coefficients of the system and parameters α and β were chosen as follows,

$$a_{11} = 3;\ a_{12} = 5;\ a_{21} = 2;\ a_{22} = 1.5 \quad (34)$$

$$b_{11} = 6;\ b_{12} = 4;\ b_{21} = 3;\ b_{22} = 5 \quad (35)$$

$$\alpha = 4;\ \beta = 2 \quad (36)$$

The simulation results are shown in Fig.9 and Fig.10. As can be seen every controller was able to stabilize outputs of the system at reference values in each approach until the time reaches 40 seconds. Until that time it seems like all three methods are suitable for the simulation. But after extending time axes, there can be seen that after the time of 45 seconds, in the first case, and in the time of 40 seconds in the second case the regulation marches simulated by the first and second method starts to become unstable. There can be seen rapid increase of action values and then the outputs of the system diverges. The third method is able to control outputs on reference values in the long run. From that point the third approach seems to be the most suitable for the modeling of such systems. On this place should be noted that this behavior was discovered only for simulation with unstable systems. For the stable ones all three approaches gave the exactly same regulation marches.

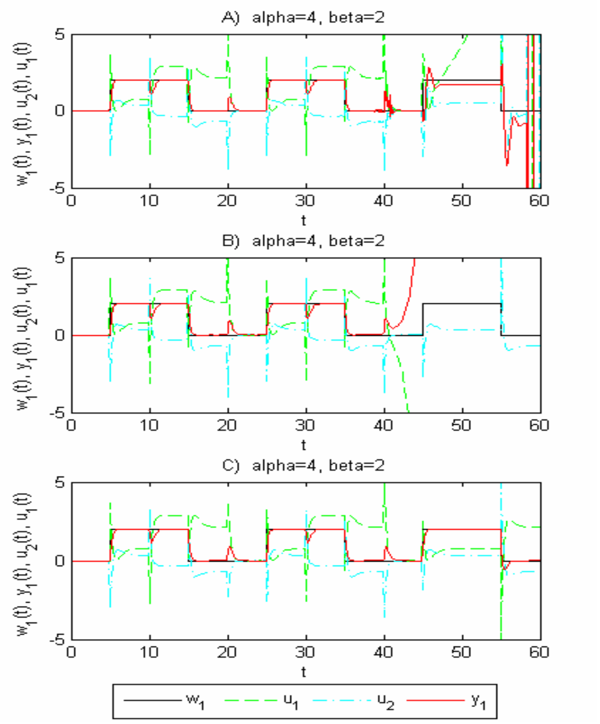

Fig.9 Regulation of the 1st output of the system A) first method, B) second method, C) third method

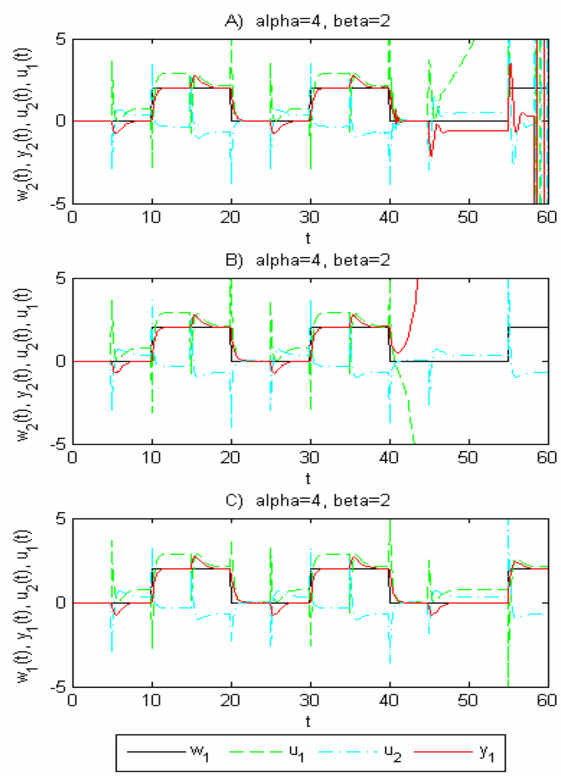

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Fig.10 Regulation of the 2nd output of the system A) first method, B) second method, C) third method

4 Conclusion

The main aim of this work was to compare behavior of three approaches for simulation of the closed-loop MIMO system. In the article there were presented three approaches for the simulation of such systems. The research has shown that the most suitable variant for simulation of the closed-loop MIMO regulation circuit is the third one, where controlled system and controller are modeled as elementary blocks and they are connected the way that represents description of the system in the form of differential equations. Representation of the system and controller in the form of transfer matrices seemed to be unreliable. It is possible that this behavior may be caused by a numerical instability of some algorithms that are implemented in Simulink.

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