Uncertainty Analysis of the Cross-sectional Area of a Structural Member

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Abstract: - The paper deals with the fuzzy analysis of the cross-sectional area within the scope of reliable design of a steel bar. The paper describes the application of the theory of fuzzy sets as a tool in the modelling of uncertainty arising due to human activities. The fuzzy numbers of the geometrical characteristics were derived with respect to the tolerance limits of standard EN 10034 and results of experimental research. The fuzzy number of the cross-sectional area was evaluated according to the general extension principle. Results of the fuzzy analysis are compared with the histogram of cross-sectional area. The paper elaborates on technical knowledge in the field of reliability assessment and design of steel structures.

Key-Words: Uncertainty, Optimization, Reliability, Random, Fuzzy, Structure, Industry

1 Introduction
Hot-rolled steel profiles are rolled with deviations in geometry according to standard EN10034. The standard prescribes tolerance limits, which should be respected and adhered to in production. Geometric characteristics of steel products are generally random variables. Their histograms and statistical characteristics are obtained from experimental research [1, 2].

Results of non-commercial experimental research provide valuable background material for stochastic studies of theoretical reliability [3-6]. The economic demand of experimental research differs depending on the type of member that is investigated. Material and geometric characteristics of sheets and hot-rolled steel members of steel grade S235 and S355 are relatively available [7]. Concrete structures are characterised by greater variability of material characteristics and reliable data from experimental research is relatively less [8-10].

A separate group consists of data dealing with initial geometric imperfections of steel structures. As corroborated by results of sensitivity analysis [11-17], the influence of initial imperfections on the ultimate limit state varies depending on the slenderness of designed elements.

Reliability analyses generally require in addition to stochastic methods the application of fuzzy approaches wherever epistemic uncertainty is present [18-21]. Study [22] shows that the fuzzy approach suitably supplements results of probabilistic analysis of reliability. A relevant variable in the reliability of structures is the failure probability. Failure probability can generally be evaluated as a fuzzy number and can be worked with using the tools of fuzzy logic [23].

The synthesis of stochastic and fuzzy approaches provide an effective tool for the complex analysis of reliability, which has found utilization in the verification of design criteria of building structures according to the EUROCODE standard. The EUROCODE standards in essence make use of semi-probabilistic methods, which are calibrated using stochastic or fuzzy stochastic methods [22]. The paper is aimed at the comparison of the results of fuzzy analysis of the cross-sectional area with results of experimental research [1].

Fig. 1 IPE 200 cross-section
2 Problem Formulation

Faced with the task of specifying the set of all profiles IPE 200, see Fig. 1, we can with certainty postulate that if the height measured in each section along the span of the beam is equal to its nominal value \( h = 200 \text{mm} \), then the degree of truth of the statement that it is the profile IPE 200 is of grade 1, i.e. absolute truth or 100 % true. However, one may ask, “What about the height of 200.001 mm?” Using our bare naked eyes we are not able to distinguish or differentiate between the heights 200 mm and 200.001 mm. We, however, intuitively feel that the degree of truth should be lower than for the height of 200 mm. The tolerance limits of deviations in dimensions and shape are given by standard EN 10034. In the classical crisp set theory if the measured height lies within the tolerance limits given by standard EN 10034 we can with certainty state that it is the height of profile IPE 200.

2.1 Fuzzification of Geometric Characteristics

Fuzzy sets of the geometric characteristics of profile IPE 200 are specified on the basis of the following assumptions:

1. Geometric characteristics, which are equal to the nominal dimensions of profile IPE 200, are with a 100 percent certainty the geometric characteristic of the profile IPE 200.
2. The truth function decreases with (in absolute values) increasing distance from the nominal dimension.
3. The grade of membership is higher, the higher the frequency of occurrence of the corresponding geometric characteristic obtained experimentally from real IPE 200 profiles.
4. Beams whose geometry does not respect the tolerance limits of standard EN 10034 are not beams of profile IPE 200.

Let us consider the membership function in the shape of the theoretical probability distribution of the occurrence of the corresponding variable. With regard to feasibility of solution, the Gaussian probability distribution with standard deviation given according to [1] will be used to approximate the histograms of geometric characteristics. Adhering to the above-mentioned assumptions, it is necessary that the mean value equals the nominal value. The standard deviation will be considered according to [1].

Although membership functions need not be scaled between zero and unity, it is usually performed so that the variables are normalized. A fuzzy set may be normalized by dividing its membership function by its largest distribution (probability) function value. The maximum values of membership functions of the nominal values of geometric characteristics, i.e. height \( h \), width \( b \), web thickness \( t_1 \), flange thickness \( t_2 \), are considered as 1. In accordance with point (3) of the aforementioned assumptions, we shall assume the membership function to be formatively identical with the Gaussian distribution. Let us for better lucidity symbolically denote the fuzzy sets of height \( h \), width \( b \), web thickness \( t_1 \), flange thickness \( t_2 \) as \( V_h, V_b, V_{t_1}, V_{t_2} \), or in short as \( V_k \), where \( k \in \{h, b, t_1, t_2\} \), see Fig. 2.

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It is apparent from [1] that the relative mean value, which was evaluated from a data set designated as the ratio of measured to nominal values, is approximately equal to 1, i.e. the nominal value. The standard deviation will be considered according to [1].

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The universe \( U \) of fuzzy set \( \overline{V_k} \) illustrated in Fig. 3 is defined by its nominal value \( \overline{V_{k,nom}} \), minimal value \( \overline{V_{k,min}} \) and maximal value \( \overline{V_{k,max}} \) according to EN 10034. The fuzzy set \( C \) satisfying requirements 1 to 4 is obtained through the realization of the intersection of sets \( V_k \) and \( \overline{V_k} \).

\[
C = V_k \cap \overline{V_k}
\]
In the crisp set theory, intersection is characterized by the operations of minimum, which in this context corresponds to the connection “and” (logical conjunction). The following is valid for the membership function $C(x)$ of fuzzy set $C$:

$$C(x) = V_k(x) \land \overline{V}_k(x)$$  \hspace{1cm} (2)

In other words: Element $k \in U$ belongs to the intersection of fuzzy sets $V_k, \overline{V}_k \subset U$ with the degree of membership given as the smaller of degrees $V_k(x)$ and $\overline{V}_k(x)$. Fuzzy set $C$ is shown in Fig. 4.

Obtained fuzzy sets of geometric characteristics $h$, $b$, $t_1$, $t_2$ are depicted in Fig. 5 to Fig. 8.

2.2 Fuzzy Analysis

Fuzzy arithmetical operations can generally be defined using Prof. L. Zadeh’s extension principle [23]. For the analysis the extension principle was utilized in the form of $\alpha$-cuts. The relative area in Fig. 9 was obtained as the division of the area value by the nominal value of 0.0027248 m$^2$. The comparison of the fuzzy set of relative area $A$ with experimentally obtained histograms is depicted in Fig. 9.
3 Conclusion

The analysis of fuzzy uncertainty of geometric characteristics was illustrated in the presented paper. Fuzzification was performed with regard to the tolerance limits of dimensions and shape of the standard EN 10034 and results of experimental research. Within the scope of analysis, the tolerance limit of the standard is understood as a prerequisite, the fulfilment of which is more or less vague. There exist a number of factors, which prevent steel manufacturers from constantly producing steel products on the boundaries of tolerance limits. The actual frequency of occurrence exhibits random deviations. The highest frequency of occurrence is relatively far from the limit values due to the requirement that majority of realizations, i.e. 95 %, lie within the tolerance limits. In the case of symmetrical tolerance limits and Gaussian distribution the highest frequency of occurrence is around its nominal value.

Despite the apparent strictness of technical regulations, they are always carried out by people, i.e. more or less vaguely [24-26]. Practically, this means that the actual production, but also sale of steel products and the realization of building structures are eventually controlled by the conventional standard, which can never be in ideal concordance with the requirements given by technical standards. Thus in order to minimize this vagueness, it is necessary that the requirements of standards are formulated with the greatest precision possible. The performed study demonstrates a lucid and specific application of the theory of fuzzy sets in the modelling of uncertainties emanating from vagueness. Whether or not we would like to admit it, this phenomenon exists virtually in all aspects of human activities.

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