Boundary layer flows induced by permeable continuous surfaces stretched with prescribed skin friction

Mohamed Ali[1] and Khaled Al-Salem

Abstract- The boundary layer flow and heat transfer on an isothermal stretched surface moving with prescribed skin friction is studied for permeable surface. Three major cases are studied namely; uniformly moving surface (m = 0), stretched surface with constant skin friction (m = 1/3) and surface moving with constant heat flux (m = 1). Where m is the index of the power law velocity exponent. Similarity solutions are obtained for the boundary layer equations subject to power law temperature and velocity variation. The effect of various governing parameters, such as Prandtl number Pr, suction/injection parameter f_w and m are studied. The results show that increasing m enhances the dimensionless heat transfer at the suction case while degrade it at the injection case at fixed f_w. However, for fixed m as f_w increases the dimensionless heat transfer coefficient increases.

Keywords- Stretching surface, Boundary layers, Prescribed skin friction, Suction or injection, similarity solutions.

I. Introduction

A continuously moving surface through an otherwise quiescent medium has many applications in manufacturing processes. Such processes are hot rolling, wire drawing, spinning of filaments, metal extrusion, crystal Since the pioneer study of Sakiadis [6] who developed a numerical solution for the boundary layer flow field of a stretched surface, many authors have attacked this problem to study the hydrodynamic and thermal boundary layers due to a moving surface [7-16].

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Recently, Magyari and Keller [17] have initiated a new research field by solving the induced boundary layer flows of impermeable stretched surfaces using prescribed skin friction boundary condition instead of the usual prescribed velocity boundary condition. Their results have shown substantial deviation of velocity profiles, temperature profiles as well as the wall heat fluxes from that of defined velocity boundary condition when compared on the actual physical scales of the coordinates.

Suction or injection of a stretched surface was introduced by Erickson et al [118] and Fox et al [19] for uniform surface velocity and temperature and by Gupta and Gupta [20] for linearly moving surface. Chen and Char [21] have studied the suction and injection on a linearly moving plate subject to uniform wall temperature and heat flux and the more general case using a power law velocity and temperature distribution at the surface was studied by Ali [22]. Recently, Magyari et al. [23] have reported analytical and computational solutions when the surface moves with rapidly decreasing velocities using the self-similar method, and the flow part of the problem was considered analytically by Magyari and Keller [24] for permeable surface moving with a decreasing velocity for velocity parameters –1/3 and –1/2.

In all papers cited earlier the effect of buoyancy force was neglected and the following papers have taken the buoyancy force into consideration however, suction or injection at the moving surface was relaxed. Such papers are Lin et al [25] for horizontal isothermal plate moving in parallel or reversibly to a free stream. Also the papers by Karwe and Jaluria [4, 5], Kang and Jaluria [26, 27] to obtain the buoyancy effects on moving plate in rolling and extrusion processes, in materials processing, casting process, and in channel flow for thermal processing respectively, and it was found that the effect of thermal buoyancy is more significant when the plate is moving vertically upward than when it is moving horizontally. Ingham [28] studied the existence of the solutions of the boundary layer equations of a uniformly moving vertical plate with temperature inversely proportional to the distance up the plate.

Laminar mixed convection of uniformly moving vertical surface for different temperature boundary conditions are considered by Ali and Al-Yousef [29, 30] and by Ali [31] for stretched surface with rapidly decreasing velocities.

The present paper extends the work of [17] for the surface to be permeable and moving with different power law velocity distribution. However, the analyses are focused on cases of uniformly (m = 0), linearly moving surface (m = 1) and m = 1/3 corresponding to fixed skin friction at the surface for isothermal surface temperature (n = 0) for different Prandtl numbers.

II. Mathematical analysis

Consider the steady two-dimensional motions of convective boundary layer flow induced by a moving surface with suction or injection at the surface. For incompressible viscous fluid environment with constant properties, the equations governing this convective flow can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \quad (3)$$

subject to the following prescribed skin friction coefficient boundary conditions:

$$\begin{aligned} &\tau(x)=\tau_w(x),\ v(x)=v_w(x) && @y=0 \\ &T-T_\infty=T_w-T_\infty=Cx^n && @y=0 \\ &u(x)=0, T(x)=T_\infty=\text{const.} && @y\to\infty \end{aligned} \quad (4)$$

The x coordinate is measured along the moving surface from the point where the surface originates, and the y coordinate is measured normal to it (Fig. 1). Positive or negative v imply injection or suction at the surface respectively, and u and v are the velocity components in x and y directions respectively. Similarity solutions arise when

$$u=U_o x^m f'(\eta), \qquad T-T_\infty=Cx^n\theta(\eta) \quad (5)$$

$$\eta=y\sqrt{\frac{m+1}{2}}\sqrt{\frac{U_o x^m}{\nu x}} = \frac{y}{x}\sqrt{\frac{m+1}{2}}\sqrt{Re_x} \quad (6)$$

$$v=-\sqrt{\frac{2\nu U_o}{m+1}}\, x^{\frac{m-1}{2}}\left(\frac{m+1}{2}f+\frac{m-1}{2}f'\eta\right), \quad m\neq-1 \quad (7)$$

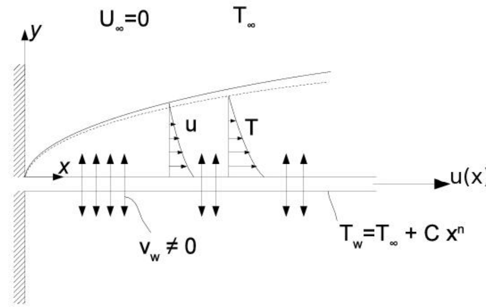


Figure 1. Schematic of boundary layers induced close to a moving surface with prescribed skin friction.

and the shear stress and the heat flux at the surface are given by respectively

$$\tau_w(x)=\mu U_o x^{m-1}\sqrt{\frac{m+1}{2}}Re_x^{1/2} f''(0)\sim x^{(3m-1)/2} f''(0) \quad (8)$$

$$q_w(x)=-kCx^{n-1}Re_x^{1/2}\sqrt{\frac{m+1}{2}}\,\theta'(0)\sim x^{(m-1+2n)/2}\,\theta'(0) \quad (9)$$

It should be mentioned that, positive or negative m indicate that the surface is accelerated or decelerated from the extruded slit respectively. Where f' and θ are the dimensionless velocity and temperature respectively, and η is the similarity variable. Substitution in the governing equations gives rise to the following two-point boundary-value problem.

$$f'''+ff''-\frac{2m}{m+1}f'^2=0 \quad (10)$$

$$\theta''+Pr\left(f\theta'-\frac{2n}{m+1}f'\theta\right)=0 \quad (11)$$

The transformed boundary conditions for prescribed skin friction coefficient following Magyari and Killer [17].

$$f''(0)=-1, \quad f(0)=f_w, \quad f'(\infty)=0 \quad (12)$$

$$\theta(0)=1, \quad \theta(\infty)\to 0 \quad (13)$$

The quantity f (0) = f_w will be referred to as the dimensionless suction/injection velocity. In this way $f_w = 0$ corresponds to an impermeable surface, $f_w < 0$ to suction (i. e. $v_w(x) < 0$) and $f_w > 0$ to lateral injection (i. e. $v_w(x) > 0$) of the fluid through a permeable surface. The temperature of the injected fluid is assumed to coincide with the local temperature $T_w(x)$ of the stretching surface.

The local skin friction coefficient and the local Reynolds and Nusselt numbers are now given at the surface by

$$C_f\sqrt{\frac{Re_x}{2}}=f''(0)\sqrt{(m+1)} \quad (14)$$

$$Re_x=\frac{u_w x}{\nu}=\frac{U_o x^{m+1}}{\nu} \quad (15)$$

$$\frac{Nu_x}{\sqrt{\frac{Re_x}{2}}} = -\sqrt{m+1}\quad \theta'(0) \qquad (16)$$

III. NUMERICAL SOLUTION PROCEDURE

The coupled nonlinear ordinary differential equations (10) and (11) are solved numerically by using the fourth order Runge-Kutta method. Solutions of the differential Eqs. (10) and (11) subject to the boundary conditions (12), (13) were solved for different values of f_w. At each f_w; guessed values for $f'(0)$ and $\theta'(0)$ are assumed and the differential equations (10) and (11) were integrated until the boundary conditions at infinity $f'(\eta)$ and $\theta(\eta)$ decay exponentially to zero (at least of order 10^{-5}). If the boundary conditions at infinity are not satisfied then the numerical routine uses a half interval method to calculate corrections to the estimated values of $f'(0)$ and $\theta'(0)$. This process is repeated iteratively until exponentially decaying solution in f' and θ is obtained. The value of η_∞ was chosen as large as possible depending upon the Prandtl number and the suction/injection parameter f_w, without causing numerical oscillations in the values of f' and θ. It should be noted that since the f' and θ must exponentially decaying to zero to satisfying the boundary conditions at infinity then implied boundary conditions which must also be satisfied are f'' and θ' approaching zeros at infinity. Numerical solutions are obtained for a range of f_w between a minimum negative value (injection) to a maximum positive value (suction) such that the boundary conditions mentioned earlier must satisfy otherwise the solutions are rejected. Comparison with Magyari and Killer [17] is given in Table 1 for uniform skin friction (m = 1/3) and for uniform surface temperature (n = 0) and for Pr = 100. This comparisons show good agreements in $f'(0)$, $f(\infty)$ and $\theta'(0)$ which gives confidence about our current results.

Table 1. Comparisons with Magyari and Killer [17] for isothermal impermeable surface stretched with constant skin friction (n = 0, m = 1/3) in an ambient fluid medium of Pr = 100

	Current results		
f_w	$f'(0)$	$\theta'(0)$	$f(\infty)$
0	1.1323120	-8.300242	1.130436
	Magyari and Killer [17]		
	$f'(0)$	$\theta'(0)$	$f(\infty)$
0	1.13231319	-8.3001	1.13030744

IV. RESULTS AND DISCUSSION

Equations (10) and (11) are solved numerically for uniformly moving surface velocity with uniform temperature (m = 0, n = 0), for constant shear stress at the surface and isothermal

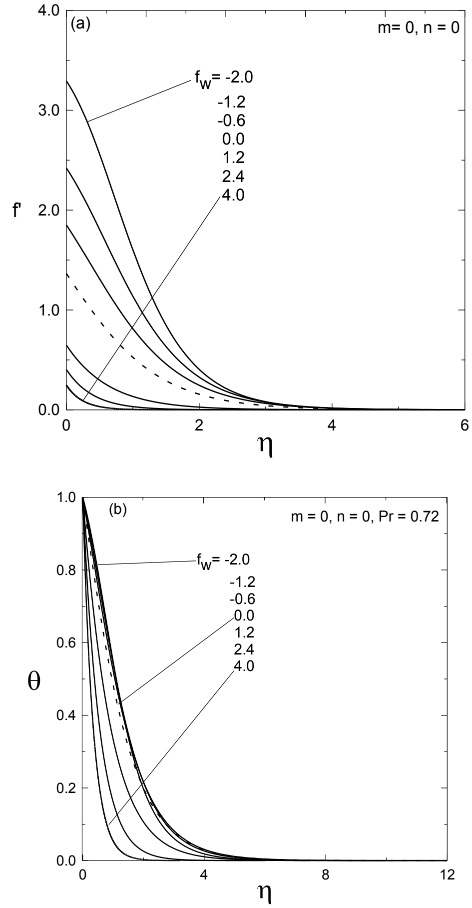


Figure 2. Profiles induced by stretched surface with constant skin friction for Pr = 0.72, m = 0 and n = 0 (a) velocity and (b) temperature

temperature (m = 1/3, n = 0) and for uniform heat flux at the surface (m = 1, n = 0) for Pr = 0.72, 3 and 7. Figure 2(a, b) shows the velocity profiles in 2(a) and the temperature profiles in 2(b) respectively for different suction/injection parameter f_w. As seen in Fig. 2(a) injection ($f_w < 0$) increases the momentum boundary layer thickness while suction thins them ($f_w > 0$). It should be noted that, the velocity at the surface is function of f_w not having a constant value of one as in the usual case of prescribed velocity at the surface and the dashed line presents the impermeable surface ($f_w = 0$). Figure 2(b) shows samples of temperature profiles for Pr = 0.72 (air) and for the same parameters shown in Fig. 2(a). It is clear that suction thins the thermal boundary layer thickness but the injection profiles are collapsed on almost one curve and there are no significant changes due to increasing the injection. It should be mentioned that the f_w used in this figure and in any following figures means that those are the minimum and maximum values such that the boundary conditions at infinity satisfied. Figure 3(a, b) shows the corresponding profiles for uniform shear stress (m = 1/3) at the surface with isothermal surface temperature. The effect of increasing m is clear on the profiles of both velocity and temperature. Figure 4(a, b) shows the velocity and temperature profiles for m = 1 and n = 0 corresponding to uniform heat flux at the surface as defined

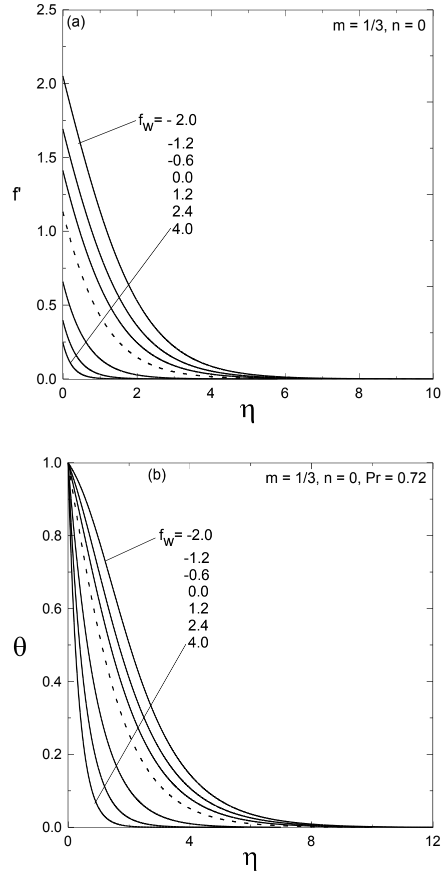
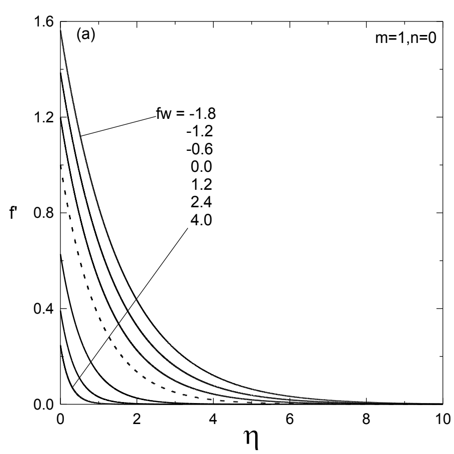


Figure 3. Profiles induced by stretched surface with constant skin friction for Pr = 0.72, m = 1/3 and n = 0 (a) velocity and (b) temperature.



by Eq. (9). Comparison between Figs. 2, 3 and 4 shows that the velocity profiles are function of m as expected and the temperature profiles are function of both m and n however, significant increase in the thickness of the thermal boundary layer occur for injection but no major changes happen in the thickness in case of suction. Figure 5 shows the velocity profiles for different m as a function of f_w. This figure shows that as m increases the velocity at the surface decreases at fixed f_w up to about f_w = 2 then no changes could be observed for changing m. The vertical dashed line presents the impermeable case.

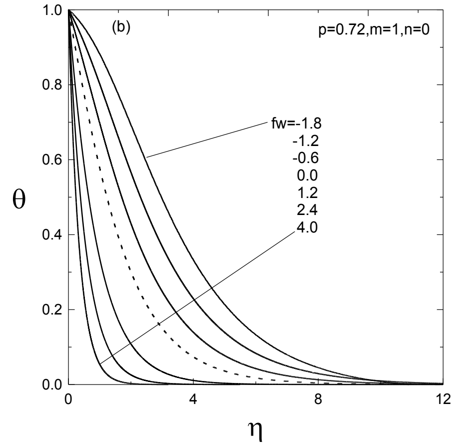


Figure 4. Profiles induced by stretched surface with constant skin friction for Pr = 0.72, m = 1 and n = 0 (a) velocity and (b) temperature

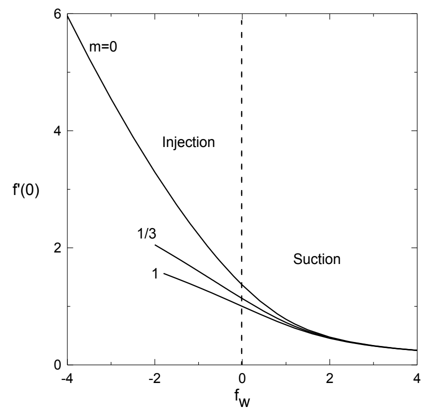


Figure 5. Profiles induced by stretched surface with constant skin friction for various values of m (a) stretching velocity and (b) entrainment velocity

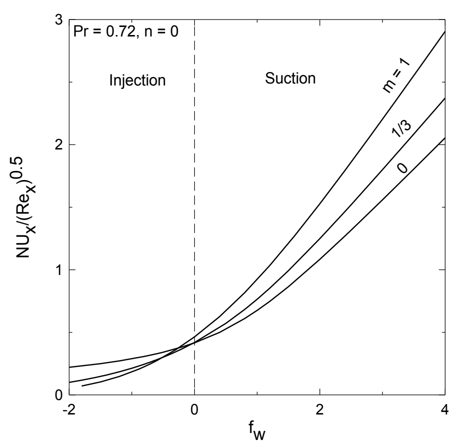


Figure 6. Dimensionless heat transfer coefficient as defined by Eq. (16) for various values of m, Pr = 0.72, uniform surface temperature (n = 0) as function of the suction/injection parameter f_w.

The dimensionless heat transfer coefficient is presented in Fig. 6 for Pr = 0.72 and for different values of m. It could be seen from this figure that for isothermal surface temperature increasing m enhances the heat transfer at the suction case while degrades it at the injection case at any fixed f_w.

However, for fixed m as f_w increases the dimensionless heat transfer coefficient increases too.

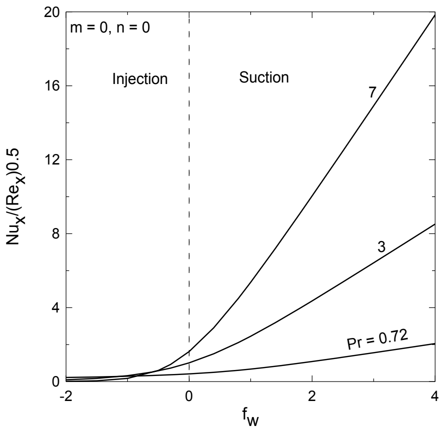


Figure 7. Dimensionless heat transfer coefficient as defined by Eq. (16) for various values of Prandtl numbers and for uniform surface temperature and for uniformly moving surface velocity (m = 0) as function of the suction/injection parameter f_w.

It should be mentioned that other Prandtl numbers give similar results and comparisons of the dimensionless heat transfer coefficient for different Prandtl numbers is shown in Fig. 7 for the uniform moving surface with uniform surface temperature. As expected increasing Prandtl number enhances the heat transfer coefficient especially in the suction case.

V. Conclusions

Heat transfer and flow field characteristics of a isothermal stretched permeable surface with prescribed skin friction are studied for uniformly moving surface, surface stretched with constant skin friction and for linearly moving surface.

The results show that at the surface the dimensionless stretching velocity decreases as the velocity index increases for any fixed suction/injection parameter f_w. It is also shown that the effect of increasing m is negligible beyond $f_w \geq 2.0$. For isothermal surface temperature increasing m enhances the dimensionless heat transfer (Eq. (16)) at the suction side and degrades it at the injection side at fixed f_w. However, for fixed m as f_w increases the dimensionless heat transfer coefficient increases too. Furthermore, the dimensionless heat transfer coefficient is function of Prandtl number and enhancements are occurred as the Pr increases for fixed values of f_w. This enhancement almost obtained in the suction region while almost no remarkable distinction at the injection side as f_w approaches -2.

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