Coverage Models in Military Operations
GEORGE ALEXANDRIS, NIKOLAOS V. KARADIMAS and NIKOLAOS DOUKAS
Informatics and Computer Science LAB
Department of Mathematics and Science Engineering
Univ. of Military Education - Hellenic Army Academy
Vari-Koropi Ave, 16673, Vari
GREECE
gpa@aueb.gr, nkaradimas@sse.gr, nikolaos@doukas.net.gr

Abstract: - The use of covering models in military operations is a key issue which can significantly affect decision making in several military scenarios concerning the formulation of the best military structure. In this paper we concentrate on the well known Maximal Coverage Location Problem in combination with the Back Up Coverage Location Problem and demonstrate that using the above covering models in order to formulate the best military structure can lead to excellent covering results. We propose the above models based on the notion of coverage and exploit the capabilities of Geographic Information Systems in order to better formulate, solve and visualize the problem of the best military coverage structure. Results of an empirical study indicate that the proposed models can provide optimal coverage structure and thus must be used extensively in military operations.

Key-Words: - Coverage, Geographic Information Systems (GIS), Location Analysis, Decision Making.

1 Introduction
Covering problems constitute an extensive set of problems in location analysis with numerous applications ranging from crew scheduling to cytological screening tests (PAP tests) for cervical cancer [1]. They deal with the proper location of servers (facilities) such that a given demand set or area is appropriately covered. Coverage is achieved when the service provided by a server is available to any point within the demand area within some predetermined distance or time.

One can identify two main classes of demand covering problems, as proposed by Daskin [2]: (a) mandatory covering problems, where all the demand area must be covered using the minimum number of servers and (b) maximal covering models, where the largest possible part of the demand area must be covered using a given number of available servers.

We concentrate on the class of maximal covering problems, whose main representative is the Maximal Covering Location Problem (MCLP), first stated by Church and ReVelle [3]. Since its original formulation, the MCLP has been extended in several directions to cater for more realistic problem situations such as the case of congested servers that arises when demand points request service by a server already busy serving other demand points. Typical applications of the MCLP and its extensions include the location of emergency facilities [4], the design of hierarchical health care systems [5], [6], the design of congested service systems [7] and many others.

In many cases where backup coverage is needed, the Maximal Backup Coverage Location Model is used. Hogan and ReVelle [8] introduced two different backup coverage models, named BACOP1 and BACOP2. BACOP1 maximizes backup coverage while requiring each demand point to have first coverage. BACOP2 relaxes the first coverage requirement and it trades off first coverage against backup coverage by assigning weights to each. These backup models can be really useful in military applications, where not only the location to place military units matters, but backing them up, is really of a great importance too.

In most practical applications of the MCLP and MBCLP both the demand area and the feasible locations of the servers are described by discrete sets. In cases where one (or both) set is continuous, a common approach is to transform this set (or sets) into a discrete set by superimposing a grid of blocks over the continuous area and specifying a single point within each block as a representative of that block. The problem is then stated as a discrete covering problem and is solved using one of the existing techniques (Greedy heuristics [2], Lagrangean relaxation [9], Lagrangean/surrogate heuristics [10].
In our paper we study the use of the above covering models in the formulation of the best military backup structure. More specifically, we consider the well known MCLP and MBCLP and we demonstrate that these models can be used effectively in cases we seek to find the best military structure during a military operation. We focus on G.I.S for representing and formulating the problem and we use optimization software for solving it. Finally we implement the solution using G.I.S and visualize the best military structure so as to help the decision makers to choose the best solution for every possible scenario.

The rest of the paper is organized as follows. We review MCLP and MBCLP and demonstrate how the MBCLP can be considered as an extension of the MCLP for the backup coverage of the demand space (area). We illustrate the applicability of the proposed models by means of a case study concerning the best military backup structure in the municipality of Athens. Finally, we draw some conclusions and discuss some future research directions.

2 Mathematical Models

The Maximal Covering Location Model seeks the maximum population that can be served within a stated service distance or time (range) given a limited number of service points. This location problem is structured mathematically in the following way:

Indices:
- i: index for demand points
- j: index for service points

Sets
- I: set of all demand points.
- J: set of all potential service points
- N(i): set of potential service points that can cover demand point i, namely: N(i) = { j ∈ J | dij ≤ S } for all i in I, where:
  - dij = distance from demand point i to service point j
  - S = the range

Parameters:
- αi: parameter representing the desirability of coverage for each demand point i.
- p: the number of branches to be located (less than phase II).

Decision variables

\[ x_j = \begin{cases} 1, & \text{if a facility is located at } j \\ 0, & \text{otherwise} \end{cases} \]
\[ y_i = \begin{cases} 1, & \text{if demand point } i \text{ is covered} \\ 0, & \text{otherwise} \end{cases} \]

The complete Maximal Coverage Location Model (MCLM) is:

\[
\text{MAXIMIZE } z = \sum_{i \in I} a_i y_i
\]

Subject to
\[
\sum_{j \in N(i)} x_j \geq y_i \text{ for all } i \in I \tag{1}
\]
\[
\sum_{j \in J} x_j = p \tag{2}
\]
\[
x_j \in \{0,1\} \text{ for all } j \in J
\]
\[
y_i \in \{0,1\} \text{ for all } i \in I
\]

Constraints (1) state that demand point i is covered whenever at least one facility is located within time or distance S whereas constraint (2) expresses the number of facilities that can be sited in each sub-region.

In military operations where backup coverage is crucial the Maximal Backup Coverage Location Model may be used. As we mentioned earlier, the Maximal Backup Coverage Location Model does not require primary coverage, but treats both primary coverage and backup coverage as objectives to be maximized, given the availability of P facilities to be placed on the network. This bi-objective model can be stated as follows. The indices and the sets are defined similarly to the MCLP but the model is bi-objective and uses one more set of decision variables ui, where:

\[ u_i = \begin{cases} 1, & \text{if demand point } i \text{ is covered by at least two facilities} \\ 0, & \text{otherwise} \end{cases} \]

The remaining decision variables are defined similarly to model MCLM.

The complete Maximal Backup Coverage Location Model (MBCLM) is:

\[
\text{MAXIMIZE } z_1 = \sum_{i \in I} a_i y_i
\]
\[
\text{MAXIMIZE } z_2 = \sum_{i \in I} a_i u_i
\]

Subject to
\[
\sum_{j \in N(i)} x_j \geq y_i + u_i \text{ for all } i \in I \tag{3}
\]
\[
u_i \leq y_i \text{ for all } i \in I \tag{4}
\]
\[
\sum_{j \in J} x_j = p
\]
x_j \in \{0,1\} \text{ for all } j \in J
y_i, u_i \in \{0,1\} \text{ for all } i \in I

Constraints (3) state that primary and backup coverage of demand point i cannot both be achieved unless at least two facilities are within time or distance (range) S from i. Constraints (4) state that backup coverage cannot be achieved unless primary coverage is first achieved.

3 Computational Implementation
We tested the models described in the previous section in a real case study, concerning the location of military units. More specifically, we applied the models for locating military units in the Municipality of Athens (Fig.1). This demand space was partitioned into sub areas (sub-polygons) by laying down a grid of square blocks of equal size.

Fig.1: The demand space (Municipality of Athens)

This demand space was broken up into a finite set of demand polygons by laying down a grid of square blocks of equal size. The centroid of each polygon was defined as a demand point representing the demand corresponding to that polygon (Fig.2).

Fig.2: Demand Points and Candidate Service Points

The objective was to find the best military structure i.e locate a number of military units (P) within the municipality in question using the above covering models, so as to cover the maximum area and/or same time achieve the best backup coverage for the military units. The models were solved using Premium Solver on a Pentium PC with a 4.2 Ghz processor and 8 Mbytes of RAM. The input files were prepared using ArcGIS of ESRI and the solutions were exported back into ArcGIS for visualization.

The following table presents the coverage solutions of the MCLP for 1 to 18 military units.

Table 1: Table of MCLP for Municipality of Athens

<table>
<thead>
<tr>
<th>MILITARY UNITS</th>
<th>MCLP COVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.31%</td>
</tr>
<tr>
<td>2</td>
<td>21.48%</td>
</tr>
<tr>
<td>3</td>
<td>30.59%</td>
</tr>
<tr>
<td>4</td>
<td>39.34%</td>
</tr>
<tr>
<td>5</td>
<td>47.63%</td>
</tr>
<tr>
<td>6</td>
<td>55.70%</td>
</tr>
<tr>
<td>7</td>
<td>63.56%</td>
</tr>
<tr>
<td>8</td>
<td>68.99%</td>
</tr>
<tr>
<td>9</td>
<td>74.13%</td>
</tr>
<tr>
<td>10</td>
<td>79.27%</td>
</tr>
<tr>
<td>11</td>
<td>83.78%</td>
</tr>
<tr>
<td>12</td>
<td>87.41%</td>
</tr>
<tr>
<td>13</td>
<td>91.33%</td>
</tr>
<tr>
<td>14</td>
<td>94.96%</td>
</tr>
<tr>
<td>15</td>
<td>96.53%</td>
</tr>
<tr>
<td>16</td>
<td>97.13%</td>
</tr>
<tr>
<td>17</td>
<td>98.32%</td>
</tr>
<tr>
<td>18</td>
<td>100%</td>
</tr>
</tbody>
</table>

Using G.I.S we can implement any of the above solutions very easily. Below we use G.I.S to present a scenario where we use MCLP for locating 13 military units in Municipality of Athens. This solution covers the 91.33% of the total area.
An interesting observation is the fact that the marginal contribution in coverage due to the location of additional servers decreases as the number of servers increases. For instance, in the municipality of Athens for range equal to 1000 meters, locating extra server increases population coverage by 9.17% when only one server is present, 4.54% when ten servers are present and only 1.19% when sixteen servers are present. Similar results are observed during the whole process of the addition of extra servers. Hence, it is worth performing this kind of parametric analysis since very good solutions in terms of coverage and structure can be identified.

As far as backup coverage is concerned, for achieving the best military back up structure, we have applied the bi-objective model presented in the previous section that considers primary as well as backup coverage. The constraint method was used to derive the compromised solutions, between the conflicting objectives of the bi-criterion problem. Adopting this method, we optimize the first objective while the other one is constrained to values that vary through a range of feasible values, as explained in Cohon [11].

The trade-offs between these two objectives for a range of 1 km and two different number of servers are presented in Table 2 and 3. Fig.4 presents the Bi-objective Trade-Off Curve of Table 3. For these solutions it is not possible to increase one objective without decreasing the other.

---

**Table 2: Trade off table of Athens, Scenario A (Range 1 km, servers 17)**

<table>
<thead>
<tr>
<th>Municipality</th>
<th>Range</th>
<th>Servers</th>
<th>Primary coverage</th>
<th>Backup coverage</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>17</td>
<td>98.32%</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>17</td>
<td>98.28%</td>
<td>11%</td>
<td></td>
</tr>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>17</td>
<td>98.20%</td>
<td>20%</td>
<td></td>
</tr>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>17</td>
<td>96.24%</td>
<td>31%</td>
<td></td>
</tr>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>17</td>
<td>91.39%</td>
<td>40%</td>
<td></td>
</tr>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>17</td>
<td>73.48%</td>
<td>50%</td>
<td>No feasible solution</td>
</tr>
</tbody>
</table>

**Table 3: Trade off table of Athens Scenario A (Range 1 km, servers 14)**

<table>
<thead>
<tr>
<th>Municipality</th>
<th>Range</th>
<th>Servers</th>
<th>Primary coverage</th>
<th>Backup coverage</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>14</td>
<td>94.96%</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>14</td>
<td>94.84%</td>
<td>12%</td>
<td></td>
</tr>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>14</td>
<td>90.04%</td>
<td>22%</td>
<td></td>
</tr>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>14</td>
<td>75.80%</td>
<td>30%</td>
<td></td>
</tr>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>14</td>
<td>56.14%</td>
<td>40%</td>
<td></td>
</tr>
<tr>
<td>Athens MBCLM</td>
<td>1000</td>
<td>14</td>
<td>53.48%</td>
<td>50%</td>
<td>No feasible solution</td>
</tr>
</tbody>
</table>

---

Fig.3: Locating 13 military units in Municipality of Athens

Fig.4: Bi-objective Trade-Off Curve of Table 6.
As one would expect, in the extreme case where no backup coverage is achieved, a small decrease in primary coverage results in a much bigger increase in backup coverage. As backup coverage increases, the necessary reduction in primary coverage progressively increases. Backup coverage cannot be increased beyond a certain limit which depends on the range and the number of available servers. For instance, in the case of 14 servers and range of 1000 meters, backup coverage cannot be increased beyond 50%.

Any of these solutions can be represented visually through GIS so that decision makers can have a clear understanding of any possible military structure - configuration and suggest possible improvements.

In Fig.5 we present the solution of a primary coverage of 96.24% and a backup coverage of 31% as displayed in GIS. The decision maker can directly observe that in cases of backup coverage models, military structure is more tight, as a result of the backup coverage and can be directly dependent on the value of the minimum distance which can be used as a constraint in cases of backup coverage.

![GIS Solution](image)

Fig.4: GIS Solution for a primary coverage of 96.24% and a backup coverage of 31%

This methodology can be extended in several ways. Firstly, it can be used to determine locations for alternative backup military services such as health centers, food kiosks etc. Secondly, it can be applied to locate administrative support centres for different types of military operations such as communication centers, etc.

4 Conclusion

In this paper we discuss the issue of locating military units in a realistic demand space, using covering models. We demonstrate that using Maximal Coverage type models, in combination with G.I.S, we can not only locate the most appropriate position for a number of available military units but we also can visualise the selected solutions using G.I.S capabilities. Furthermore, by changing the basic parameters of the models, (range of coverage, number of available military units, backup distance), we can create different scenarios of the original problem. By using G.I.S we can help decision makers to choose among the best solutions.

We demonstrate that using MCLP we can cover more space, but the need for backup coverage usually leads to less primary coverage while at the same time creates a more ‘robust’ military structure.

Empirical results from a realistic case study indicate that the proposed models in combination with G.I.S can be used to formulate, solve and visualise military coverage problems easily.

This work can be extended in several ways. Firstly, it would be interesting to investigate the possibility of having spatial objects such as areas rather than a discrete set of points as candidate locations for the servers. Secondly, it would also be an interesting challenge to experiment with backup coverage of more than one military unit, located in different backup distance values.

Furthermore using weights we can preset the importance of every space to be covered, thus continue to seek for the best military structure while covering at the beginning of the process the most important strategic places. Finally, from the implementation point of view, a natural extension would be to incorporate the GIS and the proposed model into an integrated system that will not require significant intervention by the user.

References:


