Capacity Calculation for Collaborative Communication with Imperfect Phase Synchronization, AWGN and Fading in Sensor Networks

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Abstract: Collaborative communication produces high power gain and significantly mitigates the fading if both frequency and phase synchronization are achieved. In this paper, a procedure of the capacity calculation for collaborative communication system with imperfect phase synchronization that includes the influence of noise and Rayleigh fading is proposed, modeled, theoretically analyzed and simulated. The exact formula for channel capacity as a function of channel parameters and the number of collaborative nodes is derived. It was confirmed that using collaborative communication, the capacity of the network can be increased and energy consumption can be reduced.

Key-Words: Collaborative communication, Channel capacity, Waveform channel, Noise and fading.

1 Introduction
Due to limited power of sensor nodes, an energy efficient transmission is the key requirement in wireless sensor networks. Wireless networks with collaborative communication [1]-[7] produce high power gain if phase and frequency is achieved. One of the basic problems in design of this kind of networks is how to determine their capacity.

In this paper we are investigating the channel capacity of collaborative communication system in the presence of AWGN and fading with imperfect phase synchronization. For a wireless sensor network of a star configuration, where a base station communicates directly with the sensor nodes, the capacity can be calculated using the procedure that is traditionally used in wireless cellular networks. In the case when the wireless sensor network uses collaborative nodes it achieves space diversity in signal transmission. For this kind of network the problem is how to calculate the network capacity. This problem, which was extensively investigated in the last decades, became lately a subject of research interests. An extensive survey of multi-way channels that were developed until 1976 can be found in [8]. The analysis of relay channels was a part of this survey. The capacity theorems for the Gaussian degraded, reversely degraded and feedback relay channels are presented in [9].

The relaying of signals in collaborative network can be achieved using various strategies. The compress-and-forward or decode-and-forward relay strategies are analysed for applications in relay networks assuming that the Gaussian noise and Rayleigh fading are present in the communication channel [4]. A detailed analysis of the capacity of a general wireless network with \( n \) nodes randomly located in a region of area 1 m\(^2\) is investigated and presented in [10].

The upper and lower capacity bounds for cooperative diversity are derived in [11] and forward relay networks are analyzed in [12] –[13] resulting in exact expressions for the probability of error and channel capacity over independent and non-identical Rayleigh fading channels [14] and Nakagami channel [15]. Capacity of noise channel and cooperative channels are presented and analyzed in [16] and [17].

One of the major contributions of this paper is the new mathematical model to calculate the capacity of collaborative communication system that includes the influence of the phase error, AWGN and Rayleigh fading in the channel. The second contribution is the derivation of theoretical expressions for both the received power and capacity as a function of the number of collaborative nodes. In addition a technique of power reduction using collaborative communication with preserving a high capacity of the network is proposed.

The paper is organized as follows. In Section 2 the mathematical model of the collaborative communication system is presented and capacity of...
the collaborative communication is calculated. Section 3 presents results of system simulation, analytical results and addresses problem of power consumption. Section 4 contains conclusions.

2 Mathematical Model of Collaborative Communication System

2.1 General Considerations
In distributed sensor network the accurate position of sensor nodes is generally not known and there is no central node that performs phase, time and frequency synchronization. This results in the position estimation error (called the displacement error), that is translated into phase error of the transmitted signal. To achieve a high power gain, capacity gain and reduce the BER using collaborative transmission, the frequency, time and phase synchronization needs to be achieved.

However, the perfect synchronization cannot be achieved in this system. Therefore, in this paper we are proposing a general architecture in which collaborative nodes transmit the same data to the base station by using same carrier signal with imperfect phase synchronization with the local carrier at the base station as shown in Figure 1. In Figure 1 a set of nodes that make a network to collaboratively transmit the same data towards the base station. Sensor nodes can exchange the information within the network and towards the base station. For \( m \) collaborative nodes the detailed structure of the system that is used for its theoretical modelling is shown in Figure 2.

![Figure 1 Geometry of Sensor Nodes.](image)

2.2 Capacity Calculation
Let \( m \) collaborative nodes make a network to transfer the information to the base station as shown in Figure 1. Let \( s(t) \) is the information data transmitted to the base station by the collaborative nodes. The distance between the nodes and the base station estimated with certain errors of estimation. These errors in distance estimation we will call the displacement errors. In our mathematical model of the network these displacement errors will be translated into phase errors. Let \( d_0 \) be the nominal distance between collaborative node and the base station. Let \( f_0 \) be the carrier frequency, and assuming line of sight propagation, the phase produced due to distance \( d \) can be written as \( \Theta_0 = \frac{2\pi f_0 d}{c} \), where \( c \) is the speed of light. Let \( \Delta d(i) \) be the displacement error between base station and the \( i^{th} \) collaborative node, then the phase error for the \( i^{th} \) node due to displacement error is given by \( \Theta_i = \frac{2\pi f_0 \Delta d(i)}{c} \).

Let \( \cos(2\pi f_0 t) \) be the carrier/reference signal used by all collaborative nodes. We assume that signal delay is very small with respect to the signal bit interval \( T \), so there is significant guard interval and inter symbol interference (ISI) can be ignored.

The cumulative received signal at the Base station is given by:

\[
r_r(t) = \Re \left\{ \sum_{i=1}^{m} h_i s(t)e^{j(2\pi f_0 t + \Theta_i(i))} \right\} + n(t) ,
\]

where \( n(t) \) is AWGN and \( h_i \) is the Rayleigh fading. At the base station the demodulated signal is integrated and its output is given by

\[
R = \sum_{i=1}^{m} h_i S \cos(\Theta_j(i)) + n.
\]

where \( S = \pm \sqrt{E_s} \) is the signal amplitude and \( n \) is the noise amplitude at sampling time \( T \). Power of \( R \) is given by

\[
P_R = \left[ \sum_{i=1}^{m} h_i S \cos(\Theta_j(i)) + n \right]^2.
\]

As \( \Theta_i(i) \), \( h_i \) and \( n \) are the independent random variables and \( n \) is zero-mean, we have to calculate the mean value of received power as

\[
E[P_R] = E \left[ \left[ \sum_{i=1}^{m} h_i S \cos(\Theta_j(i)) + n \right]^2 \right] + \sigma^2_n ,
\]

where \( \sigma^2_n \) and is the variance of noise and \( \sigma^2_n = N \) (Noise power). Equation (4) can also be developed as follows.
Fig. 2 Mathematical model of a collaborative communication system.

\[ E[P_R] = \sum_{i=1}^{\infty} E[S^2_i] E[\cos^2(\Theta_f(i))] E[h_i^2] + \sum_{i=1}^{\infty} \sum_{j=i}^{\infty} E[S^2_i] E[\cos(\Theta_f(i))\cos(\Theta_f(j))] E[h_i h_j] + N \]

As \( E[S^2]=P \), all \( \Theta_f(i) \) are i.i.d random variables, therefore \( E[\Theta_f(i)] \sim E[\Theta_f] \) and all \( h_i \) are i.i.d. random variables, therefore, \( E[h_i]=E[h] \). The equation (5) can now be written as

\[ E[P_R] = mPE[\cos^2(\Theta_f)]E[h_i^2] + m(m-1)PE[\cos(\Theta_f)]E[\cos(\Theta_f)]E[h_i h_j] + N \]  

(6)

Using the values of equations (A.1), (A.2), mean value of \( h \) and \( E[h^2]=1 \) equation (6) becomes

\[ E[P_R] = mP \left[ \frac{1}{2} + \frac{\sin(2\phi)}{4\phi} \right] + m(m-1)b^2\pi P \left[ \frac{\sin(\phi)}{\phi} \right]^2 + N \]  

(7)

where \( \phi \) is distribution limit of phase error and \( b \) is the mode of Rayleigh random variable \( h \).

The power of the received signal is the sum of the signal part and noise part. Thus the capacity of the collaborative communication system can be found as

\[ C = \frac{1}{2} \log_2 \left( 1 + \frac{S}{N} \right) = \frac{1}{2} \log_2 \left( 1 + \left[ \frac{1}{2} + \frac{\sin(2\phi)}{4\phi} \right] + \frac{m(m-1)b^2\pi}{2} \left[ \frac{\sin(\phi)}{\phi} \right]^2 \right) \frac{P}{N} \]  

(8)

In the absence of phase error i.e., \( \phi = 0 \), the channel capacity of collaborative communication system may be found as

\[ C = \frac{1}{2} \log_2 \left( 1 + \frac{S}{N} \right) = \frac{1}{2} \log_2 \left( 1 + \left[ \frac{1}{2} + \frac{\sin(2\phi)}{4\phi} \right] + \frac{m(m-1)b^2\pi}{2} \right) \frac{P}{N} \]  

(9)

**Special Case:**

If the transmitted power of each collaborative node is \( P/m \), equation (8) can be written as,

\[ C = \frac{1}{2} \log_2 \left( 1 + \frac{S}{N} \right) = \frac{1}{2} \log_2 \left( 1 + \left[ \frac{1}{2} + \frac{\sin(2\phi)}{4\phi} \right] + \frac{m(m-1)b^2\pi}{2} \right) \frac{P}{N} \]  

(10)

This is the case when we reduce the power of each collaborative node by factor \( m \), i.e., the total transmitted power is \( P \). We will use this reduction to investigate the capacity with reduced power consumption of each collaborative node. This case can be used to investigate the effect of total transmitted power on capacity. The results obtained...
from equations (8), (10) and Monte Carlo simulation is discussed in section 3.

3 Analysis of a Collaborative Communication Channel Capacity

We have performed Monte Carlo simulation to analyze the capacity of collaborative communication system in the presence of phase error using SIMULINK and MATLAB. The wireless communication environment used in simulation is established using SIMULINK’s AWGN and Rayleigh channel blocks. Results show that the analytical results match very well with the simulation results.

Figures 3 and 4 present analytical and simulation results of our model in the presence of Rayleigh fading for phase error distributed over \([-0.1\pi~-0.1\pi]\) and \([-0.4\pi~-0.4\pi]\) respectively, and assuming that the power of each collaborative node is \(P\), so the total transmitted power of the network is equal to \(mP\). Figures 5 and 6 present analytical and simulation results for phase error distributed over \([-0.1\pi~-0.1\pi]\) and \([-0.4\pi~-0.4\pi]\) respectively for the case when the transmitted power of each collaborative node is \(P/m\), so the total transmitted power of the network is \(P\).

From results shown in figures 3, 4, 5 and 6, it is confirmed that the capacity of the system increases as number of collaborative nodes increases. It is also shown that capacity decreases as the phase error increases. From figures 3, 4, 5 and 6 it is obvious that capacity decreases by approx. 0.3 bits/sec/Hz, when the phase error distribution increases from \([-0.1\pi~-0.1\pi]\) to \([-0.4\pi~-0.4\pi]\).

3.1 Capacity in the presence of AWGN and Rayleigh fading

From figures 5 and 6 it is analyzed that if the total transmitted power of the network is decreased by factor \(m\), where \(m\) is number of collaborative nodes, we can still achieve substantial capacity gain. From equations (8), (10) and results shown it is also analyzed that if the total transmitted power is decreased by factor \(m\), capacity will decreased by only factor \(0.5\log_2(m)\) in the presence of AWGN, fading and phase errors.

The proposed collaborative communication system modelling and capacity calculation can be very useful tool in practice, because the required capacity of network can be maintained with the reduced energy consumption in the network. Thus, by using collaborative communications inside complex networks can be used as a method for reducing overall power consumption in the network.
Fig 6. Capacity for phase error distributed over \{-0.4π \leq \varphi \leq 0.4π\} and each node transmits power $P/m$, so total transmitted power by the network is $P$.

4 Conclusions

Theoretical analysis and simulation confirmed that the capacity of a collaborative communication channel in the presence of AWGN, fading and phase error can be found in closed form and expressed as a function of signal to noise ratio for the number of collaborative nodes as a parameter. It is concluded that significant capacity gain can be achieved using collaborative communication system with imperfect phase synchronization. This is the consequence of the collaborative communication system that can be considered as a space diversity system. In particular we analysed the case when the power of the transmitted signal is reduced by $m$ times where $m$ is the number of collaborative nodes. It is concluded that capacity will decreased by only factor $0.5\log_2(m)$ if the total transmitted power is decreased by $m$ times. It was shown that the capacity can be maintained high in this case, which leads to the substantial power saving in the system.

References:


**Appendix**

As \( \Theta_f \) has uniform distribution from \( \{-\varphi \text{ to } \varphi\} \).

The mean value of \( \cos(\Theta_f) \)

\[
E[\cos(\Theta_f)] = \int_{-\infty}^{\infty} \cos(\Theta_f) p(\Theta_f) d\Theta_f
\]

\[
= \int_{-\varphi}^{\varphi} \cos(\Theta_f) \frac{1}{2\varphi} d\Theta_f
\]

\[
= \frac{\sin(\varphi)}{\varphi}.
\]  

(A.1)

The mean value of \( \cos^2(\Theta_f) \)

\[
E[\cos^2(\Theta_f)] = \int_{-\infty}^{\infty} \cos^2(\Theta_f) p(\Theta_f) d\Theta_f
\]

\[
= \int_{-\varphi}^{\varphi} \cos^2(\Theta_f) \frac{1}{2\varphi} d\Theta_f
\]

\[
= \frac{1}{2} + \frac{\sin(2\varphi)}{4\varphi}.
\]  

(A.2)