

Complex Dynamics of a Memristor Based Chua's Canonical Circuit

CHRISTOS K. VOLOS

Department of Mathematics and Engineering Sciences
Univ. of Military Education - Hellenic Army Academy
Athens, GR16673
GREECE
chvolos@gmail.com

IOANNIS M. KYPRIANIDIS AND IOANNIS N. STOUBOULOS

Department of Physics
Aristotle University of Thessaloniki
Thessaloniki, GR54124
GREECE
imkypr@auth.gr, stouboulos@physics.auth.gr

COSTIN CEPIȘCĂ,

Faculty of Electrical Engineering,
Politehnica University of Bucharest,
Bucharest, ROMANIA
costin.cepisca@upb.ro

Abstract: - In this paper the complex dynamics of Chua's canonical circuit with a memristor instead of a nonlinear resistor, was studied. The proposed memristor is a flux controlled memristor, where the relation between flux and charge is a smooth continuous cubic function. A very important phenomenon concerning Chaos theory, such as, the great sensitivity of the circuit on initial conditions, was studied thoroughly by using various techniques via computer simulations.

Key-Words: - Memristor, nonlinear circuit, Chua's canonical circuit, sensitivity on initial conditions, bifurcation diagram.

1 Introduction

In the last decade the research activities in the field of systems with chaotic behavior have triggered a lot of investigation on possible military applications of such systems. Especially, the design and implementation of chaotic circuits became a very interesting subject due to their applications in secure communications [1], cryptography [2], random number generators [3-4], radars [5] and robots [6]. Also, the recent confirmation of the existence of the memristor, which is the fourth fundamental component in electronic circuit theory, established a new approach in nonlinear circuits design.

In 1971 Chua claimed that a fourth element should have been added to the list of the three fundamental elements in electronic circuit theory: resistor (R), capacitor (C), and inductor (L) [7]. This element was named memristor (short for memory-resistor). Even though such an element would have useful potential applications, nobody had presented a physical device of this element. Recently, researchers at Hewlett-Packard Laboratories

published a paper announcing the invention of the memristor [8].

In that work, Strukov et al. presented a physical device to illustrate their invention. It was a new nanometer-size solid-state two-terminal device, which "remembers" its state, after its power is turned off, that explains several phenomena in nanoscale systems, such as in thermistor [9] and spintronic devices [10]. Using memristors, the researchers at the HP Laboratories succeeded to create a simple data storage device with a great storage density. Also, the memristor could lead to the next generation of computers and to the development of cellular neural networks [11].

In more details, Chua noted that there must be six mathematical relations connecting pairs of the four fundamental circuit variables, current (i), voltage (v), charge (q) and magnetic flux (ϕ), but a link between charge and flux was missing. As it is known from the electronic circuit theory, the three fundamental elements, resistor (R), capacitor (C)

and inductor (L), connect pairs of the four circuit variables, in such a way, that:

$$R = \frac{dv}{di}, \quad C = \frac{dq}{dv}, \quad L = \frac{d\phi}{di} \quad (1)$$

So, Chua came to the conclusion that there should be a fourth fundamental element, the memristor, which satisfies Eq. (2) between charge and flux. $M(q)$ is called memristance and associates voltage (v_M) and current (i_M) of the memristor with the Eq.(3).

$$M(q) = \frac{d\phi(q)}{dq} \quad (2)$$

$$v_M = M(q) \cdot i_M \quad (3)$$

In the case of a linear element, M is a constant, and the memristor is identical to resistor. However, in the case of M being itself a function of q , produced by a nonlinear circuit element, then no combination of the fundamental circuit elements reproduces the same results as the memristor. Also, in the relation

$$i_M = W(\phi) \cdot v_M \quad (4)$$

$W(\phi)$, is called memductance and is determined by Eq. (5).

$$W(\phi) = \frac{dq(\phi)}{d\phi} \quad (5)$$

In Ref. [12] a memristor-based Chua's canonical circuit has been studied. Various phenomena, which are related to Chaos theory, such as the sensitivity on initial conditions, the route to chaos through the mechanism of period doubling, the phenomenon of coexisting attractors and the antimonotonicity, were observed.

In this work, we have focused our attention to the great sensitivity on initial conditions of the above mentioned circuit. This phenomenon is probably the most important in nonlinear systems and especially in memristors, which are nonlinear systems with memory. The memory in the case of such systems has the meaning of the initial conditions. So, in this work, the effect of initial conditions in the circuit's behavior was studied thoroughly, via computer simulations. The proposed memristor has a cubic nonlinear relation between flux (ϕ) and charge (q). In the next sections, the proposed nonlinear circuit is described followed by the simulation analysis. In this procedure we have used various techniques,

based on bifurcation diagrams, for observing the great sensitivity of the system on initial conditions. Conclusions remarks are included in the last section.

2 The Proposed Circuit

In 2008, Itoh and Chua proposed several nonlinear oscillators based on Chua's circuit, in which the Chua diode was replaced by monotone increasing piecewise-linear memristors [13]. Muthuswamy and Kokate proposed other memristor based chaotic circuits [14]. In 2010, Muthuswamy and Chua proposed an autonomous circuit that uses only three circuit elements in series: a linear passive inductor, a linear passive capacitor and a memristor [15]. Furthermore, in Refs. [16-19] cubic memristors have replaced the nonlinear elements in well known family of Chua's circuits.

In this work, our study was based on Chua's canonical circuit [20-23]. This circuit is a nonlinear autonomous 3rd-order electric circuit, where G_n is a linear negative conductance, while it's nonlinear resistor has been replaced by a memristor (Fig. 1). The proposed memristor M is a flux-controlled memristor described by the function $W(\phi(t))$, where $q(\phi)$ in Eq. (6) is a smooth continuous cubic function of the form:

$$q(\phi) = -a \cdot \phi + b \cdot \phi^3 \quad (6)$$

with $a, b > 0$. As a result, in this case the memductance $W(\phi)$ is provided by the following expression:

$$W = \frac{dq(\phi)}{d\phi} = -a + 3 \cdot b \cdot \phi^2 \quad (7)$$

By applying Kirchhoff's circuit laws to the memristor-based Chua's canonical circuit, we obtain the following state equations (8),

$$\begin{cases} \frac{d\phi}{dt} = v_1 \\ \frac{dv_1}{dt} = \frac{1}{C_1}(i_L - W(\phi)v_1) \\ \frac{dv_2}{dt} = -\frac{1}{C_2}(i_L + G_N v_2) \\ \frac{di_L}{dt} = \frac{1}{L}(-v_1 + v_2 - i_L R) \end{cases} \quad (8)$$

where, v_1 and v_2 represent the voltages across the capacitors C_1 and C_2 , while i_L is the current through the inductor L . In the present paper we have chosen the following values for the circuit parameters: $R = 300 \, \Omega$, $L = 100 \, \text{mH}$, $G_n = -0.40 \, \text{mS}$, while C_1 and C_2 are the control parameters. Also, α and b have the following values: $\alpha = 0.5 \cdot 10^{-4} \, \text{C/Wb}$ and $b = 4 \cdot 10^4 \, \text{C/Wb}^3$. In Fig. 2, the $\phi - q$ characteristic curve of Eq. (6) for the chosen set of parameters α and b is plotted.

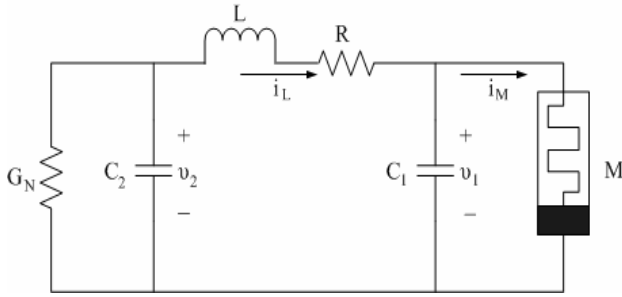


Fig. 1. Memristor-based Chua canonical circuit.

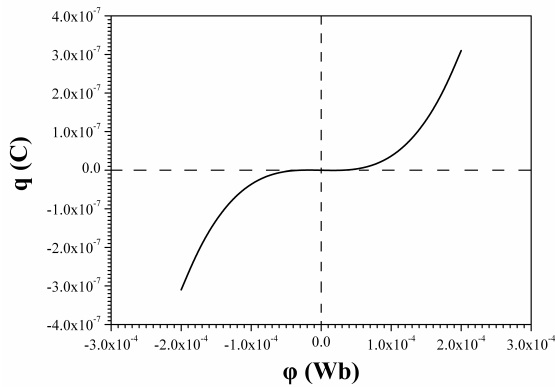


Fig. 2. The flux–charge smooth continuous cubic function of the memristor.

3 The Effect of Initial Conditions in Circuit's Dynamic

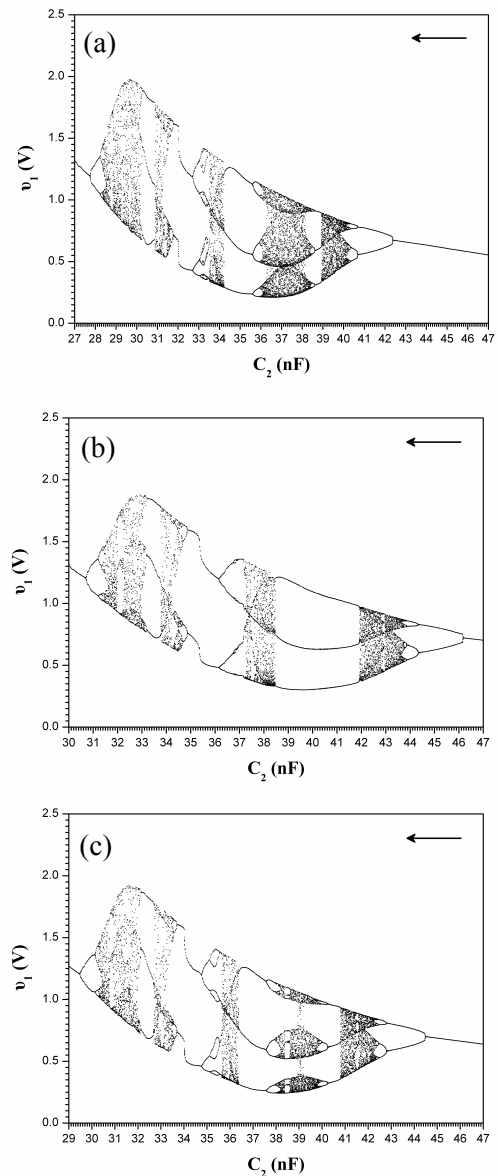
In this section, the study of the dynamic behavior of the system's state equations (8) was investigated numerically by employing a fourth order Runge–Kutta algorithm.

It is known, that systems which exhibit chaotic behavior are very sensitive to the changes of the initial conditions. Different initial conditions will probably create totally different dynamic behavior. The main tool of dynamic's investigation, in this work, are the bifurcation diagrams of voltage (v_1) versus capacitance (C_2), which were plotted by giving constant values to capacitance C_1 .

The study of system's sensitivity on initial conditions was based on the use of three investigation approaches following different production methods of the bifurcation diagrams. In the next paragraphs the above mentioned approaches with the corresponding remarks are presented.

3.1 First Approach

In the first approach, the bifurcation diagrams were produced by decreasing the value of the capacitor C_2 from $C_2 = 47 \, \text{nF}$ with step $\Delta C_2 = 0.01 \, \text{nF}$, which is shown by the left arrow in each bifurcation diagram, while the value of C_1 remained the same ($C_1 = 28 \, \text{nF}$). Also, the system had different initial conditions in each iteration, in such a way that the last set of initial conditions in previous iteration became the first set in the next iteration. Furthermore, the initial value of the parameter (ϕ) changed for each diagram, while the other initial conditions remained the same: $(v_1)_0 = 0.005 \, \text{V}$, $(v_2)_0 = 0.015 \, \text{V}$ and $(i_L)_0 = 0.001 \, \text{A}$.



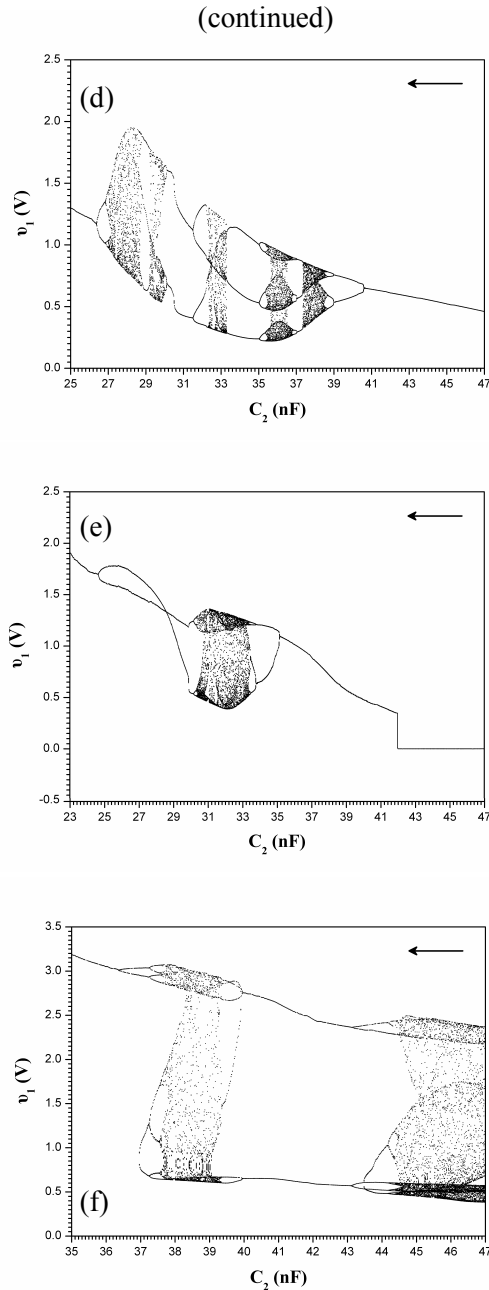


Fig. 3. Bifurcation diagrams of v_1 versus C_2 , for $C_1 = 28$ nF, with initial conditions: $(v_1)_0 = 0.005$ V, $(v_2)_0 = 0.015$ V, $(i_L)_0 = 0.001$ A and (a) $(\varphi)_0 = 0$ Wb, (b) $(\varphi)_0 = 0.00015$ Wb, (c) $(\varphi)_0 = 0.00018$ Wb, (d) $(\varphi)_0 = 0.00010$ Wb, (e) $(\varphi)_0 = 0.00005$ Wb and (f) $(\varphi)_0 = 0.00012$ Wb.

The comparative study of the six bifurcation diagrams of Fig. 3 shows the qualitative change of the dynamic behavior of the circuit with memristor, as C_2 takes different discrete values. As it is shown in Fig. 3, a slight change of the initial value of a variable, (φ) , can cause a dramatically different bifurcation diagram. For our circuit, this is the first proof of its great sensitivity on initial conditions.

3.2 Second Approach

According to this approach, a comparison between two different bifurcation diagrams was made. The first bifurcation diagram (Fig. 4a) of v_1 versus C_2 , for $C_1 = 28.5$ nF, with initial conditions, $(v_1)_0 = 0.83987$ V, $(v_2)_0 = 0.95493$ V, $(i_L)_0 = 0.00139$ A and $(\varphi)_0 = 0.00013$ Wb, was produced by increasing the value of the capacitor C_2 from $C_2 = 29$ nF to $C_2 = 47$ nF with step $\Delta C_2 = 0.01$ nF.

The second bifurcation diagram of v_1 versus C_2 (Fig. 4b) is the reverse of the first one, with the same value of C_1 ($C_1 = 28.5$ nF). This means that this diagram was produced by decreasing the value of the capacitor C_2 from $C_2 = 47$ nF to $C_2 = 29$ nF, with the same step $\Delta C_2 = 0.01$. In this case, the last set of conditions of the first diagram was used as initial conditions, $((v_1)_0 = 0.66708$ V, $(v_2)_0 = 0.75844$ V, $(i_L)_0 = 0.00172$ A and $(\varphi)_0 = 0.00014$ Wb).

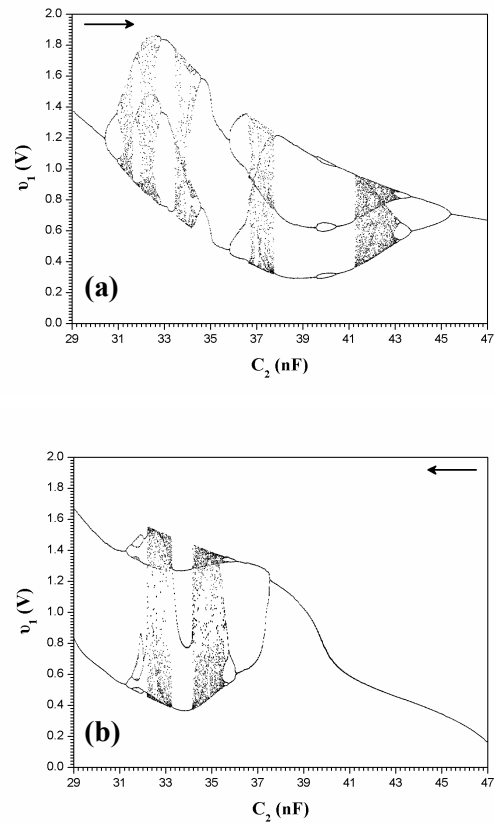


Fig. 4. Bifurcation diagrams of v_1 versus C_2 , for $C_1 = 28.5$ nF, which in case (a) the diagram was produced by increasing the value of the capacitor C_2 while in (b) the diagram was produced by decreasing the value of the capacitor C_2 .

This method of study of the dynamic behavior is very useful, especially in real systems in which a variable resistor or capacitor may play the role of the

control parameter, as happens in this case. The bifurcation diagrams of Fig. 4 reveal in a most clear way the great sensitivity of the system on initial conditions. Each one of the two bifurcation diagrams is completely different from the other one.

As we can observe, in these two bifurcation diagrams chaotic regions coexist with windows of periodic behavior (i.e. for $C_2 = 42$ nF), while in other regions different periodic limit cycles are shown (i.e. for $C_2 = 40$ nF, where a period-6 limit cycle of the first diagram coexists with a period-1 limit cycle of the second diagram).

3.3 Third Approach

Finally, in this approach two seemingly same bifurcation diagrams were compared again. These two bifurcation diagrams (Fig. 5) of v_1 versus C_2 for $C_1 = 29$ nF were produced by decreasing the value of the capacitor C_2 from $C_2 = 47$ nF to $C_2 = 27$ nF with step $\Delta C_2 = 0.01$ nF. Also, the initial conditions were: $(v_1)_0 = 0.005$ V, $(v_2)_0 = 0.015$ V, $(i_L)_0 = 0.001$ A and $(\phi)_0 = 0.0001$ Wb. The only, but very important difference between these two diagrams was the nonidentical set of initial conditions in each iteration.

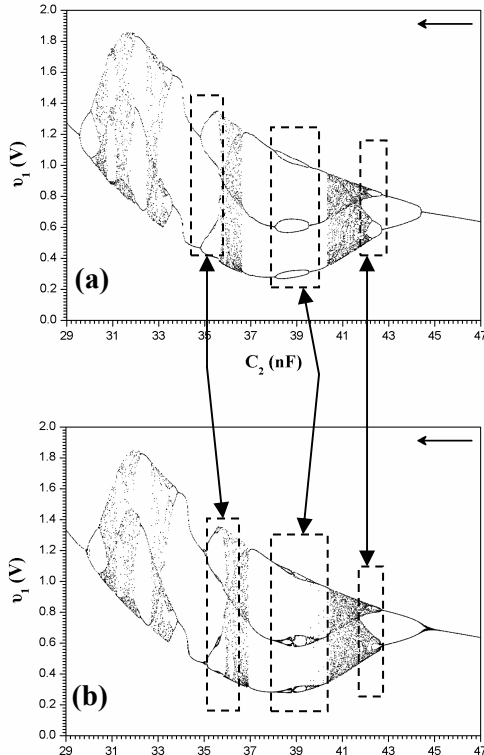


Fig. 5. Bifurcation diagrams of v_1 versus C_2 , for $C_1 = 29$ nF, with (a) different set of initial conditions and (b) with the same set of initial conditions in each iteration.

In the first diagram, the system had different initial conditions in each iteration, in such a way that the last set of initial conditions in previous iteration became the first set in next iteration, as it has been mentioned in the previous approaches. However, the second diagram is produced with the same set of initial conditions in each iteration.

The two bifurcation diagrams of Fig. 5 have almost the same form with a slight difference especially in the regions of periodic behavior. This difference is more obvious in the regions inside the dotted frames of Fig. 5. In each one of these three regions, different periodic limit cycles coexist with each other. Especially, the triple primary bubble in the middle dotted frame of Fig. 5(a) was converted to a more complex structure, as it is clearly shown in Fig. 6.

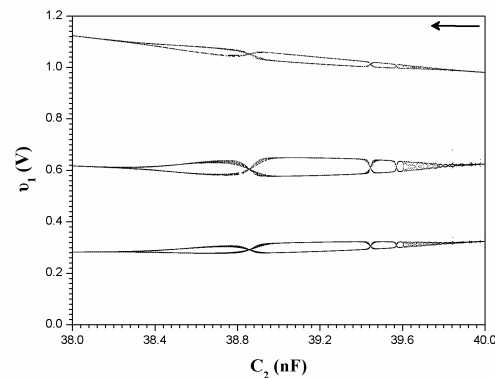


Fig. 6. The enlargement of region in the second dotted frame of the bifurcation diagram of Fig. 5(b).

4 Conclusion

In this paper, the great sensitivity on initial conditions of a memristor based Chua's canonical circuit has been studied. The nonlinear resistor of the initial Chua's circuit, [20], has been replaced by a memristor, which had a continuous cubic function between flux (ϕ) and charge (q). Using the bifurcation diagram as the basic tool of our study, we observed the qualitative changes of the dynamic behavior of the circuit using different sets of initial conditions. For a more thoroughly examination of the influence of initial conditions to the system's behavior we apposed three different approaches, in which the bifurcation diagrams were produced with different techniques.

Generally, this work is the first attempt to examine the phenomenon of sensitivity on initial conditions in a case of a nonlinear circuit which contains a memristor. As a result we have to mention, that the existence of the memristor augments this phenomenon, as we can observe,

especially in the first approach. Furthermore, this phenomenon is very important, due to the fact that initial conditions of a system with memristor play the role of the memory, for which such systems are characterized. Finally, the next step in the study of the influence of initial conditions to the dynamic behavior, would be the use of various proposed memristors in other well known nonlinear circuits.

References:

- [1] C. K. Tse and F. Lau, Chaos-based Digital Communication System: Operating Principles, Analysis Methods, and Performance Evaluation, Berlin, New York: Springer Verlag, 2003.
- [2] Ch. K. Volos, I. M. Kyprianidis and I. N. Stouboulos, Experimental Demonstration of a chaotic Cryptographic Scheme, *WSEAS Trans. Circuits Syst.*, Vol. 5, 2006, pp. 1654–1661.
- [3] Ch. K. Volos, I. M. Kyprianidis and I. N. Stouboulos, Image Encryption Process Based on a Chaotic True Random Bit Generator, *Proc. 16th IEEE Int. Conference on Digital Signal Processing*, Vols. 1 and 2, 2009, pp. 1091–1094.
- [4] Ch. K. Volos, I. M. Kyprianidis and I. N. Stouboulos, Fingerprint Images Encryption Process based on a Chaotic True Random Bits Generator, *Int. J. Multimedia Intelligence and Security*, Vol. 1, 2010, pp. 320–335.
- [5] T. Jiang, S. Qiao, Z. Shi, L. Peng and J. Huangfu, Simulation and Experimental Evaluation of the Radar Signal Performance of Chaotic Signals Generated from a Microwave Colpitts Oscillator, *Progress In Electromagnetics Research*, Vol. 90, 2009, pp. 15–30.
- [6] D. I. Curiac and C. Volosencu, Developing 2D Chaotic Trajectories for Monitoring an Area with Two Points of Interest, *Proc. of the 10th WSEAS Int. Conference on Automation and Information*, 2009, pp. 366–369.
- [7] L. O. Chua, Memristor - The Missing Circuit Element, *IEEE Trans. on Circuit Theory*, Vol. CT-18, 1971, pp. 507–519.
- [8] D. Strukov, G. Snider, G. Stewart and R. Williams, The Missing Memristor Found, *Nature*, Vol. 453, 2008, pp. 80–83.
- [9] M. Sapoff and R. M. Oppenheim, Theory and Application of Self-heated Thermistors, *Proc. IEEE*, Vol. 51, 1963, pp. 1292–1305.
- [10] Y. Pershin and M. Di Ventra, Spin Memristive Systems: Spin Memory Effects in Semiconductor Spintronics, *Phys. Rev. B, Condens. Matter*, Vol. 78, 2008, pp. 113309/1–4.
- [11] M. Itoh and L. O. Chua, Memristor Cellular Automata and Memristor Discrete Time Cellular Neural Networks, *Int. J. Bifurcation and Chaos*, Vol. 19, 2009, pp. 3605–3656.
- [12] Ch. K. Volos, I. M. Kyprianidis and I. N. Stouboulos, Memristors: A New Approach in Nonlinear Circuits Design, *Proc. of the 14th WSEAS Int. Conference on Communication*, 2010, pp. 25–30.
- [13] M. Itoh and L. O. Chua, Memristor Oscillators, *Int. J. Bifurcation and Chaos*, Vol. 18, 2008, pp. 3183–3206.
- [14] B. Muthuswamy and P. P. Kokate, Memristor based Chaotic Circuits, *IETE Techn. Rev.*, Vol. 26, 2009, pp. 415–426.
- [15] B. Muthuswamy and L. O. Chua, Simplest Chaotic Circuit, *Int. J. Bifurcation and Chaos*, Vol. 20, 2010, pp. 1567–1580.
- [16] I. M. Kyprianidis, Ch. K. Volos and I. N. Stouboulos, Chaotic Dynamics from a Nonlinear Circuit based on Memristor with Cubic Nonlinearity, *Proc. AIP*, Vol. 1203, 2010, pp. 626–631.
- [17] B. Muthuswamy, Implementing Memristor based Chaotic Circuits, *Int. J. Bifurcation and Chaos*, Vol. 20, 2010, pp. 1335–1350.
- [18] B. Bo-Cheng, L. Zhong and X. Jian-Ping, Transient Chaos in Smooth Memristor Oscillator, *Chin. Phys. B*, Vol. 19, 2010, pp. 030510–1–6.
- [19] L. Chua, Resistance Switching Memories are Memristors, *Appl. Phys A*, Vol. 102, 2011, pp. 765–783.
- [20] L. O. Chua and G. N. Lin, Canonical Realization of Chua's Circuit Family, *IEEE Trans. Circuits Syst.*, Vol. 37, 1990, pp. 885–902.
- [21] I. M. Kyprianidis, M. L. Petrani, J. A. Kalomirois and A. N. Anagnostopoulos, Crisis-induced Intermittency in a Third-order Electrical Circuit, *Phys. Rev. E*, Vol. 52, 1995, pp. 2268–2273.
- [22] I. M. Kyprianidis and M. E. Fotiadou, Complex Dynamics in Chua's Canonical Circuit with a Cubic Nonlinearity, *WSEAS Trans. Circ. Syst.*, Vol. 5, 2006, pp. 1036–1043.
- [23] I. M. Kyprianidis, New Chaotic Dynamics in Chua's Canonical Circuit, *WSEAS Trans. Circ. Syst.*, Vol. 5, 2006, pp. 1626–1633.