Accelerated Analysis of Low-Level Injection Operation for Transistor-Based Oscillating Amplifiers

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Abstract: - The problem of the accelerated analysis of oscillating amplifiers is addressed. To achieve both computational efficiency and adequate accuracy also in case of transmission-type, transistor-based, circuit structures, a novel approach is proposed which makes use of a general reduced-order model of the injected oscillator. A perturbation-refined analysis method is thus applied, which permits to derive the first-order exact set of differential equations that describes the system behavior in the fundamental-frequency complex-envelope domain. As illustrated by the RF Meissner driven oscillator example presented, the devised approach achieves the stated goals, and lends itself as a convenient, design-oriented, tool for the fast analysis of steady-state, locking-stability and transient response of this class of nonlinear dynamical circuits.

Key-Words: Oscillating amplifiers, Injection-locking, Nonlinear circuit simulation, Adler equation.

1 Introduction

Oscillating amplifiers, also known as injection-locked amplifiers, are a class of nonlinear circuits with peculiar features. They are adopted in the RF and microwave frequency ranges when a highly-saturated, narrow-band, amplification of a weak signal is required. They can also be adopted to obtain, from a high-power high-efficiency but noisy oscillator and a low-power low-noise source, a sinusoidal signal with excellent phase noise performances [1-2].

Because of the nonlinear resonant nature of the equations characterizing such circuits, conventional analysis or simulation techniques in the time-domain are extremely inefficient, especially if global behavior quality indexes are of interest, e.g., for design purposes. As example, we can consider the evaluation of the locking bandwidth (LBW, in what follows), i.e., the range of frequencies where the phase-lock condition is achieved. Especially in case of low-level injection operation, where the LBW is a small fraction of the carrier frequency, this search, if carried out numerically, can become extremely time consuming. For this reason, a number of methods have been developed in the past for the study of oscillating amplifiers in the stroboscopic time-domain, directly in terms of amplitude and phase of the complex envelope of the various harmonics of state variables. In case the analysis is developed in a completely numerical manner (e.g., using “Circuit-Envelope” algorithms [3]), there is a significant advantage in terms of computational efficiency, but the problem of the lack of a design-oriented tool remains. Also, while the steady-state and transient operation are efficiently simulated, the same does not occur for the LBW evaluation, which still involves a time-consuming, man-assisted, iterative search procedure via bracketing of stable and unstable solutions in the surroundings of the unlock frequency limits. As to the fully analytical approaches, while potentially extremely powerful, they have to cope with the difficulties of such a stiff dynamical nonlinear problem [1-2,4-8]. Therefore, they usually ground on substantial approximations of the problem, which either limit the class of treatable systems, or reduce the accuracy of the analysis. In particular, it can be noticed that while the frequency-domain theory of oscillating amplifiers equipped with negative-resistance microwave diodes is relatively complete [4], the same does not hold for more up-to-date circuit configurations using RF transistors as active element(s) and a transmission-type topology [2].

In this paper, a novel approach to the above said problem is proposed. A general, reduced-order, model of the injected oscillator is firstly introduced. A perturbation-refined analysis method is then applied, which permits to derive the first-order exact set of differential equations describing the circuit behavior in the fundamental-frequency complex-envelope domain. As shown below, this semi-analytical method achieves the goal of combining high computational efficiency and good accuracy.
2 Deriving Dynamical System Model

The circuit block structure of the transmission-type, transistor-equipped, oscillating amplifiers we are considering here is illustrated in Fig.1. There is evidenced the single-loop topology comprising the ideal summing network and the feedback block “β”, the amplifying transistor, and the selective tank and load-coupling network. Although not arguable from the figure, it is assumed here that a generic circuit with the shown topology belongs to the class of treatable systems only if it satisfies appropriate conditions. In particular, we assume here that the scheme represents a properly designed, self-starting monochromatic oscillator, when no input locking signal is applied ($V_s=0$). This means that the resonator must possess adequately high selectivity characteristics. Consequently, we can assume that all node voltages will be quasi-sinusoidal quasi-static waveforms under transient operation. They can thus be characterized in terms of the relevant first-harmonic components (amplitude and phase: $V_k=V_{k}(t)\cdot \exp[j\theta_k(t)]$; $k=0\div4$), which result slowly varying functions of time. Therefore, we can develop our analysis in the dynamical fundamental-frequency complex-envelope domain. In particular, after deep investigation of the problem, we selected the equivalent system block structure depicted in Fig.2, that can be derived from its circuital counterpart of Fig.1 after proper identification of the various functional blocks. For the purpose of our analysis, it is important to evidence the presence of the “$Y_N$” block which represents the active device. Unlike previous treatments in which the nonlinearity is modeled using a Sinusoidal-Input Describing Function (SIDF) [9] or other equivalent single I/O element [4-7], here we adopt a Two-Sinusoidal-Input Describing-Function (TSIDF) [9]. In fact, while a SIDF model is adequate to describe in the frequency-domain the instantaneous nonlinear relationship between voltage and current of a one-port active element (such as a negative resistance diode), this is not the case when two-port active elements (such as transistors) are involved. Such variation of the system block scheme is the key point that will permit, in the end, to achieve the desired accuracy in the simulation of the oscillating amplifier response. In fact, the use of the TSIDF permits to account for the nonlinear dependence of the current $I_N$, not only on the input phasor $V_1$, but also on the output phasor $V_2$. In formulas, we have:

$$I_N(V_1, V_2) = Y_N(V_1, V_2, \theta_2 - \theta_1) \cdot V_1,$$  \hspace{1cm} (1)

which recalls that the dependence of $Y_N$ on node voltage phases is a differential and not an absolute one and, more important, that the TSIDF associated to a memoryless nonlinearity is a complex quantity.

Now, since the voltage phasor $V_2$ is related to the transistor output current via the input impedance $Z_i$ of the resonator:

$$V_2 = -Z_i(\omega) \cdot I_N(V_1, V_2),$$  \hspace{1cm} (2)

we can combine (1) and (2) into an implicit set of equations which defines an overall, equivalent, mutual admittance $Y_M(V_1, \omega)$ of the active block (the dashed box in Fig.2), implicitly defined through the relationship:

$$I_N(V_1, \omega) = Y_M(V_1, \omega) \cdot V_1.$$  \hspace{1cm} (3)

Such nonlinear and frequency-dependent mutual admittance $Y_M$ is capable of accounting, in an unabridged way, for all the nonlinear interaction phenomena occurring between the active device and its passive, resonant, load. In particular, it can model the practically observed dependence of the open-loop voltage gain (OLVG):
on the drive voltage amplitude $V_1$ not only in terms of its mid-band magnitude but also of its selectivity characteristics (see subsequent Section 4, for a numerical example of such effect). By combining (4) with the summing element constitutive equation:

$$V_1 = V_0 + V_3,$$  \hspace{1cm} (5)

after setting $\theta_0=0$ and rearranging, we get:

$$(1 - A(V_1, \omega)) \cdot V_1 = V_0.$$  \hspace{1cm} (6)

At this point, we can particularize the subsequent steps of our analysis to the specific case of low-level injection here considered. If the injection signal $V_0$ is “small”, we can take advantage of perturbation analysis methods and develop further calculations using incremental quantities (with respect to the free-running oscillating regime). In particular, for the drive voltage amplitude $V_1$ we set:

$$V_1(t) = V_{1, osc} + \Delta V_1(t).$$  \hspace{1cm} (7)

In (7), and elsewhere below, the subscript “osc” indicates evaluation in correspondence of the free-running oscillating condition, as calculable from (6) setting there $V_0=0$, and then solving the resulting nonlinear equation $A(V_1, \omega)=1$, either analytically or numerically. If we make the additional assumption that the transistor nonlinearity, while causing a marked dependence on $V_1$ of the OLVG’s mid-band magnitude and selectivity, does not appreciably changes its resonant frequency, we can adopt for it the following abridged relationship:

$$A(V_1, \omega)|_{V_1=V_{1, osc} + \Delta V_1} = \frac{\omega_{osc} \cdot (1 + \Delta V_1 \cdot A'_{osc})}{\omega_{osc} + 2 j(Q_{osc} + \Delta V_1 \cdot Q'_{osc}) (\omega - \omega_{osc})},$$  \hspace{1cm} (8)

where:

$$A'_{osc} = \frac{dA}{dV_1}|_{V_1=V_{1, osc}, \ \omega=\omega_{osc}}; \ \ \ Q'_{osc} = \frac{dQ}{dV_1}|_{V_1=V_{1, osc}}.$$  \hspace{1cm} (9)

As can be easily recognized, this approximation corresponds to a “single-tuned like”, variable Q-factor, reduced-order model of the open-loop transfer function, and has been found to be adequate in most practical cases. Replacing (8) into (6) provides the incremental algebraic model which describes, in the frequency-domain, the oscillating amplifier regime under continuous-wave (non-modulated) low-level injection operation.

On this basis, we can now derive the incremental differential model which describes, in the complex-envelope domain, the oscillating amplifier dynamics under general low-level injection operation. To this purpose, we make use here of a perturbation-refined approach based on the band-limited differential operator algebra introduced in [4].

As a first step, we quantitatively specify the smallness of the injection signal, setting for it the order defining condition:

$$V_0 = \mathcal{O} \left[ \frac{V_{1, osc} \cdot A'_{osc}}{Q_{osc}} \right].$$  \hspace{1cm} (10)

Under above assumptions, and taking for grant also that $A'_{osc}$ and $Q'_{osc}/Q_{osc}$ will be both $\mathcal{O}(1)$, as commonly occurs, the consequential order defining relationships can be shown to hold:

$$\frac{\Delta V_1}{V_{1, osc}} = \mathcal{O} \left[ \frac{1}{Q_{osc}} \right]; \ \ \ \frac{\omega - \omega_{osc}}{\omega_{osc}} = \mathcal{O} \left[ \frac{1}{Q_{osc}^2} \right].$$  \hspace{1cm} (11)

Making use of (10) and (11) to truncate, to the same order of magnitude, all terms appearing in the unabridged CW regime equation (6), provides its first-approximation-exact abridged counterpart:

$$2 j Q_{osc} \frac{\omega - \omega_{osc}}{\omega_{osc}} \cdot V_{1, osc} \cdot e^{j \theta_1} = V_0,$$  \hspace{1cm} (12)

which analytically defines steady-state values of $\Delta V_1$ and $\theta_1$ under CW injection ($\Delta V_{1, ss}$ and $\theta_{1, ss}$), as a function of the injection signal amplitude ($V_0$) and frequency ($\omega$), and of the abridged system parameter set $\{\omega_{osc}, Q_{osc}, A'_{osc}\}$.

To obtain the differential system model we can follow an analogous perturbation-refined procedure, starting from the dynamical analogue of (8) that is obtained, in view of the theory presented in [4], simply by replacing the angular frequency $\{\omega\}$ with its symbolic counterpart $\{\omega - j \frac{d}{dt}\}$ and then performing the necessary calculations and higher order terms truncations. This way, we firstly obtain:

$$\frac{1}{V_{1, osc}} \cdot \frac{d\Delta V_1(t)}{dt} + \left( \omega - \omega_{osc} + \left[ 1 + \frac{\Delta V_1(t)}{V_{1, osc}} \right] \cdot \frac{d\theta_1(t)}{dt} \right) +$$

$$- \frac{\omega_{osc} \cdot A'_{osc}}{2 Q_{osc}} \cdot \Delta V_1(t) = \frac{\omega_{osc}}{2 Q_{osc}} \cdot \frac{V_0}{V_{1, osc}} \cdot e^{-j \theta_1},$$  \hspace{1cm} (13)

and then, after manipulation and truncation, the normal form differential set of equations:
where:

\[ \frac{d\Delta V_1(t)}{dt} = \frac{\omega_{osc}}{2Q_{osc}} \cdot (V_0 \cos[\theta_1(t)] + A'_{osc} V_{1,osc} \Delta V_1(t)) \] (14a)

\[ \frac{d\theta_1(t)}{dt} = -(\omega - \omega_{osc}) - \frac{\omega_{osc}}{2Q_{osc}} \cdot \frac{V_0}{V_{1,osc}} \sin[\theta_1(t)] . \] (14b)

Equations (14) do provide solution to the stated analysis problem. In fact, they not only permit to simulate with great computational efficiency the dynamical response of the driven oscillator directly in terms of amplitude and phase transients (in a scaled time-domain), but also provide the mean to perform the phase-lock stability investigation, i.e., to evaluate the LBW, in a fully analytical manner.

3 Locking Stability Analysis

Steady-state equation (12) provides, in general, more than one solution, i.e., more than one possible regime. Whether a given equilibrium point is stable or not has to be ascertained via a dynamical stability analysis. Having at one’s disposal the differential system equations directly in terms of the complex-envelope components, as here provided by (14a,b), such analysis can straightforwardly carried out via a local linearization technique. More precisely, we firstly evaluate the 2x2 Jacobian matrix related to (14) in the equilibrium point considered:

\[
J = \begin{bmatrix}
\frac{\omega_{osc}}{2Q_{osc}} A'_{osc} V_{1,osc} & (\omega - \omega_{osc}) V_{1,osc} \\
0 & \frac{\omega_{osc}}{2Q_{osc}} A'_{osc} \Delta V_{1,ss}
\end{bmatrix}.
\] (15)

The associated characteristic polynomial is then constructed:

\[ \lambda^2 + \Delta_1 \lambda + \Delta_0 = 0 , \] (16)

where:

\[ \Delta_1 \equiv -\text{tr}(J) = -\frac{\omega_{osc}}{2Q_{osc}} A'_{osc} \cdot (V_{1,osc} + \Delta V_{1,ss}) ; \] (17a)

\[ \Delta_2 \equiv \text{det}(J) = \frac{\omega_{osc}}{2Q_{osc}} A'_{osc}^2 V_{1,osc} \cdot \Delta V_{1,ss} . \] (17b)

The locking stability criteria are eventually obtained by setting the condition that both the zero (\(\Delta_0\)) and first degree (\(\Delta_1\)) coefficients must be positive. Since the coefficient \(A'_{osc}\) is required to be negative (for the free-running oscillation stability), and the locked oscillation amplitude \((V_{1,osc} + \Delta V_{1,ss})\) positive, the unique condition remains: \(\Delta V_{1,ss} > 0\). The minimum/maximum (angular) frequency for which phase-lock can occur at the given value of injection signal is thus provided by the limit condition: \(\Delta V_{1,ss} = 0\). In view of equation (12), this is tantamount to saying that:

\[ 2jQ_{osc} \frac{\omega - \omega_{osc}}{\omega_{osc}} = \frac{V_0}{V_{1,osc}} , \] (18)

from which we get:

\[ \omega_{min} = \omega_{osc} - \frac{\omega_{osc}}{2Q_{osc}} \cdot \frac{V_0}{V_{1,osc}} , \] (19a)

\[ \omega_{max} = \omega_{osc} + \frac{\omega_{osc}}{2Q_{osc}} \cdot \frac{V_0}{V_{1,osc}} . \] (19b)

Notice that the band limits expressed by (19), and the corresponding value of LBW\(\equiv (\omega_{max} - \omega_{min})\), are similar to the well known expressions derived by Adler [6], and their most recent extension [7-8], with one significant difference: the fact the OLVG quality factor is evaluated at the oscillation amplitude \(V_{1,osc}\) rather than at \(V_1\) (i.e., coincident with the loaded quality-factor of the linearized transfer function). This fact explains the better numerical agreement achieved by this theory with respect to previous methods. It can also be remarked that our treatment has derived stability borders (19) without requiring the fictitious assumption of a hard-limiting of the oscillation amplitude in order to eliminate (14a) from calculations, as done in [6] and most of the other subsequent related works.

4 Example of Method Application

To better illustrate the features of the devised approach, a lumped-elements Meissner oscillating amplifier is analyzed here as an example of application. The circuit structure is illustrated in Fig.3, while the elements values are indicated in Tab.1. For the sake of simplicity a purely resistive S-H nonlinear model has been adopted for the JFET. The resonant frequency and the loaded Q of the tank circuit were set to 160MHz and to 100, respectively. The turn ratio of the coils was set to 10 and the OLVG margin for oscillation buildup set to +1.6dB.

To start with, the nonlinear transfer function \(A(V_1, \omega)\) has to be evaluated. Notwithstanding the simplicity of model at hand, its analytical derivation is not practicable. Therefore a numerical approach
has been adopted, resorting to a frequency-domain, Harmonic-Balance based, RF circuit simulator (the Advanced Design System by Agilent-EEsof [3]) to analyze the open-loop counterpart of the circuit of Fig.3, by sweeping both frequency and amplitude of the sinusoidal “input” drive signal $V_1 (= V_{gs})$, and recording the “output” voltage $V_3 (= V_{L1})$. The corresponding graphs of the mid-band magnitude and Q-factor of the OLVG $A(V_1, \omega)$ are reported in Fig.4a and Fig.4b, respectively. There are evidenced the free-running oscillation point (dot) and the associated derivatives (dashed line). The relevant numerical values turn out to be: $V_{1,osc} \approx 0.235\,V$ (which corresponds to an output oscillation amplitude $V_{4,osc} \approx 2.35\,V$); $Q_{osc} \approx 84.4$; $A'_{osc} \approx -2.77$; $Q'_{osc} / Q_{osc} \approx -2.50$.

With such numerical values, solution of (12) as function of injection signal amplitude $V_0$, provides the family of steady-state response curves illustrated in Fig.5, where the shaded region indicates the unstable locking region. The stable regime is thus unique for given pair $\{V_0, \omega\}$, and corresponds to the points on the top most solution branch.

In Fig.5 are also reported a number of dots which indicate the LBW limits, as calculated with a full numerical simulation of the circuit of Fig.3. Aside from observing that they are practically coincident with the analytical solution provided by (19), i.e., the vertical tangent points of elliptical curves defined by (12), it must be remarked that such evaluation has been extremely time-consuming. In fact, to numerically determine LBW borders, we had to adopt the “Circuit-Envelope” option of ADS (ADS/CE) [3] in a man-assisted iterative search procedure, based on bracketing stable and unstable operating conditions, discriminated via long-term run phase-locking transients. The simulation time spent, of course, was orders of magnitude greater than the one necessary to apply our formulas.

Before going on, it can be remarked (see Fig.4b) the non-negligible difference between the “linear” value of the Q-factor ($=100$) with respect to the “nonlinear” one ($Q_{osc} \approx 84$). Using the former instead of the second, would have caused an error in the evaluated locking bandwidth of more than 15%.

As example of use of the dynamical equations (14), the effect of a residual FM of 12kHz (peak) on the transient response of the example oscillating amplifier to an OQPSK input signal is illustrated. The evolution of the amplitude and phase error corresponding to a $\pm 90^\circ$ phase transition is shown in Fig.6. While analogous curves could have been obtained via ADS/CE the convenience and the insightfulness of a semi-analytical approach, as the one provided by (14), has to be anyhow remarked, especially in view of design optimization purposes.
5 Conclusion

In this paper, the problem of the accelerated analysis of oscillating amplifiers under low-level injection operating conditions was addressed. In order to achieve good computational efficiency and adequate accuracy also in the case of transmission-type, transistor-based circuit structures, a semi-analytical approach was proposed. The starting point was a general reduced-order model of the driven oscillator in the fundamental-frequency complex-envelope domain. A perturbation-refined method was then applied to derive a first-order exact set of equations which describe the system behavior under steady-state and transient operation, while fully accounting for the nonlinear loading effects associated to the presence of the two-port active device. A stability analysis was also developed, deriving generalized explicit formulas for the locking borders which extend previous theories on the subject.

As illustrated by the example presented, the devised approach provides results in excellent agreement with those obtained using numerical ECAD tools, but at a fraction of simulation time. Therefore, it lends itself as a convenient, design-oriented tool for the study of this stiff class of dynamical nonlinear systems.

References: