Self-tuning Control of Nonlinear Servomotor with Disturbance Rejection

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Abstract — The contribution is focused on a design of a control algorithm for a nonlinear servomotor with disturbance rejection using adaptive control strategy. It is obvious that for a rejection of the measurable disturbance is a suitable control strategy based on polynomial approach. The regression (output error) models are used in the identification part, the parameter estimates of the process and disturbance models were computed using the least squares method extended by a directional (adaptive) forgetting. The controller synthesis is based on polynomial theory – pole assignment method. The designed controller was applied to a laboratory servo system Amira DR300 in real-time conditions.

Keywords — Disturbance rejection, Nonlinear system, Pole assignment, Real-time control, Self-tuning control, Servomotor.

I. INTRODUCTION

SERVOMOTOR is a typical equipment which is characterized by nonlinear behaviour (varying gain with dead zone and hysteresis). Therefore classical control approaches (e.g. PID with fixed parameters) does not produce optimal control. Laboratory servo system DR300 can be successfully controlled by various adaptive control strategies including dual control [1]. Model Predictive Control (MPC) based on Generalized Predictive Control (GPC) method can also be used to control this laboratory equipment [2]. A comparison of the standard self-tuning LQ control and a predictive control was presented in [3]. Results of several identification methods and pole assignment non-adaptive control of DR300 laboratory model is designed in [4]. An explicit MPC for similar system using hybrid model was proposed by Herceg et al. [5]. A hybrid formulation and design of the MPC for similar servo system was used by Zabiri and Samyudia [6]. A measurable disturbance, which can influence output of the given laboratory equipment, was not considered in the above-mentioned contributions.

One approach to adaptive control is based on recursive estimations of unknown system characteristics and controller synthesis. This kind of adaptive controller (adaptive control with recursive identification), is referred to as a self-tuning controller (STC) in the literature [7], [8], [9] and [10]. The self-tuning controllers use the combination of the recursive process identification based on a selected model and the controller synthesis based on knowledge of parameter estimates of the controlled process.

This approach was applied to control of the DR300 servo system in this paper. The controller includes disturbance rejection of a sinusoidal disturbance signal. This type of disturbance can occur for example in electrical system where electromagnetic field of AC power lines is superimposed on electromagnetic field of control lines. The proposed algorithm is designed using polynomial theory developed for linear controlled systems. The adaptive algorithm respects nonlinear characteristics of controlled system.

This paper is organized in the following way. Theoretical background is described in Section 2. The description of DR300 laboratory servo model and analysis of its steady state properties are introduced in Section 3. Section 4 contains process identification of the DR300 laboratory model. The control algorithm is derived in Section 5. Real-time experimental results are presented in Section 6 and results are summed up in Section 7.

II. THEORETICAL BACKROUND

Controllers applied further in this paper were designed using a polynomial approach. Polynomial control theory is based on the apparatus and methods of a linear algebra (see e.g. [11], [12], [13]). The polynomials are the basic tool for a description of the transfer functions. They are expressed as the finite sequence of figures – the coefficients of a polynomial. Thus, the signals are expressed as infinite sequences of figures. The controller synthesis consists in solving of linear polynomial (Diophantine) equations in a general form [14].

![Fig. 1 Block diagram of a closed loop 2DOF control system](image)

The design of the controller algorithm is based on the general block scheme of a closed-loop with two degrees of freedom (2DOF) according to Fig. 1.

The controlled process is given by the transfer function in the form of proper polynomial fractions

\[
G_r(z) = \frac{Y_r(z)}{U(z)} = \frac{B(z^{-1})}{A(z^{-1})}
\]  

(1)
where \( A \) and \( B \) are coprime polynomials that fulfill the inequality \( \deg B \leq \deg A \). The controller contains the feedback part \( G_f \) and the feedforward part \( G_n \). \( y \) is the controlled output, \( u \) is the control input, \( w \) is the reference signal and \( v \) is the load disturbance with transfer function

\[
G_r(z) = \frac{Y(z)}{V(z)} = \frac{C(z^{-1})}{A(z^{-1})}
\]  

(2)

Then the digital controllers can be expressed in the form of a discrete transfer functions:

\[
G_r(z) = \frac{R(z^{-1})}{P(z^{-1})}; \quad G_n(z) = \frac{Q(z^{-1})}{P(z^{-1})}
\]  

(3)

A polynomial approach to the design of a control system with the disturbance rejection is used in [15], [16], [17], [18].

The control algorithm is used for reference signal tracking and rejection of sinusoidal disturbance whose frequency must be known. Step changes of the reference signal are usually used in practice and the sinusoidal disturbance is supposed in this case. Then the step of height \( w \) can be expressed as

\[
W(z^{-1}) = \frac{N_r(z^{-1})}{D_r(z^{-1})} = \frac{w_1}{1-z^{-1}}
\]  

(4)

and harmonic disturbance signal can be expressed as

\[
V(z^{-1}) = \frac{N_i(z^{-1})}{D_i(z^{-1})} = \frac{A_i z^{-1} + 1}{1-\alpha z^{-1} + z^{-2}}
\]  

(5)

where \( A \) is amplitude of sinusoidal signal, \( \beta = \sin \omega T_o \) and \( \alpha = 2 \cos \omega T_o \); \( \omega \) and \( T_o \) are angular frequency and sampling period respectively.

According to the scheme presented in Fig. 1, the output \( y \) can be expressed as:

\[
Y(z^{-1}) = \frac{G_r(z)G_n(z)}{1+G_r(z)G_n(z)} W(z^{-1}) + \frac{G_n(z)}{1+G_r(z)G_n(z)} V(z^{-1})
\]  

(6)

By combining (1), (2), (3) and (6), expression for the control error can be derived

\[
E(z^{-1}) = W(z^{-1}) - Y(z^{-1}) = \frac{A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1})}{A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1})} W(z^{-1}) - \frac{A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1})}{A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1})} V(z^{-1})
\]  

(7)

To ensure disturbance rejection, all poles of \( V(z^{-1}) \) which are not stable must be included in polynomial \( P(z) \). Thus \( P \) contains second order polynomial

\[
P(z^{-1}) = \tilde{P}(z^{-1}) = 1 - \alpha z^{-1} + z^{-2}
\]  

(8)

The procedure leading to determination of polynomials \( Q, R \) and \( \tilde{P} \) in (7) and (8) can be briefly described as follows. The feedback part of the controller is given by a solution of the polynomial Diophantine equation

\[
A(z^{-1})D_r(z^{-1}) = \tilde{P}(z^{-1}) + B(z^{-1})Q(z^{-1}) = D(z^{-1})
\]  

(9)

A stable polynomial on the right side along with stable polynomial \( \tilde{P} \) ensures the control stability and load disturbance attenuation. An asymptotic tracking is provided by a feedforward part of the controller given by a solution of the polynomial Diophantine equation

\[
S(z^{-1})D_n(z^{-1}) + B(z^{-1})R(z^{-1}) = D(z^{-1})
\]  

(10)

where \( S \) is an auxiliary polynomial which does not affect controller design but which is necessary for calculation of (10). The degrees of individual controller polynomials must fulfill following equalities:

\[
\deg Q = \deg A + \deg D_r - 1 \quad \deg \tilde{P} = \deg A - 1 \quad \deg R = \deg D_n - 1 \
\deg S = 2 \deg A + \deg D_r - \deg D_n - 1 \quad \deg D = 2 \deg A + \deg D_r - 1
\]  

(11)

The controller parameters then result from solutions of the polynomial equations (9) and (10) and depend upon coefficients of polynomial \( D \) that enables to obtain the acceptable stabilizing and stable controllers.

III. DESCRIPTION OF LABORATORY MODEL DR300

The pole assignment algorithms were designed for a real-time control of the laboratory model DR300 (see Fig. 2). A block scheme of the DR300 system is presented in Fig. 3. The plant is represented by a permanently exited DC motor (M1) whose input signal (armature current) is provided by a current control loop. Its servo amplifier operates in 4 quadrant mode, so that the orientation of the current and correspondingly the orientation of the rotation of the motor is arbitrarily adjustable.
The sensors for the output signal (speed) are a tachogenerator (T) and an incremental encoder (I). The free end of the motor shaft is fixedly coupled (K) to the shaft of a second, identical motor (M2). This motor is used as a generator. Its output current is freely adjustable. The rotation speed of the motor M1 is driven by voltage $u$. The motor shaft rotations per minute (rpm) are measured by tachogenerator T. The aim of the control process is to control the rotation speed of the shaft $\omega$ by the control voltage $u$.

![Fig. 3 Block scheme of Amira DR300 servomotor](image)

From the control point of view, the Amira DR300 is a non-linear system. Its nonlinear steady state characteristics (varying gain, dead zone, and hysteresis) are shown in Fig. 4. The figure presents dependence of shaft rotations on control voltage of motor M1 while second motor is not controlled – its control voltage was zero. The steady state characteristics were measured by applying a sequence of increasing and decreasing steps to control signal.

Even in the parts of steady state characteristics where the plant output changes (approximately -2V to -1V and 1V to 2V) the gain of the system is not constant. The gain of the plant varies approximately from 3600 rpm/V to 6900 rpm/V.

![Fig. 4 Steady state characteristics of Amira DR300 servomotor](image)

### IV. PROCESS IDENTIFICATION

The block schema of an adaptive controller with disturbance rejection is shown in Fig. 5, where PE is the process parameters estimator, CD is block for the controller design, $n$ is the term describing stochastic influences.

ARX model in the following form was used for identification of the laboratory equipment DR300

$$y(k) = \Theta^T(k) \Phi(k) + n(k)$$  \hspace{1cm} (12)

where

$$\Theta^T(k) = \begin{bmatrix} a_1 & a_2 & b_1 & b_2 & c_1 & c_2 \end{bmatrix}$$  \hspace{1cm} (13)

is the vector of the parameter and

$$\Phi^T(k) = \begin{bmatrix} -y(k-1) - y(k-2) & u(k-1) & u(k-2) \\ v(k-1) & v(k-2) \end{bmatrix}$$ \hspace{1cm} (14)

is the regression vector, $n(k)$ is the white noise and $y_s$ is absolute term, which corresponds to a dead zone of the system. The dead zone of the DR300 plant is caused by friction which can be observed from Fig. 4.

![Fig. 5 Block diagram of an adaptive control-loop](image)

The DR300 plant was modelled by a linear second order system with disturbance rejection. Recursive identification was performed and the parameter estimates (13) were computed using the least squares method extended by exponential forgetting or directional (adaptive) forgetting techniques [19].

### V. DESIGN OF CONTROLLER ALGORITHM

Design of the controller consists of two parts. First, linear controller for the second order discrete system is derived. The considered system does not include compensation of friction. Therefore absolute term ($y_s$) of ARX models (15) is not considered in this part. Derivation of the linear controller is described in subsection 5.1.

The second part of controller design introduces friction compensation. The compensation is carried out by additive constant which is added to the output of linear controller derived in the first part. This compensation is introduced in subsection 5.2.

#### A. Design of the Linear Controller

It results from identification experiments that both motor and generator can be modelled by second order systems:
The control law, which ensues from Fig. 1 and transfer functions (3), is then given as

\[ u(k) = y_a(k) - y(k) - \alpha y(k) - q_1 y(k-1) - q_2 y(k-2) - q_3 y(k-3) - (\alpha + p_1) u(k-1) - (1 + \alpha p_1) u(k-2) - p_1 u(k-3) \]

\[ \text{V. EXPERIMENTAL RESULTS} \]

Even though presented control design is based on relatively complex methods, resulting control algorithm is relatively simple. Whole control algorithm is represented by equations (22) and (23). Computation of control output consists of just a small number of multiplication and additions. If on-line parameters estimation is used then the demands to computing power are higher. A personal computer equipped with the MATLAB/Simulink system was used for laboratory testing of proposed control algorithm.

Sinusoidal disturbance was used in real-time experiments described in this section. Sinusoid of angular frequency \( \omega = \pi \) (frequency 0.5 Hz) and amplitude \( A_s = 2 \) V was applied to the motor M2 of the DR300 plant (Fig. 3) in time range \( \{45 \text{ s}; 85 \text{ s}\} \).

Poles of the characteristics polynomial are defined by \( D(z^{-1}) \) in equation (9). A sole multiple pole of \( z_p = 0.65 \) was used in all subsequent experiments. Control signal in range \( u \in (-10 \text{ V}; 10 \text{ V}) \) is admissible for the DR300 plant. Sampling period of \( T_s = 0.05 \text{ s} \) was used in the identification part and all subsequent control experiments.

Initial estimates of parameters of the controlled system were computed off-line using data obtained by exciting the system by pseudorandom signal.

A. Control without Adaptation

Initial experiments were performed without incorporating on-line identification of the controlled system to obtain nominal courses for comparison with adaptive controllers.

The first controller referenced as no_adapt was designed according to the theory presented in the previous chapters. The absolute term \( y_a \) has approximately opposite values for positive and negative outputs, the value of \( u_0 = 0 \) was used. Resulting courses are presented in Fig. 7.

It can be seen that the controller copes very well with the sinusoidal disturbance. However, a steady state error is
This behaviour is caused by a difference between the controlled nonlinear DR300 plant and the linear ARX model used to derive the controller. The main difference consists in presence of friction. The effect of friction is the same as presence of step disturbance.

Resulting equation for computation of controller output has the following form

\[
\begin{align*}
\dot{u}(k) &= e(k) - g_3 y(k-1) - g_2 y(k-2) - g_1 y(k-3) - g_0 y(k-4) - \\
& \quad (\alpha + \beta) u(k-1) - (1 + \alpha \beta - \alpha) u(k-2) - \\
& \quad (\beta - 1 - \alpha \beta) u(k-3) + \mu u(k-4)
\end{align*}
\]

(26)

As can be seen from Fig. 8, steady state control error was suppressed. However oscillations occur when changing the sign of reference signal (i.e. changing the direction of shaft rotation).

To reach zero steady state error, the controller algorithm was revised to have integral behaviour – i.e. to be able to suppress steady state control error. Then (8) is superseded by

\[
P(z^{-1}) = \hat{P}(z^{-1})(1 - \alpha z^{-1} + z^{-2})(1 - z^{-1})
\]

(25)

and degree of polynomials \(Q, S\) and \(D\) in (11) increases by one.

Further derivation of the control law is similar to the one presented in section V. A. This controller is referenced as \(\text{no\_adapt\_step}\) and resulting control courses are presented in Fig. 8.

A good reference tracking can be observed, but still some oscillations occur when reference signal is crossing zero. As can be seen in Fig. 10, changes of parameter estimates correspond to changes of reference signal. Compensation of friction has similar effect to all estimates while the value of \(y_a\) should be affected more than other parameters.

The control performance can be further improved by using a posteriori information from measurement of the steady state characteristics (Fig. 4). If the direction of the rotation changes the value of absolute term \(y_a\) changes more dramatically than the values of the other terms (\(a_1, a_2, b_1, b_2, c_1, c_2\)). The identification algorithm was enhanced by increasing the values in its covariance matrix when reference signal changes its sign. This results in greater change of \(y_a\) comparing to other parameter estimates. Controller using this modification is referenced as \(\text{adapt\_cov}\) and the control and parameter courses are presented in Fig. 11 and Fig. 12 respectively.

Changes of parameters are smoother when comparing Fig. 12 and Fig. 10 because friction is compensated by changes of \(y_a\). This also leads to smoother courses of control and controlled signals.
I can be also observed from Fig. 12 that behaviour of motors M1 and M2 is not the same. Although the initial estimations of their transfer functions were the same (i.e. $\hat{h}_1(0) = \hat{c}_1(0)$ and $\hat{h}_2(0) = \hat{c}_2(0)$) the final estimations are quite distant.

In spite of statement of DR300 manufacturer that motors M1 and M2 are the same, differences in their transfer functions were discovered by adaptive controllers. This led to smaller control error as can be seen in Table I. Designed controllers were successfully tested in real-time laboratory conditions.

**REFERENCES**


